Channel Estimation for FSO Channels Subject to Gamma-Gamma Turbulence

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***Abstract*—The problem of estimating the signal intensity level in a turbulent optical channel when the channel is subject to Gamma-Gamma fading (GGF) is investigated. Such an estimation process is of paramount importance to a direct-detection, on-off-keying (OOK), free-space optical (FSO) communication receiver where a knowledge of the received signal intensity level is required to set the optimum receiver threshold. An estimator, motivated by the maximum a posterior (MAP) rule, is proposed. The proposed estimator is a computationally-efficient linear estimator. The performance of the proposed estimator is investigated using analytical tools. Furthermore, Cramer-Rao bound (CRB) for the above scenario is computed and is compared to the performance of the proposed estimator.**

***Index Terms*—FSO,estimation,Gamma-Gamma,turbulence**

I. Introduction

Optical communications through atmosphere has been the subject of numerous studies in the past few decades, for example see [1]. A major stumbling block for a successful communication via free-space optical channels is the harsh channel condition, caused by *optical* turbulence. In general, optical turbulence results in frequent and significant signal fades of more than 20 dB. It is important to note that this impairment is responsible for a non-negligible signal loss in a perfect weather condition where a line-of-sight (LOS) communication with a high level of visibility is available. This impairment, which is known as 'clear air' turbulence, cannot be overcome through conventional means, such as interleaving, as the duration of fades range from a few to several hundreds of milliseconds (msec), rendering any interleaving operation ineffective for multi-gigabit per second (Gbps) communications. Even in the absence of fading, for on-off-keying (OOK) systems, one requires a knowledge of the signal level to render a decision or to set the optimum receiver threshold. This requirement necessitates the use of channel estimation techniques. To that end, a number of recent studies have focused on channel estimation for FSO links [2]-[3]. It is important to note that the problem of maximum-likelihood sequence estimation (MLSE) has been extensively studied in the literature for FSO channels in order to yield receiver architectures which perform near the optimum region in the presence of channel effects, for example see [4]–[7]. However, such techniques are indeed complicated, posing a serious hardware and data rate restrictions on multi-Giga bits per second (Gbps) communication transceivers. Indeed, such complication can be avoided with the aid of channel estimation. To elaborate, one can readily perform channel estimation using a preamble of tens of bits (as is commonly done in most wireless communication systems), and use the estimated values for hundreds, if not thousands, of ensuing data packets as channel remains stationary over sever millions of bits at Gbps data rates in the FSO links. This implies that the complication can be restricted to the preamble and not to the entire receiver architecture, which for OOK modulation amounts to a simple threshold test once the channel state is known. It is noteworthy that one means of ameliorating the performance of such links is to use re-transmission techniques [8]. In that scenario, however, one also requires a knowledge of the channel state to set the receiver threshold.

FSO channels can assume several turbulent states; weak turbulence, modeled accurately using log-normal probability density function (pdf), long integrated path scenario (or deep saturation region), which is observed when the propagation length is large (hence, resulting in an exponentially-distributed signal intensity), and moderate to strong turbulence, which has been modeled using the K-distribution [9], [10]. In recent years, a new model

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has been proposed for modeling the FSO channels for moderate to strong turbulence, known as Gamma-Gamma fading (GGF) [11]. This model combines the small and large scale scattering elements as multiplicative components of the fading phenomenon, resulting in a fairly accurate modeling of the turbulent FSO channels in the moderate to strong regime [11]. In this paper, we present channel estimator motivated by the MAP rule for GGF channels and demonstrate that such an estimator achieves nearly optimum performance for an observation interval not exceeding 50 bits. It is important to note that, in a recent investigation, the problem of MLE for GGF channel was studied without arriving at a close form solution [12]. In that study, the focus remained on parameter estimation and not on channel state estimation, which is the subject of the present investigation.

II. System Model

In this study, we are concerned with an FSO channel that is operating in a Gaussian regime where the receiver thermal noise levels and/or background radiation levels are large enough to insure a Gaussian received signal. Without loss of generality, we assume that the current at the photo-detector resistor for the above scenario can be modeled as

$$i(t) = \zeta(t)\, x(t) + n(t) \tag{1}$$

where $\zeta(t)$ denotes channel impact, $x(t)$ is the time-domain representation of the digital modulation (in this case OOK), and $n(t)$ denotes the receiver thermal and/or back ground noise, modeled as an additive white Gaussian noise (AWGN) with the two-sided power spectrum density of $\frac{N_0}{2}$ watts/Hz. For the problem at hand, it is assumed that the channel correlation time is much larger than the data rate, and hence we assume that $\zeta(t) \approx \zeta$ for the observation interval of interest. Furthermore,

$$x(t) = \sqrt{\frac{E_b}{T_b}} \sum_{n=-\infty}^{\infty} d_k p(t - nT_b) \tag{2}$$

with $d_k \in [0,1]$ (OOK modulation), $p(t)$ denoting a unit amplitude, non-return-to-zero (NRZ) pulse of duration T_b, and T_b signifying the bit duration in sec. Furthermore, E_b denotes the received energy per bit in the absence of channel effects. E_b can be estimated using the transmitter power, optical characteristics of the transmitter and receiver optics, and geometric losses. At the receiver, an integration and dump operation is performed, where it is assumed that the symbol timing has been acquired. In that event,

$$i_k = \frac{1}{\sqrt{T_b}} \int_{(k-1)T_b}^{kT_b} i(t)\, dt = \sqrt{E_b}\zeta d_k + n_k \tag{3}$$

where $n_k = \frac{1}{\sqrt{T_b}} \int_{(k-1)T_b}^{kT_b} n(t)\, dt$ is a zero-mean Gaussian random variable with variance $\frac{N_0}{2}$. Normalizing i_k by $\sqrt{E_b}$, we can use the following observable for performing estimation:

$$y_k = \frac{1}{\sqrt{E_b}} i_k = \zeta d_k + w_k \tag{4}$$

where now w_k denotes a zero-mean Gaussian random variable with variance

$$\sigma^2 = \frac{1}{\text{SNR}} = \left(\frac{2E_b}{N_0}\right)^{-1} \tag{5}$$

with SNR denoting the signal-to-noise ratio. Furthermore, let $Y = [y_1, y_2, ..., y_N]'$ denote the observed vector for an $N-$sample observation of the channel. Without the loss of generality, we assume that the header used to provide the channel estimation consist of all 1's. Hence,

$$Y = \zeta\,[1, 1, ..., 1]' + [w_1, w_2, ..., w_N]'. \tag{6}$$

III. Channel Model

For a GGF channel model, the pdf of ζ is given by

$$f(\zeta) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \zeta^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\zeta}\right) \tag{7}$$

where it is assumed that the mean of ζ is 1 (normalized channel effect). In this equation, $\Gamma(x)$ is the Gamma function and $K_n(x)$ is the modified Bessel function of the second kind of order n. Furthermore, $\alpha = \frac{1}{\sigma_l^2}$ and $\beta = \frac{1}{\sigma_s^2}$ where σ_s^2 and σ_l^2 are the normalized variances of ζ_s and ζ_l (the impacts of turbulence caused by large and small-scale eddies) with $\zeta = \zeta_s \zeta_l$. As can be seen, the key parameters that define this pdf are α and β. For plane wave scenario and when inner scale of atmosphere can be neglected,

$$\sigma_l^2 \approx \exp\left[\frac{0.54\sigma_1^2}{\left(1 + 1.22\sigma_1^{12/5}\right)^{7/6}}\right] - 1 \tag{8}$$

$$\sigma_s^2 \approx \exp\left[\frac{0.51\sigma_1^2}{\left(1+0.69\sigma_1^{12/5}\right)^{5/6}}\right]-1 \tag{9}$$

with $\sigma_1^2 = 1.23C_n^2 k^{7/6} L^{11/6}$ denoting the Rytov variance [13]. In this equation, C_n^2, k, and L denote the index of refraction structure constant, wave number, and the propagation distance. In general, it is customary to use Rytov variance as a measure of turbulence level. In particular, $\sigma_1^2 \le 0.3$ denotes weak turbulence, whereas $0.3 < \sigma_1^2 < 5$ denotes moderate to strong turbulence region. For $\sigma_1^2 \to \infty$ (deep saturation), $\sigma_s^2 \to 1$ ($\beta \to 1$), resulting in an exponentially distributed ζ (as expected).

Since $E\{\zeta\} = 1$, indeed $\frac{2E_b}{N_0}$ may be viewed as the average SNR. Furthermore, since the scintillation index is given by [13]

$$\sigma_I^2 = \frac{E\left(\zeta^2\right)}{E^2\left(\zeta\right)} - 1, \tag{10}$$

the variance of ζ ($E\{\zeta\} = 1$) may be viewed as the scintillation index. It can readily be shown that the variance of ζ (or scintillation index) is given by

$$\sigma_I^2 = \sigma_\zeta^2 = \frac{(\alpha+1)(\beta+1)}{\alpha\beta} - 1 \tag{11}$$

Fig. 1 depicts the relationship between the Rytov variance, α, β, and σ_ζ^2. Note that for the Rytov variance of 5, $\beta \to 1$ and $\sigma_\zeta^2 \to 1$ (scintillation index of an exponentially distributed random variable).

IV. MAP ESTIMATOR

The above formulation suggest that one requires the knowledge of ζ to set the receiver threshold (in the case of OOK, that threshold will be $\frac{\zeta}{2}$). Hence, for the purpose of realizing an optimum receiver, one requires to estimate the random parameter ζ using the observed vector Y. For the channel model considered here, we have the following likelihood function for estimating the random parameter ζ:

$$\begin{aligned}\ln f(Y,\zeta) &= \ln f(Y|\zeta) + \ln f(\zeta)\\ &= -\sum_{k=1}^{N}\frac{(y_k-\zeta)^2}{2\sigma^2} + \ln f(\zeta)\\ &\quad +C\end{aligned} \tag{12}$$

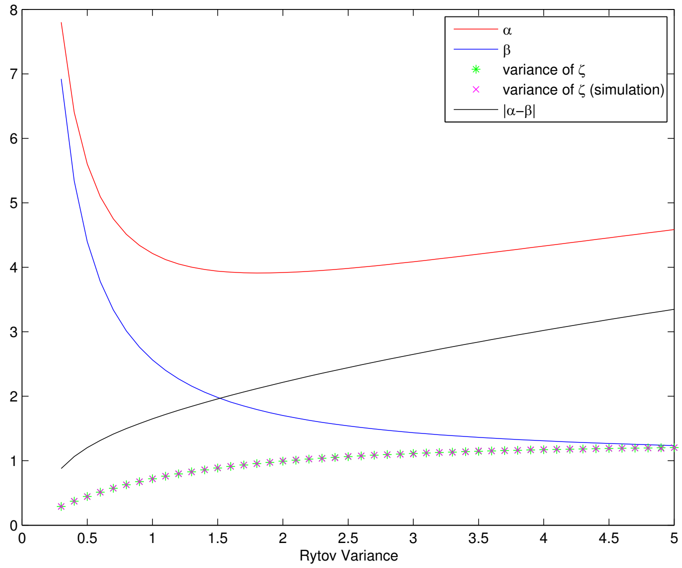

Fig. 1. The impact of Rytov variance on α, β, and the scintillation index (or variance of ζ).

where C is a constant and

$$f(Y|\zeta) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}}\exp\left\{-\sum_{k=1}^{N}\frac{(y_k-\zeta)^2}{2\sigma^2}\right\}.$$

The MAP estimator may be obtained as follows:

$$\hat{\zeta}_{MAP} = \max\arg\{\ln[f(Y,\zeta)]\} \tag{13}$$

where the maximization is obtained with respect to ζ. In other words, the likelihood function $\Lambda(\zeta.Y)$, given by

$$\Lambda(\zeta,Y) = \frac{\partial \ln f(Y,\zeta)}{\zeta}, \tag{14}$$

must be set to zero in order to obtain $\hat{\zeta}_{MAP}$. This implies solving

$$\Lambda(\zeta,Y) = \sum_{k=1}^{N}\frac{(y_k-\zeta)}{\sigma^2} + \frac{\partial \ln f(\zeta)}{\partial\zeta} = 0 \tag{15}$$

The difficulty in obtaining $\hat{\zeta}_{MAP}$ is in simplifying the second term $\frac{\partial \ln f(\zeta)}{\partial\zeta}$. Note that

$$\Lambda(\zeta,Y) = \sum_{k=1}^{N}\frac{(y_k-\zeta)}{\sigma^2} + \eta(\zeta) \tag{16}$$

where

$$\eta(\zeta) = \left(\frac{\alpha+\beta}{2}-1\right)\frac{1}{\zeta} + \frac{\partial \ln K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\zeta}\right)}{\partial\zeta}. \tag{17}$$

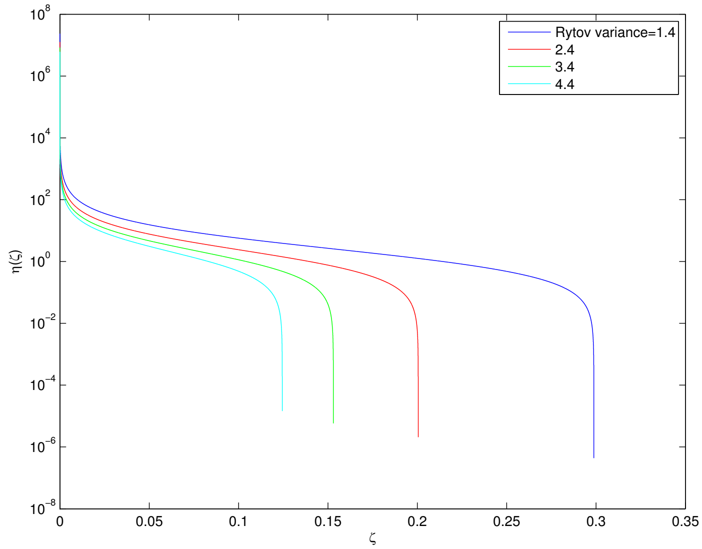

Fig. 2. $\eta(\zeta)$ for various levels of Rytov variance.

In order to arrive at the MAP estimator, then one is required to solve $\Lambda(\zeta, Y) = 0$. Due to the complexity of $\eta(\zeta)$, the task of acquiring the MAP estimate is a daunting one. This is a particularly difficult task at multi-Gbps data rates. However, an approximation to this estimate can be found. Furthermore, one can space the pilot symbols by a substantial number of symbols (as channel is 'frozen'), resulting in a sub-GHz processing requirement.

V. Linear Estimator

The core of the complexity of the estimator resides in computing $\eta(\zeta)$. To gain an insight into the behavior of $\eta(\zeta)$, this function is depicted in Fig. 2 for various levels of the Rytov variance. Rather surprisingly, this function is only significant for fairly small values of ζ, essentially dropping to zero for values of $\zeta > 0.35$. It is also critical to rearrange the likelihood function as follows:

$$\Lambda(\zeta, Y) = -N\left\{\zeta - \frac{1}{N}\sum_{k=1}^{N} y_k - \frac{\sigma^2}{N}\eta(\zeta)\right\} \quad (18)$$

Note that the last term involving $\eta(\zeta)$ is now scaled by $\frac{1}{N\text{SNR}}$. This is a significant development, stating that for a typical SNR and a value of $N >> 1$, the last term in the likelihood function will have very little impact on the estimation process. Hence, a linear estimator, given by

$$\hat{\zeta} = \frac{1}{N}\sum_{k=1}^{N} y_k, \quad (19)$$

is proposed. This estimator is shown to yield a performance that is almost identical to the optimum estimator for a reasonable set of parameters. To that end, we use CRB as a means of assessing the performance of the optimum estimator.

VI. Cramer-Rao Bound

To assess the performance of the optimum receiver, we resort to the CR bound, which is given by

$$E\left\{\left(\hat{\zeta} - \zeta\right)^2\right\} \geq \text{CRB} = \frac{-1}{E_{Y,\zeta}\left\{\frac{\partial^2 \ln f(Y,\zeta)}{\partial \zeta^2}\right\}} \quad (20)$$

where the expectation $E_{Y,\zeta}\{\}$ is performed with respect to Y and ζ. For the problem at hand,

$$\begin{aligned} \text{CRB}^{-1} &= -E_{Y,\zeta}\left\{\frac{\partial^2 \ln f(Y/\zeta)}{\partial \zeta^2}\right\} \\ &\quad -E_{\zeta}\left\{\frac{\partial^2 \ln f(\zeta)}{\partial \zeta^2}\right\} \end{aligned} \quad (21)$$

Note that

$$\begin{aligned} & -E_{Y,\zeta}\left\{\frac{\partial^2 \ln f(Y/\zeta)}{\partial \zeta^2}\right\} \\ = & -E_{Y,\zeta}\left\{\frac{\partial}{\partial \zeta}\sum_{k=1}^{N}\frac{(y_k - \zeta)}{\sigma^2}\right\} \\ = & \; E\left\{\frac{N}{\sigma^2}\right\} = \frac{N}{\sigma^2}, \end{aligned} \quad (22)$$

whereas the second term is an expectation with respect of ζ only. Let

$$J = -E_{\zeta}\left\{\frac{\partial^2 \ln f(\zeta)}{\partial \zeta^2}\right\} \quad (23)$$

denote the portion of the CRB related to the random parameter (this is similar to the *data* portion of the *Fisher's information matrix*). Hence,

$$\text{CRB} = \frac{1}{\frac{N}{\sigma^2} + J} = \frac{1}{N(\text{SNR}) + J} \quad (24)$$

Unfortunately, a closed-form solution for J does not exist, and hence, one has to resort to numerical integration to evaluate CRB. The CRB for the problem at hand is plotted in Figs. 3 and 4. A

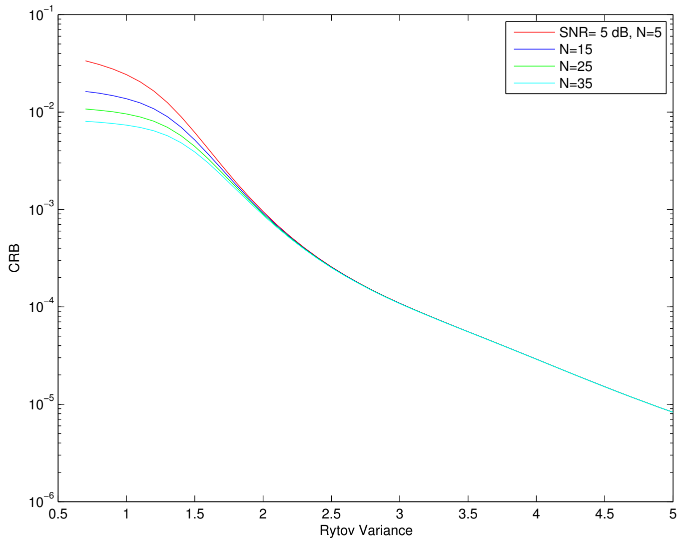

Fig. 3. Cramer-Rao Bound for MAP estimation of ζ as a function of Rytov variance for GGF.

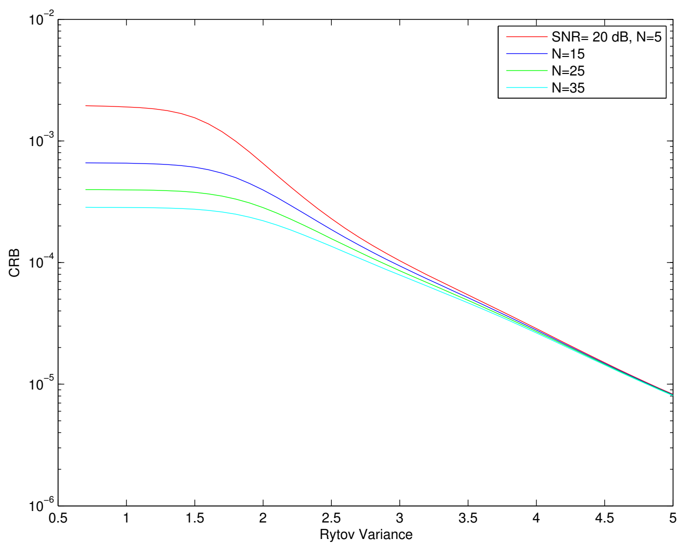

Fig. 4. Cramer-Rao Bound for MAP estimation of ζ as a function of Rytov variance for GGF.

somewhat curious observation can be made from this figure, which points to a decreasing CRB when the Rytov variance is increased. This observation, although counterintuitive, is a direct consequence of the shape of the GGF pdf. To elaborate, let us examine $f(\zeta)$ for the Rytov variances of 1, 3, and 5.

Note that $f(\zeta)$ assumes a value of zero for $\zeta = 0$ (see Fig. 5) for large values of the Rytov variance. In fact, for a large Rytov variance, $\alpha - \beta > 1$ with β approaching 1. These conditions justify the

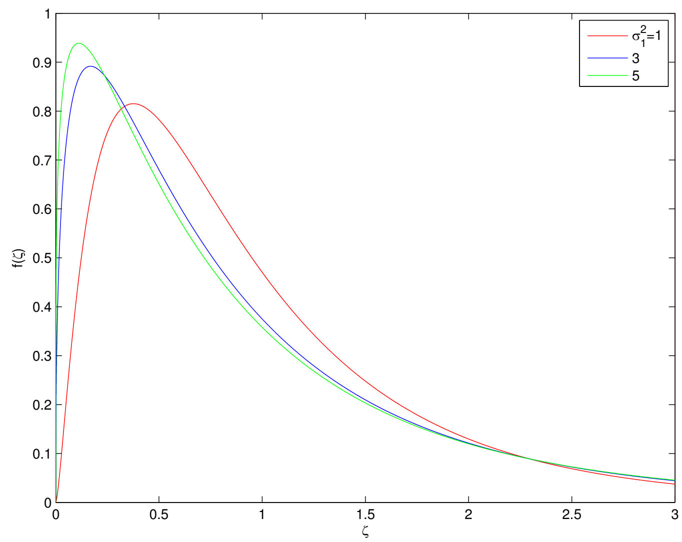

Fig. 5. The probability density function of ζ as a function of σ_1^2.

following approximation:

$$K_{\alpha-\beta}\left(2\sqrt{\alpha\beta\zeta}\right) \approx \frac{\Gamma(\alpha-\beta)}{2}\left(\frac{1}{\sqrt{\alpha\beta\zeta}}\right)^{\alpha-\beta}; \zeta \to 0. \tag{25}$$

This leads to the following approximation for $f(\zeta)$ in the vicinity of $\zeta = 0$:

$$f(\zeta) \approx \frac{(\alpha\beta)^{\beta}}{\Gamma(\alpha)\Gamma(\beta)}\zeta^{\beta-1}; \zeta \to 0. \tag{26}$$

Although the pdf approaches an exponential shape, indeed it is not exponential. In particular, at $\zeta = 0$ an exponential pdf with a mean of 1 assumes a value of 1, whereas the GGF pdf approaches 0. Due to the fact that the slope of the pdf around $\zeta = 0$ becomes increasingly large with an increase in the Rytov variance (note that the location of the peak shifts toward the y axis), $E_{\zeta}\left\{\frac{\partial^2 \ln f(\zeta)}{\partial\zeta^2}\right\}$ is an increasing function of the Rytov variance for the cases considered here. This leads to a decreasing CRB as a function of σ_1^2 when the estimation involves estimating the signal intensity (and not the parameters of the GGF pdf). A similar observation can be made when ζ is Gaussian distributed. That is, if

$$f(\zeta) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\zeta^2}{2\sigma^2}\right), \tag{27}$$

then

$$-E_{\zeta}\left\{\frac{\partial^2 \ln f(\zeta)}{\partial\zeta^2}\right\} = \frac{1}{\sigma^2}. \tag{28}$$

Now, if $\sigma^2 \rightarrow 0$, $-E_\zeta\left\{\frac{\partial^2 \ln f(\zeta)}{\partial\zeta^2}\right\} \rightarrow \infty$. In this event, the pdf assumes a slope that is increasing without bound at $\zeta = 0$ as $\sigma^2 \rightarrow 0$. In fact, the exponential pdf $\exp(\zeta)\,U(\zeta)$, due to its infinite slope at $\zeta = 0$, produces $-E_\zeta\left\{\frac{\partial^2 \ln f(\zeta)}{\partial\zeta^2}\right\} = \infty$. Hence, a decreasing CRB with σ_1^2 is not a surprising result as the limiting pdf for the GGF case as $\sigma_1^2 \rightarrow \infty$ is exponential.

VII. Performance Analysis

For the linear estimator, it is rather easy to establish performance. That is, one can define the estimate error as $\zeta_e = \hat{\zeta} - \zeta$. In that event, the variance of the error is given by

$$\begin{aligned}\sigma_{\zeta_e}^2 &= E\left\{\zeta_e^2\right\} - E^2\left\{\zeta_e\right\} \\ &= E_\zeta\left\{E\left\{\zeta_e^2|\zeta\right\}\right\}\end{aligned} \tag{29}$$

as this estimator is unbiased.

$$\begin{aligned}\sigma_{\zeta_e}^2 &= E_\zeta\left\{\frac{\left(N^2-N\right)\zeta^2 + N\left(\sigma^2+\zeta^2\right)}{N^2} - \zeta^2\right\} \\ &= \frac{\sigma^2}{N} \\ &= \frac{1}{N\,(\mathrm{SNR})}\end{aligned} \tag{30}$$

Note that this estimate is larger than the CRB by

$$\begin{aligned}\Delta &= \sigma_{\zeta_e}^2 - \mathrm{CBR} = \frac{1}{N\,(\mathrm{SNR})} - \frac{1}{N\,(\mathrm{SNR}) + J} \\ &= \frac{J}{N\,(\mathrm{SNR})\left(J + N\,(\mathrm{SNR})\right)}.\end{aligned} \tag{31}$$

Provided that the actual signal estimation error (and not the percentage deviation from the optimum value) is of importance in assessing the performance of the FSO communication links, Δ is used as a performance comparison metric. This difference is plotted in Fig. 6. To investigate this matter further, the difference in the variance of the estimation error between the linear estimator and its optimum counterpart is depicted in Fig. 7 and Fig. 8 for SNR=20 dB and 5 dB, respectively. For SNR of even 5 dB, and an observation interval of 35 symbols, the linear estimator's performance closely parallels the performance of the optimum estimator.

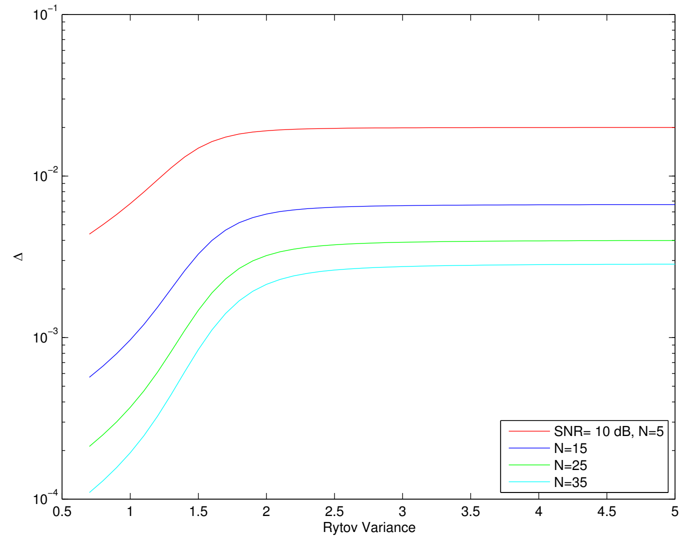

Fig. 6. The difference between the estimation error variance of the linear estimator and its optimum counterpart (SNR= 10 dB).

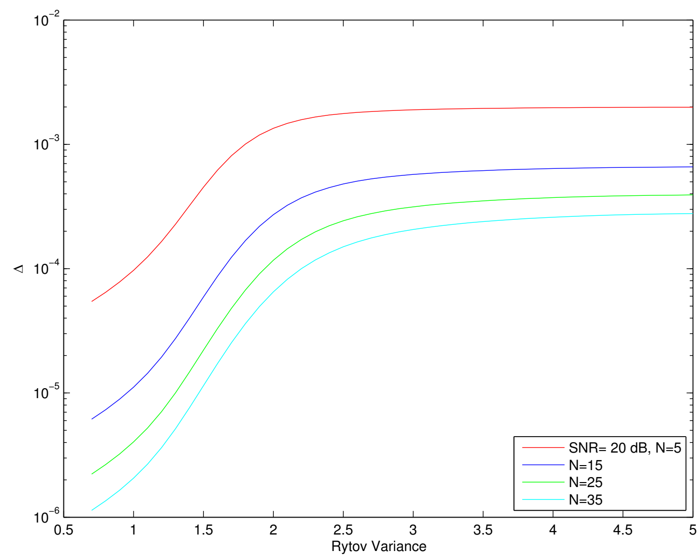

Fig. 7. The difference between the estimation error variance of the linear estimator and its optimum counterpart (SNR= 20 dB).

VIII. Concluding Remarks

In this paper, a linear estimator, motivated by MAP rule, with a small computational complexity was proposed for the estimation of the signal level for GGF channels. It was shown that the performance of this estimator closely follows that of the optimum estimator for a wide range of Rytov variance. Furthermore, it was shown that an observation interval of less than 50 symbols is needed to achieve the desired performance.

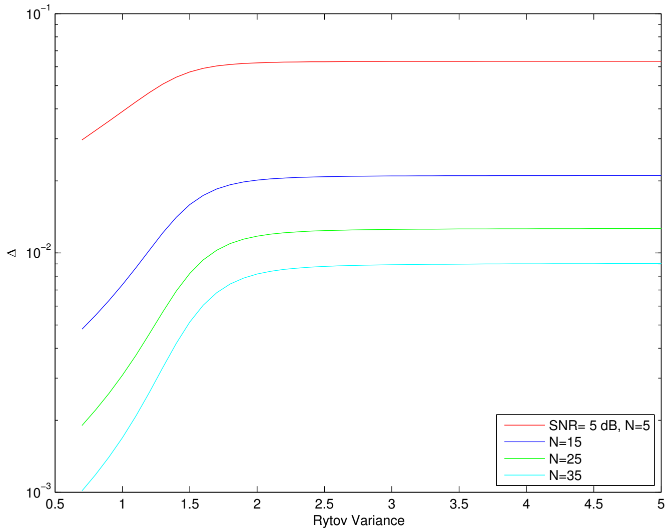

Fig. 8. The difference between the estimation error variance of the linear estimator and its optimum counterpart (SNR= 5 dB).

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