Joint Detection and Tracking of Satellite via Bernoulli Filtering

B.S. Wei
Department of EECE
UWA
Perth, Australia
21177381@student.uwa.edu.au

B. Nener
Department of EECE
UWA
Perth, Australia
Brett.nener@uwa.edu.au

***Abstract*—In general, a satellite can randomly enters or exit the field of view of the sensor. Moreover, even when the satellite is within the field of view, mis-detections can occur. In addition, the sensor also receives spurious detection or false alarming due to the large population of space debris. Standard dynamic state estimation techniques are not adequate to cope with this scenario. This paper presents a method for joint detection and tracking of satellite in the presence of mis-detection or false alarming.**

Keywords—satellite tracking; Bernoulli filter; joint detection and tracking;

I. INTRODUCTION

In many practical satellite tracking problems the standard Kalman filter or even the Bayes filter is not adequate. One of the more general problems is tracking in clutter where the measurement or observation at each time is a set of points [1, 2, 3, 4]. Such a problem requires a (state space) model with finite set observations [1, 17]. Another problem is joint detection and tracking (in clutter), where the target of interest may not always be present and exact knowledge of target existence cannot be determined from observations [1]. A target can enter and exit the surveillance region at random instances. Moreover, due to background clutter, exact knowledge of target existence in the surveillance area cannot be assumed. A filter that does not account for existence of targets may follow spurious measurements when the target is not in the scene, and when the target enters the scene the tracker may not be able to lock-on to the target. Thus, it is crucial that the filter detects the presence of the target as well as tracking it. A Bernoulli (state space) model (with finite set observations) is required to accommodate presence and absence of the target [24, 25].

Yet another basic problem, but far more challenging, is tracking multiple satellites, where the number of objects varies randomly in time, obscured by clutter, detection uncertainty and data association uncertainty. In essence, this problem involves two concurrent objectives; the detection of the presence of objects in field of view, and the tracking of each detected object which evolves in space and time [1, 2, 3, 4]. This is an important theoretical and practical problem which arises in many application areas. Traditionally, its development has been motivated by aerospace and maritime applications such as radar, sonar, and guidance and navigation [2]. In general, multi-target tracking is a very difficult problem; since it is not known how many targets there are on the scene (if any), the number of which can vary in time, and nor is there any knowledge of which target generated which part of the sensor measurement (if any), which in any case is corrupted by noise and spurious returns.

Conventional workhorses for performing multi-target tracking are based the concept of estimating *data associations*, and usually implement variations of standard multiple hypothesis tracking (MHT) techniques [2] or probabilistic data association (PDA) techniques [3]. The primary limitation of using data association based techniques is however that they are very computationally expensive. This is because, in general, they have exponential complexities, which in practice require drastic approximation. In addition, it has been recently noted that there may exist some theoretical irregularities in regards to the observability of the data association variables which features in the majority of traditional approaches [1]. This paper proposes to explore an alternative paradigm to multi-target tracking/filtering, which can presently be used to complement and further enhance existing workhorses, and ultimately pave the way forward for the next generation systems.

In recent years, there has been a rapid paradigm shift away towards what is now known as the finite set approach to multi-target tracking/filtering [1]. The premise of this approach is essentially a principled top-down characterization of multi-target systems via an appropriate mathematical formalism [1, 13, 14]. Such an approach correspondingly enables the principled development of practical multi-target trackers/filters. Furthermore, despite the relatively short timeframe since the inception of finite set approaches, the resulting algorithms have generated substantial worldwide interest from academia to defense and technology industries. Indeed, these new algorithms have also been shown to outperform traditional approaches, in terms of their performance and efficiency [4]. A suite of multiple object tracking algorithms of varying complexities have been developed in the last decade [5, 13, 14, 17, 18, 19, 20, 21].

These algorithms offer trade-offs between complexity and accuracy, from the least expensive [13] to the most expensive [21].

The current state of the art in random set based multi-target trackers/filters can be seen via the following thousand target demonstration. In certain settings, random-set-based multi-target trackers/filters can be engineered to simultaneously track in excess of 1000 moving targets, where the entire tracking/filtering algorithm and hence computational processing is performed solely on a basic laptop cpmputer.

In this paper, we described a random finite set technique for joint detection and tracking of satellite. Satellite tracking are important in space surveillance systems [23]. The dynamic model and measurement model of satellite tracking system are nonlinear. The state of the satellite is only partially observed and is obscured by false alarms and misdetections. Target tracking involves the joint detection and its individual state from observations in the presence of detection uncertainty, association uncertainty and clutter [3, 18]. A filter that does not account for existence of targets may follow spurious measurements when the target is not in the scene, and when the target enters the scene the tracker may not be able to lock-on to the target.

The paper is organized as follows. In section II, we give the state dynamic model and measurement model. The formulation of the Bernoulli filter is given in section III. Section IV introduces the implementation details of the Bernoulli filter. The conclusion of this study is in Section V.

II. PROBLEM FORMULATION

In order to take the presence/absence of target and its dynamic state into consideration, we represent the target state as a random finite set that can be either empty or a singleton [4]. The uncertainty of this is modeled as a Bernoulli random finite set (RFS). The joint detection and tracking problem can be described as an optimal Bayes filtering problem with finite-set-valued states and measurements [4].

Particle filter methods have recently emerged as a powerful tool for solving numerically complex dynamic estimation problems with high nonlinearities [15]. The particle filter approximates the posterior state probability density function (pdf) by a set of random samples. We use particle filter as the implementation of Bernoulli filter in this paper.

A. Equations of Motion

The relative movement of the Earth and a satellite is a two-body movement without perturbation according to the law of universal gravitations [27]. The Earth and the satellite can be seen as two particles. We use the position and velocity vectors in the ECI coordinate system as the state vector $X = (x, y, z, \dot{x}, \dot{y}, \dot{z})$. According to the law of universal gravitation, the motion equation of the satellite can be described as:

$$\begin{cases} \ddot{x} = -\mu \dfrac{x}{(x^2+y^2+z^2)^{3/2}} \\ \ddot{y} = -\mu \dfrac{y}{(x^2+y^2+z^2)^{3/2}} \\ \ddot{z} = -\mu \dfrac{z}{(x^2+y^2+z^2)^{3/2}} \end{cases} \tag{1}$$

Where $\mu \underline{\underline{\Delta}} 398{,}600 km^3/s^2$ is the Earth's gravitational parameter.

The motion state differential equation is:

$$\begin{aligned} \dot{X}(k) &= (\dot{x}(k)\,\dot{y}(k)\,\dot{z}(k)\,\ddot{x}(k)\,\ddot{y}(k)\,\ddot{z}(k))^T \\ &= f(X(k), k) \end{aligned} \tag{2}$$

B. Measurement Equation

Suppose there is a receiver in the center of the Earth, which can get the azimuth and pitching angle between the satellite and the Earth. The angles are described as [23]:

$$\begin{cases} \theta(k) = \tan^{-1}\left(\dfrac{y_b(k) - y_a(k)}{x_b(k) - x_a(k)}\right) \\ \varepsilon(k) = \tan^{-1}\left(\dfrac{z_b(k) - z_a(k)}{\sqrt{(x_b(k) - x_a(k))^2 + (y_b(k) - y_a(k))^2}}\right) \end{cases} \tag{3}$$

The measurement equation is defined as:

$$Y(k) = \begin{bmatrix} \theta(k) \\ \varepsilon(k) \end{bmatrix} = h(X_a(k), X_b(k)) + V(k) \tag{4}$$

Where $V(k)$ is the measurement noise.

III. BERNOULLI FILTER

The Bernoulli filter models the target state as a Bernoulli RFS [1, 24, 25]. Bernoulli RFS has the probability of $1 - q_{k|k}$ to be empty and probability of $q_{k|k}$ to be singleton. The only element of the singleton is distributed according to the pdf of $s_{k|k}(x)$ defined over the target state space. The Bernoulli filter RFS can be totally described by ($q_{k|k}, s_{k|k}(x)$). The posterior pdf of target is defined as:

$$f_{k|k}(X \mid Z_{1:k}) = \begin{cases} 1-q_{k|k} & if\ X=\phi \\ q_{k|k} \bullet s_{k|k}(x) & if\ X=\{x\} \\ 0 & if\ |X|>1 \end{cases} \quad (5)$$

The Bernoulli filter propagates the posterior pdf $f_{k|k}(X \mid Z_{1:k})$ in two steps: prediction and update. The prediction of the Bernoulli filter is [24, 25]

$$q_{k+1|k} = p_b \bullet (1-q_{k|k}) + p_s \bullet q_{k|k} \quad (6)$$

$$s_{k+1|k}(x) = \frac{p_b \bullet (1-q_{k|k}) \bullet b_{k+1|k}(x) + p_s \bullet q_{k|k} \bullet \int p_{k+1|k}(x \mid x') \bullet s_{k|k}(x')dx'}{p_b(1-q_{k|k}) + p_s q_{k|k}} \quad (7)$$

Where

p_s is the probability that the target at time k will survive until time k+1,

p_b is the probability of target birth,

$b_{k+1|k}(x)$ is the spatial distribution of predicted "target birth",

$p_{k+1|k}(x \mid x')$ is the target transitional density.

We assume p_s is constant over the state space.

The probability of existence is updated by the measurement set as

$$q_{k+1|k+1} = \frac{1-\delta_{k+1}}{1-\delta_{k+1} \bullet q_{k+1|k}} \bullet q_{k+1|k} \quad (8)$$

Where

$$\delta_{k+1} = p_D(1 - \sum_{z \in Z_{k+1}} \frac{\int g_{k+1}(z \mid x) s_{k+1|k}(x)dx}{\lambda c(z)}) \quad (9)$$

Where $g_{k+1}(z \mid x)$ is the measurement likelihood function,

λ and $c(z)$ are already defined clutter parameters,

The target spatial pdf is defined as

$$s_{k+1|k+1}(x) = \frac{1 - p_D + p_D \sum_{z \in Z_{k+1}} \frac{g_{k+1}(z \mid x)}{\lambda c(z)}}{1-\delta_{k+1}} s_{k+1|k}(x) \quad (10)$$

where p_D is the probability of detection,

IV. IMPLEMENTATION OF BERNOULLI FILTER

The computations of Bernoulli filter are carried out by sequential Monte Carlo method.

The sequential Monte Carlo method provides a general framework for the implementation of the Bernoulli filter. The Bernoulli Particle filter can approximate the spatial pdf by a set of weighted random samples or particles [24, 25]. The approximation can be defined as

$$s_{k|k}(x) \approx \sum_{i=1}^{N} w_{k|k}^i \delta_{x_{k|k}^i}(x) \quad (11)$$

Where $x_{k|k}^i$ is the state of particle and $w_{k|k}^i$ is its corresponding weight.

The steps of the Bernoulli Particle filter are [24]

The input are $q_{k|k}, \{w_{k|k}^i, x_{k|k}^i\}_{i=1}^N, Z_k, Z_{k+1}$

1. Compute the prediction of $q_{k+1|k}$
2. Draw a weighted sample of "target birth" particles $\{w_k^{i,b}, x_k^{i,b}\}_{i=1}^N$ at k from $b_k(x) = \frac{1}{|Z_k|} \sum_{z \in Z_k} \beta(x \mid z)$
3. Draw "persisting target" particles at time $k+1$: $x_{k+1}^{i,p} \sim p_{k+1|k}(x \mid x_{k|k}^i)$ for $i = 1,..., N$
4. Draw "target birth" particles at $k+1$: $x_{k+1}^{i,b} \sim p_{k+1|k}(x \mid x_{k|k}^{i,b})$ for $i = 1,..., N$
5. Compute the weights of particles at $k+1$: $w_{k+1}^{i,p} = p_s q_{k|k} w_{k|k}^i / q_{k+1|k}$; for $i = 1,..., N$

   $w_{k+1}^{i,b} = p_b(1-q_{k|k}) w_k^{i,b} / q_{k+1|k}$; for $i = 1,..., N$
6. Union of particles: $\{w_{k+1|k}^i, x_{k+1|k}^i\}_{i=1}^{2N} =$

   $\{w_{k+1}^{i,p}, x_{k+1}^{i,p}\}_{i=1}^N \cup \{w_{k+1}^{i,b}, x_{k+1}^{i,b}\}_{i=1}^N$
7. For every $z \in Z_{k+1}$,compute likelihood $g_{k+1}(z \mid x_{k+1|k}^i)$, for $i = 1,..., N$
8. For every $z \in Z_{k+1}$ compute $\phi_{k+1}(z) =$

   $\sum_{i=1}^{2N} w_{k+1|k}^i g_{k+1}(z \mid x_{k+1|k}^i)$
9. Compute δ_{k+1} from (9) using the specified $c(z)$ and $\phi_{k+1}(z) \approx \int g_{k+1}(z \mid x) s_{k+1|k}(x)dx$

10. Compute $q_{k+1|k+1}$ using (8)

11. Compute new weights $(i = 1,...,2N)$:

$$w_{k+1|k+1}^{i,*} = \frac{1 - p_D + p_D \sum_{z \in Z_{k+1}} \frac{g_{k+1}(z \mid x_{k+1|k}^i)}{\lambda c(z)}}{1 - \delta_{k+1}} w_{k+1|k}^i$$

12. Resample N times from $\{w_{k+1|k+1}^{i,*}, x_{k+1|k}^i\}_{i=1}^{2N}$ to obtain $\{w_{k+1|k+1}^i = 1/N, x_{k+1|k+1}^i\}_{i=1}^N$

Output : $q_{k+1|k+1}, \{w_{k+1|k+1}^i, x_{k+1|k+1}^i\}_{i=1}^N$

Numerical results for this algorithm will be presented later. The Bernoulli filter is the first step towards the more challenging and realistic problem of tracking multiple space objects, including satellites. The labeled random finite set filter [21] is an suitable candidate, due to its efficiency and accuracy. This filter is inspired from the notion of Bernoulli random finite sets and is based on the notion of labeled or marked point processes with distinct marks [21]. The key breakthrough was a family of conjugate priors that are closed under the Chapman-Kolmogorov equation. This admits an analytic solution to the Bayes multi-target tracking filter.

REFERENCES

[1] R. Mahler, "Statistical Multitarget Multisource Data Fusion," Artech House 2007

[2] S. S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Norwood, MA: Artech

[3] Y. Bar-Shalom and T. E. Fortmann, *Tracking and Data Association*. San Diego: Academic Press, 1988.

[4] R. Mahler, Advances in Statistical Multisource-Multitarget Information Fusion, Artech House, 2014.

[5] B. Vo, S. Singh, and A. Doucet, "Sequential Monte Carlo methods for Bayesian Multi-target filtering with Random Finite Sets," *IEEE Trans. Aerospace and Electronic Systems,* vol. 41, pp. 1224-1245, 2005.

[6] D. Stoyan, D. Kendall, and J. Mecke, *Stochastic Geometry and its applications*: John Wiley & Sons, 1995.

[7] D. Daley and D. Vere-Jones, *An introduction to the theory of point processes*: Springer-Verlag, 1988.

[8] J. Moller and R. Waagepetersen, Statistical Inference and Simulation for Spatial Point Processes. Chapman & Hall CRC, 2004.

[9] A. Karr, Point processes and their statistical inference. Marcel Dekker, 1991.

[10] I. Molchanov, *Theory of Random sets*. Spring-Verlag, 2005.

[11] O. Kallenberg, *Random measures, 4th Ed.* Akademie-Verlag, 1986.

[12] M. N. van Lieshout, *Markov Point Processes and their Applications*. Imperial College Press, 2000.

[13] R. Mahler, "Multi-target Bayes filtering via first-order multi-target moments," *IEEE Trans. Aerospace and Electronic Systems,* vol. 39, pp. 1152–1178, 2003.

[14] R. Mahler, "PHD filters of higher order in target number," *IEEE Trans. Aerospace and Electronic Systems,* vol. 43, pp. 1523–1532, 2007.

[15] A. Doucet, N. d. Freitas, and N. Gordon, *Sequential Monte-Carlo Methods in Practice*, New York: Springer-Verlag, 2001.

[16] B. Ristic, S. Arulampalam, and N. Gordon, Beyond the Kalman Filter: Particle Filters for Tracking Applications. Artec House, 2004.

[17] B.-T. Vo, B.-N. Vo, A. Cantoni, "Bayesian Filtering with Random Finite Set Observations," *IEEE Trans. on Signal Processing*, Vol. 56, No. 4, pp. 1313-1326, April 2008.

[18] B. Vo, B. Vo and A. Cantoni, "The Cardinality Balanced Multi-Target Multi-Bernoulli Filter and its Implementations," *IEEE Trans. on Signal Processing*, Vol. 57, No. 2, pp. 409-423, Feb 2009.

[19] B. Vo and W. K. Ma, "The Gaussian mixture Probability Hypothesis Density filter," *IEEE Trans. Signal Processing,* vol. 54, pp. 4091-4104, 2006.

[20] B. Vo, B. Vo, and A. Cantoni, "Analytic implementations of the Cardinalized Probability Hypothesis Density Filter," *IEEE Trans. Signal Processing,* vol 55, pp 3533-3567, 2007.

[21] B.-T. Vo, and B.-N. Vo, "Labeled Random Finite Sets and M ulti-Object Conjugate Priors," *IEEE Trans. Signal Processing*, vol. 61, no. 13, pp. 3460-3475, 2013.

[22] D. Schuhmacher, B. Vo, B.-N. Vo, "A Consistent Metric for Performance Evaluation of Multi-Object Filters," *IEEE Trans. on Signal Processing*, Vol. 56, No. 8, Part 1, pp. 3447-3457, Aug 2008.

[23] Wu, Panlong, Jianshou Kong, and Yuming Bo. "Modified iterated extended Kalman particle filter for single satellite passive tracking." *Turkish Journal of Electrical Engineering & Computer Sciences* 21.1 (2013): 120-130.

[24] B.-T. Vo, C. M. See, N. Ma, and W.-T Ng. "Multi-sensor joint detection and tracking with the Bernoulli filter." *Aerospace and Electronic Systems, IEEE Transactions on* 48.2 (2012): 1385-1402.

[25] B.-T. Vo, D. Clark, B.-N. Vo, and B. Ristic "Bernoulli forward-backward smoothing for joint target detection and tracking" *Signal Processing, IEEE Transactions on* 59.9(2011), 4473-4477.

[26] Gning, Amadou, Branko Ristic, and Lyudmila Mihaylova. "Bernoulli particle/box-particle filters for detection and tracking in the presence of triple measurement uncertainty." *Signal Processing, IEEE Transactions on* 60.5 (2012): 2138-2151.

[27] Teixeira, Bruno OS, et al. "Spacecraft tracking using sampled-data Kalman filters." *Control Systems, IEEE* 28.4 (2008): 78-94.

[28] Ristic, Branko, and Sanjeev Arulampalam. "Bernoulli particle filter with observer control for bearings-only tracking in clutter." *Aerospace and Electronic Systems, IEEE Transactions on* 48.3 (2012): 2405-2415.