Relativistic VLBI model for Finite Distance Radio Source

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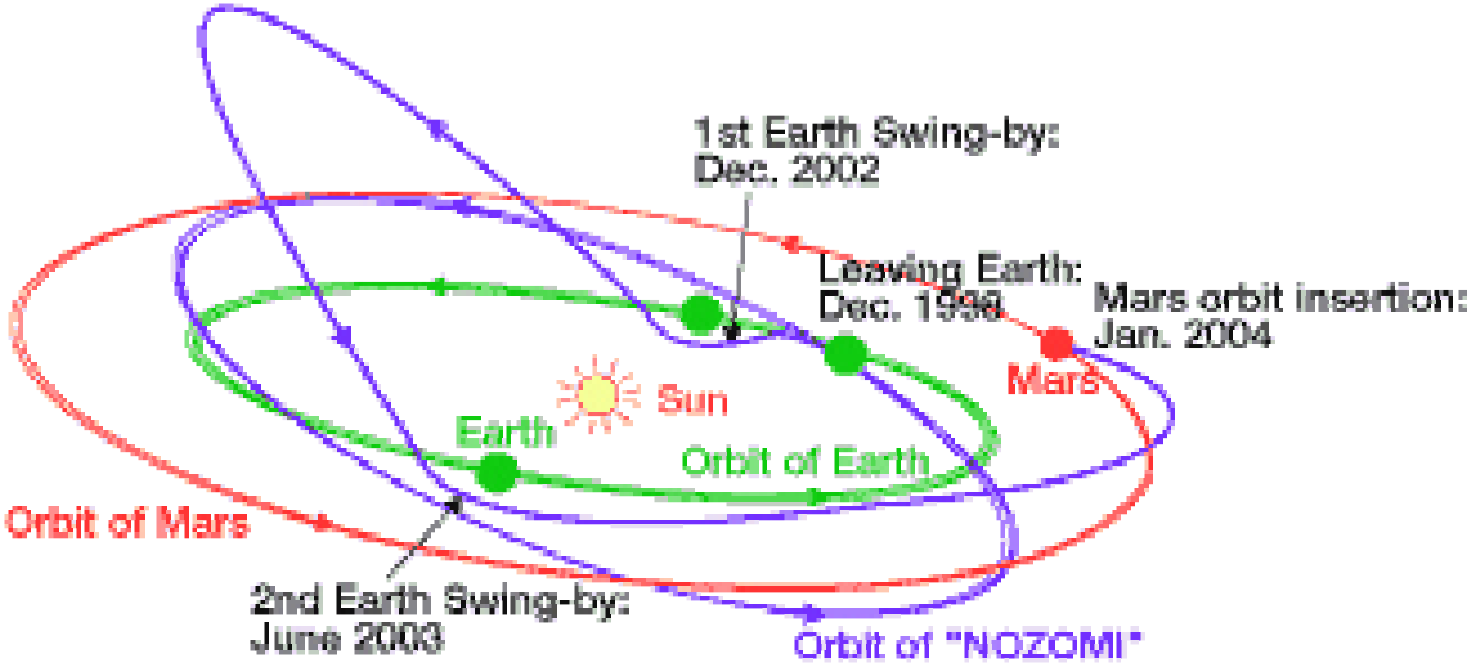
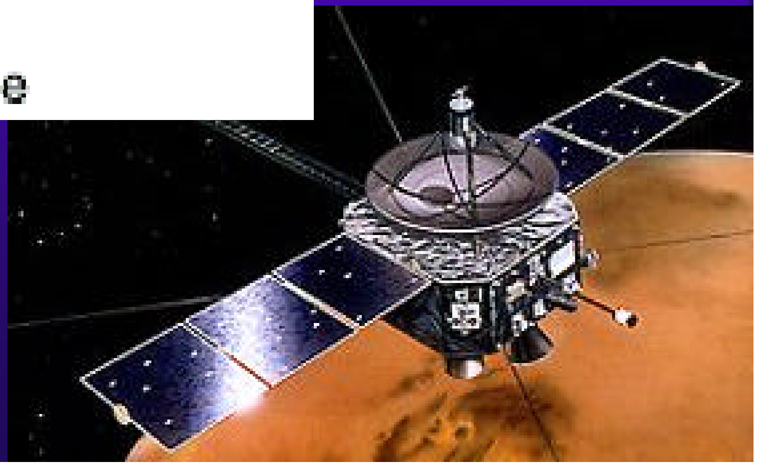
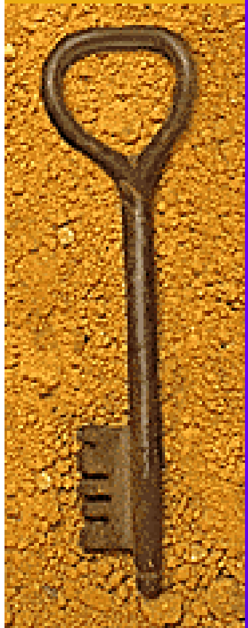
VLBI技術の応用 :宇宙飛翔体のナビゲーション •位置計測



"NOZOMI"(PLANET-B) New Mission Sequence

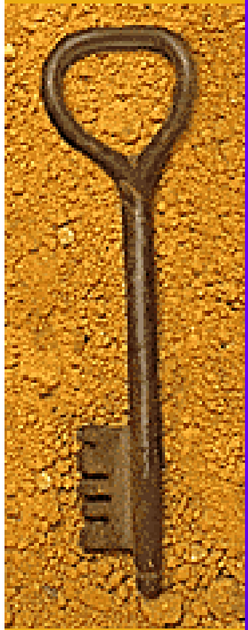




Necessity of new VLBI model

- Needs:Spacecraft Navigation (ex. NOZOMI)
- Spherical wave cannot be ignored when radio source is nearer than 30 light years.

- New VLBI delay model for finite distance radio source corresponding to the Consensus Model



VLBI Observation Equation

- was discussed around 1980-1990s. Several models proposed by (Fanselow, Herrings, Shapiro, et al…) were summarized as "**Consensus Model**" by T.M.Eubanks.
- Its precision is 10^{-12} seconds.

$$t_{v1} - t_{v2} = \left[1 + \frac{\vec{K}\bullet(\vec{V}_\oplus + \vec{w}_2)}{c}\right]^{-1}$$

$$\left\{\Delta t_g - \frac{\vec{K}\bullet\vec{b}}{c}\left[1-(1+g)U - \frac{V_\oplus^2 + 2\vec{V}_\oplus\bullet\vec{w}_2}{2c^2}\right] - \frac{\vec{V}_\oplus\bullet\vec{b}}{2c^2}\left(\vec{V}_e+\vec{w}_2\right)\bullet\vec{K}\right\} - \frac{\vec{V}_e\bullet\vec{b}}{c^2}$$

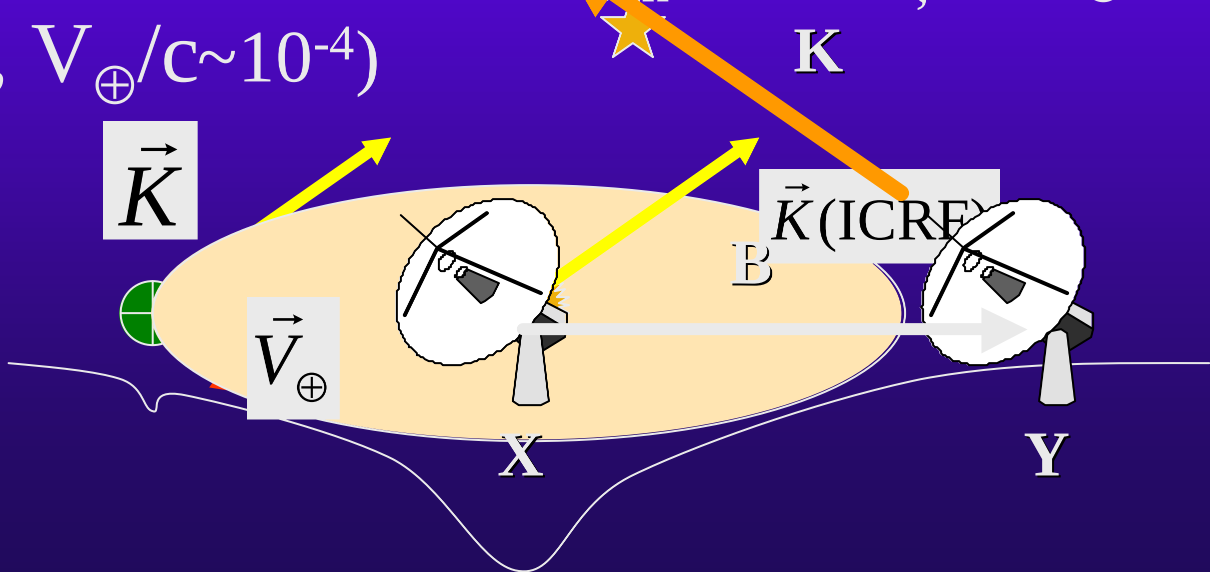
VLBI observation model:

simple principle, but complicated due to gravity and motion of the observer.

- Observable is express on Barycentric Ref. Frame

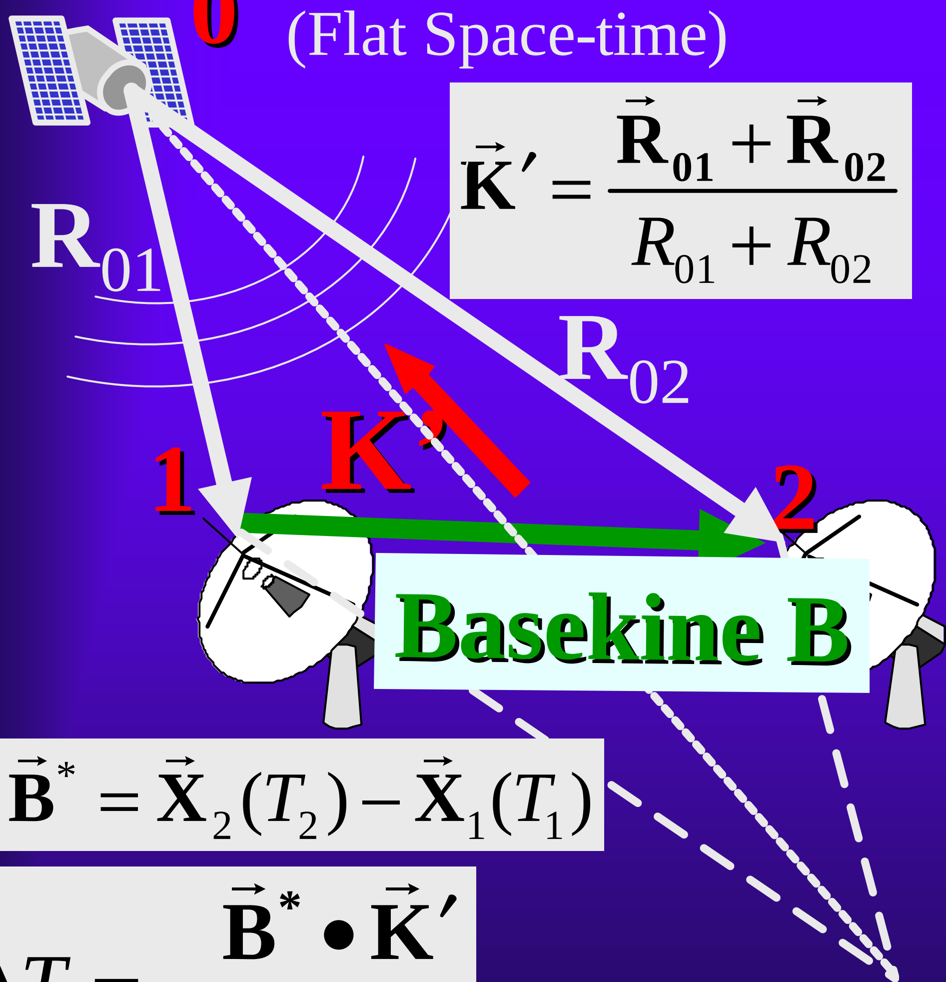
$$c(T_2 - T_1) = -\vec{K} \bullet \vec{B} \qquad \vec{B} = \vec{X}_2(T_2) - \vec{X}_1(T_1)$$

- But, on measurements on the Earth is affected by …($GM_{sun}/c^2 \sim 10^{-8}$, $GM_{\oplus}/c^2 \sim 7x10^{-10}$, $V_{\oplus}/c \sim 10^{-4}$)



VLBI model for Finite Distance radio source

Think about VLBI observation on Barycentric Ref. Frame (Flat Space-time)



$$\vec{\mathbf{K}}' = \frac{\vec{\mathbf{R}}_{01} + \vec{\mathbf{R}}_{02}}{R_{01} + R_{02}}$$

$$\vec{\mathbf{B}}^{*} = \vec{\mathbf{X}}_2(T_2) - \vec{\mathbf{X}}_1(T_1)$$

$$\Delta T = -\frac{\vec{\mathbf{B}}^{*} \bullet \vec{\mathbf{K}}'}{c}$$

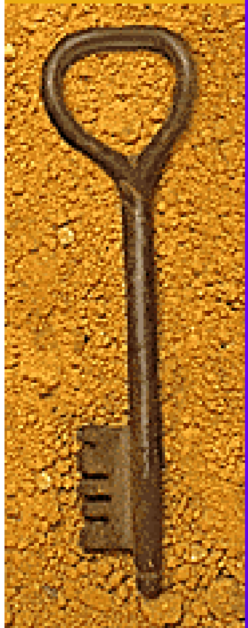
(Fukuhisma 1994 A&A)

If baseline vector B defined in simultaneity is used,

$$\vec{\mathbf{B}} = \vec{\mathbf{X}}_2(T_1) - \vec{\mathbf{X}}_1(T_1)$$

$$\Delta T = \frac{-\vec{\mathbf{B}} \bullet \vec{\mathbf{K}}'}{\left(1 + \frac{\hat{\vec{\mathbf{R}}}_{02} \bullet \vec{\mathbf{V}}_2}{c}\right)c}$$

Coordinates Transformation



Coordinates Transformation

(Barycentric R. Frame→Geocentric R.Frame)

Linearized Post Newtonian

$$ds^2 = g_{mn}\,dx^m dx^n$$

$$= (1-2f)c^2 dT^2 - (1+2gf)dX^2$$

$$g_{00} = (1-2f),\quad g_{0i} = g_{i0} = 0$$

$$g_{ij} = \begin{cases} (1+2gf), & (i=j) \\ 0, & (i \neq j) \end{cases}$$

$$\text{where,}\quad f = \sum_p \frac{GM_p}{r_p c^2}$$

$$t_{v1} - t_{v2} = \left[1 + \frac{\hat{\vec{R}}_{02} \bullet \vec{V}_2}{c}\right]^{-1}$$

$$\left\{ \Delta t_g - \frac{\vec{K}' \bullet \vec{b}}{c}\left[1-(1+g)U - \frac{V_\oplus^2 + 2\vec{V}_\oplus \bullet \vec{w}_2}{2c^2}\right] - \frac{\vec{V}_\oplus \bullet \vec{b}}{2c^2}\left(\vec{V}_e + \vec{w}_2\right)\bullet \vec{K} \right\} - \frac{\vec{V}_e \bullet \vec{b}}{c^2}$$

Finite VLBI Model

$$t_{v1} - t_{v2} = \left[1 + \frac{\hat{\vec{R}}_{02} \bullet \vec{V}_2}{c}\right]^{-1}$$

$$\left\{\Delta t_g - \frac{\vec{K}' \bullet \vec{b}}{c}\left[1 - (1+g)U - \frac{V_\oplus^2 + 2\vec{V}_\oplus \bullet \vec{w}_2}{2c^2}\right] - \frac{\vec{V}_\oplus \bullet \vec{b}}{2c^2}\left(\vec{V}_e + \vec{w}_2\right) \bullet \vec{K}'\right\} - \frac{\vec{V}_e \bullet \vec{b}}{c^2}$$

Consensus Model

$$t_{v1} - t_{v2} = \left[1 + \frac{\vec{K} \bullet (\vec{V}_\oplus + \vec{w}_2)}{c}\right]^{-1}$$

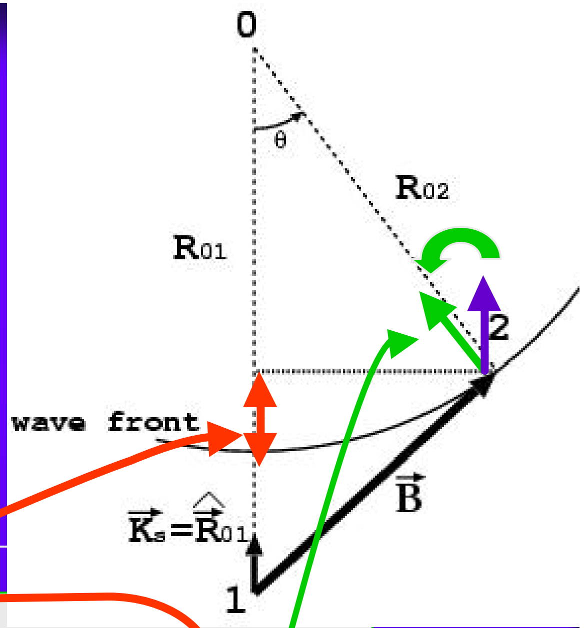
$$\left\{\Delta t_g - \frac{\vec{K} \bullet \vec{b}}{c}\left[1 - (1+g)U - \frac{V_\oplus^2 + 2\vec{V}_\oplus \bullet \vec{w}_2}{2c^2}\right] - \frac{\vec{V}_\oplus \bullet \vec{b}}{2c^2}\left(\vec{V}_e + \vec{w}_2\right) \bullet \vec{K}\right\} - \frac{\vec{V}_e \bullet \vec{b}}{c^2}$$

Difference:

Finite - Consensus

Assumption:

$$\hat{\vec{\mathbf{R}}}_{01}\,\text{(Finite distance)} = \vec{\mathbf{K}}_s\,\text{(Infinite distance)}$$

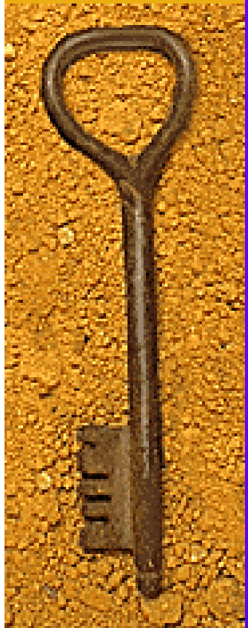


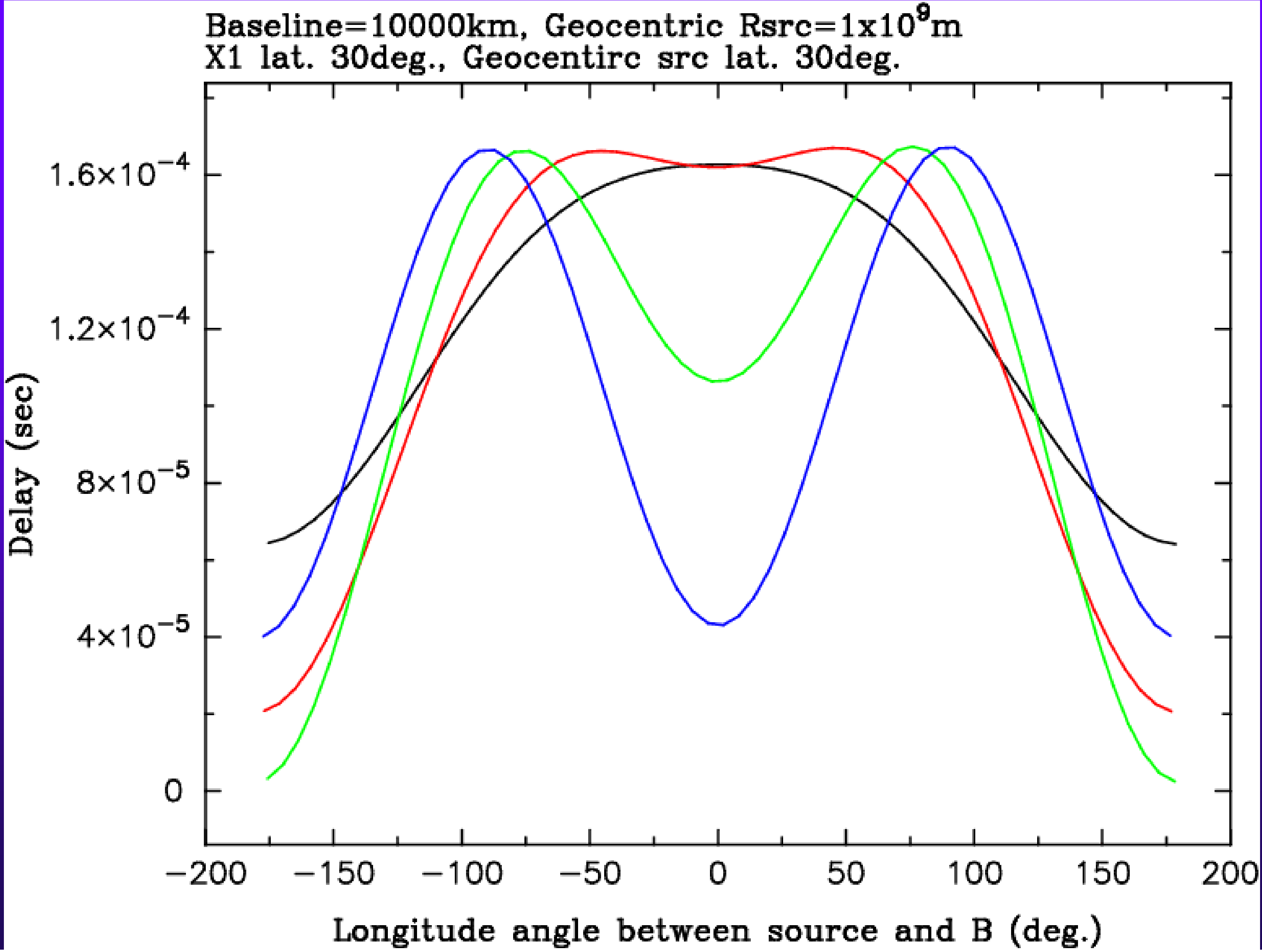
$$t_{\text{Finite}} - t_{\text{Infinite}} = \frac{R_{02}}{c}\frac{1-U-\dfrac{V_e^2+2\vec{\mathbf{w}}_2\bullet\vec{\mathbf{V}}_e}{2c^2}}{1+\dfrac{\hat{\vec{\mathbf{R}}}_{02}\bullet\vec{\mathbf{V}}_2}{c}}\left(\frac{|\vec{\mathbf{K}}_s\times\vec{\mathbf{B}}'|^2}{2}+\frac{|\vec{\mathbf{K}}_s\times\vec{\mathbf{B}}'|^4}{8}+\cdots\right)$$

$$-\frac{\vec{\mathbf{K}}_s\bullet\vec{\mathbf{B}}}{c}\frac{\left[\vec{\mathbf{K}}_s\times\left(\vec{\mathbf{K}}_s\times\vec{\mathbf{B}}\right)-\left(\dfrac{|\vec{\mathbf{K}}_s\times\vec{\mathbf{B}}'|^2}{2}+\dfrac{|\vec{\mathbf{K}}_s\times\vec{\mathbf{B}}'|^4}{8}+\cdots\right)\vec{\mathbf{K}}_s\right]}{\left(1+\dfrac{\vec{\mathbf{K}}_{02}\bullet(\vec{\mathbf{V}}_e+\vec{\mathbf{w}}_2)}{c}\right)^2}\bullet\frac{\vec{\mathbf{V}}_2}{c}$$

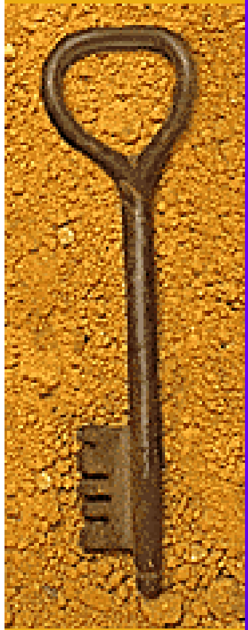
$$\text{Where, }\vec{\mathbf{B}}' \equiv \frac{\vec{\mathbf{B}}}{R_{02}} \le 10^{-2}\ (\text{at}\ R_{02} \ge 10^9\,m)$$

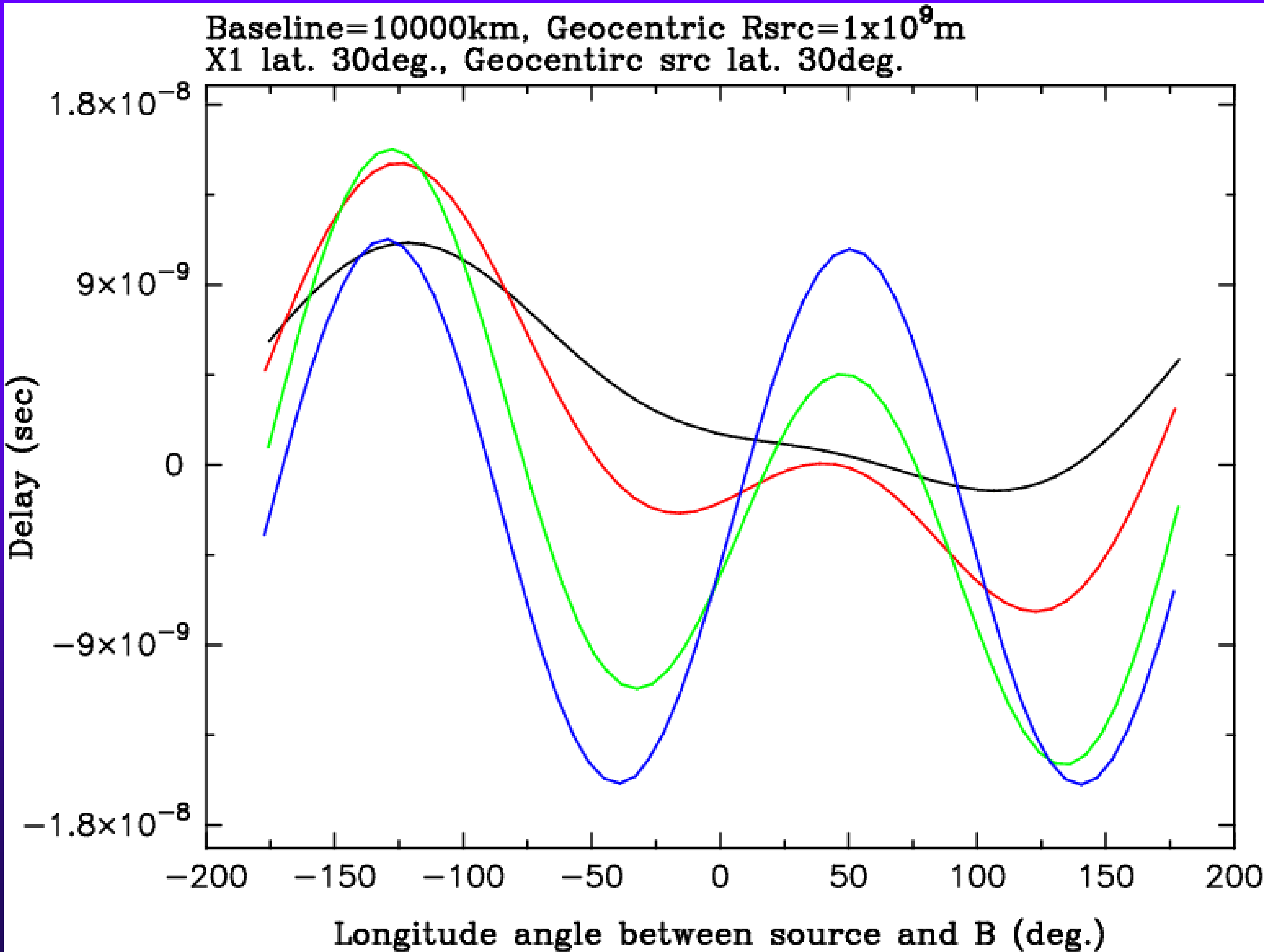
Effect of curved wavefront



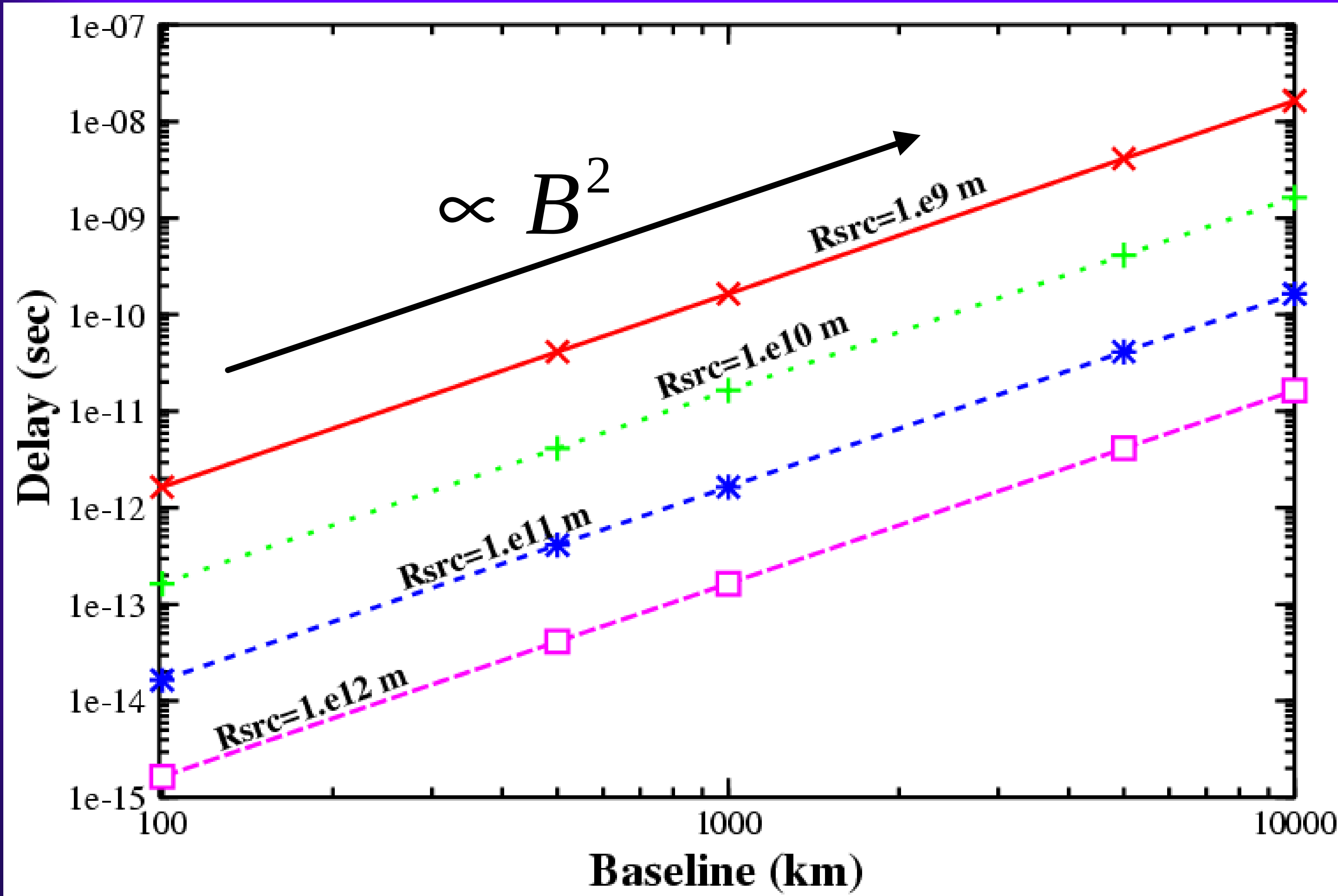


Effect of parallax

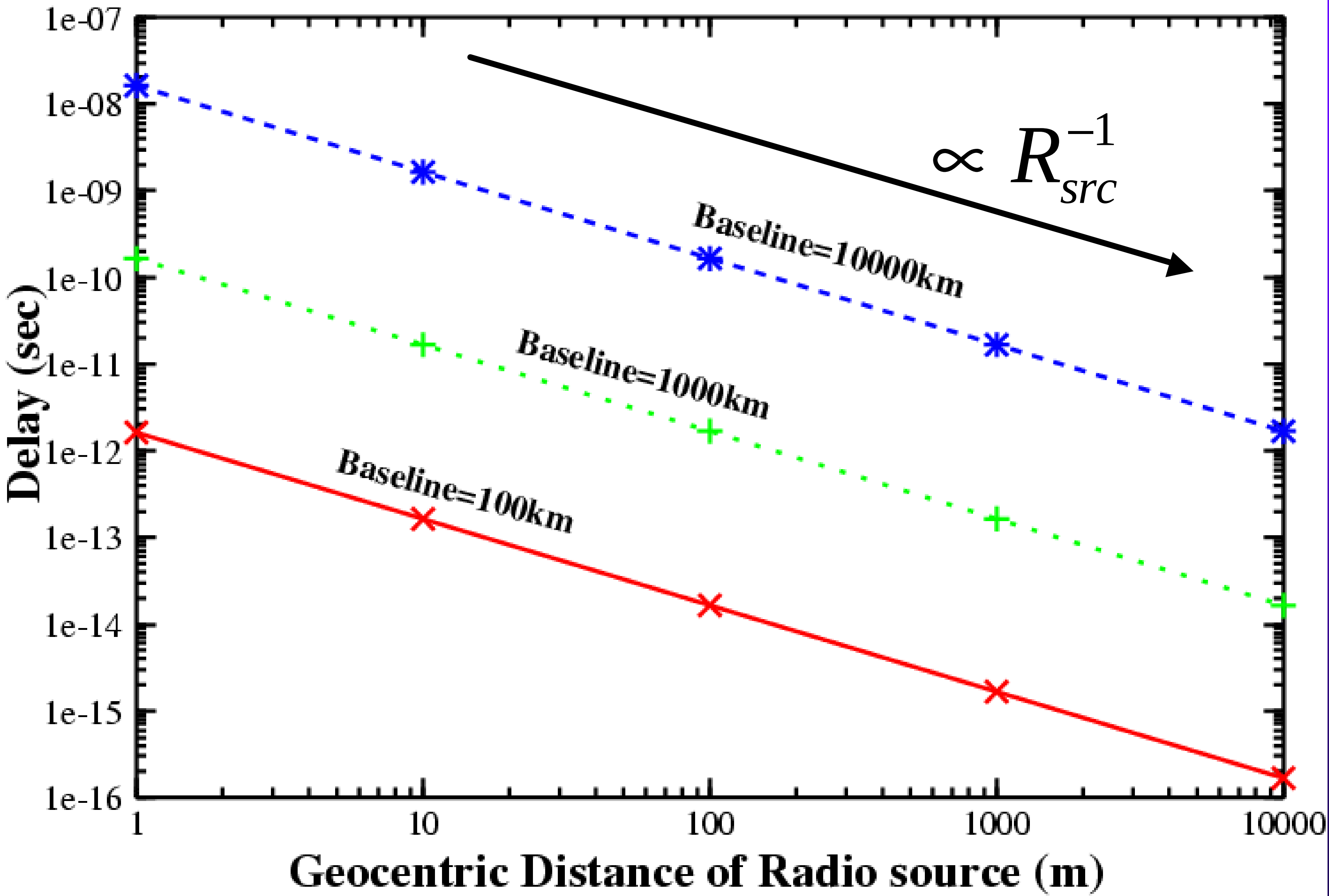




Dependence on Baseline



Dependence on Source Distance



Summary

- VLBI delay model for Finite distance radio source was derived. (accuracy 5ps,$R_{src}>10^9$m)
- The difference from Consensus model comes from Curvature of wavefront and parallax.
- The model was implemented in modifued CALC ver.9 package for spacecraft navigation (NOZOMI)