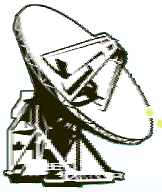

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Relativistic VLBI Delay Model for Finite Distance Radio Source

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Abstract When very long baseline interferometry (VLBI) is applied to observation of radio source in the solar system, the curvature of the the wave front cannot be approximated as plane wave, then current standard VLBI model (`consensus model') does not have enough accuracy. An alternative precise VLBI delay model for finite distance radio source corresponding to the `consensus model' is required in such a case (e.g. planets, asteroids, and spacecraft). We derived a formula of relativistic VLBI delay model for finite distance radio source by taking into account coordinates transformation by using linearized PPN metric. This model is valid in accuracy of several pico seconds when the radio source is at distance beyond 10^9 m. Our model include the `consensus model' as a special case that radio source is at infinite distance form the observer. This model was implemented by modification of CALC ver.9 and used for correlation processing and analysis of NOZOMI.

1. Delay in the rest frame of Solar System Barycenter

Variables of large capital indicate quantity in the rest frame of solar system barycenter (hereafter referred as FCB) and small ones represent those of geocentric reference frame (hereafter referred as FCG). Then Barycentric Coordinate Time (TCB) is represented by T and Geocentric Coordinate Time (TCG) is represented by t. Suffix 0,1,2 corresponds to radio source and two VLBI observation stations, respectively. Position vector of i^{th} station in the FCB is expressed by $\vec{\mathbf{X}}_i$. Radio signal is supposed to be emitted at T_0 from the radio source and to arrive at i^{th} observation station at T_i. Relative vector and magnitude of it is expressed by

$$\vec{\mathbf{R}}_{ij} = \vec{\mathbf{X}}_i - \vec{\mathbf{X}}_j, \quad R_{ij} = \left|\vec{\mathbf{R}}_{ij}\right|$$

Let us define a pseudo baseline vector $\vec{\mathbf{B}}^*$ and pseudo source vector $\vec{\mathbf{K}}^*$ as follows:

$$\vec{\mathbf{B}}^* = \vec{\mathbf{X}}_2(T_2) - \vec{\mathbf{X}}_1(T_1) = \vec{\mathbf{R}}_{20}(T_2) - \vec{\mathbf{R}}_{10}(T_1)$$

$$K^* = \frac{\vec{\mathbf{R}}_{02}(T_2) - \vec{\mathbf{R}}_{01}(T_1)}{R_{02}(T_2) - R_{01}(T_1)} \qquad (1)$$

Time interval between arrival of signal to two stations is expressed with these vectors as follows (Fukushima, 1994) :

$$c(T_2 - T_1) = R_{02}(T_2) - R_{01}(T_1) + c\Delta t_g$$
$$= -\vec{\mathbf{K}}^* \bullet \vec{\mathbf{B}}^* + c\Delta t_g \qquad (2)$$

where Δt_g indicates difference of delay due to gravitational refraction of light between two VLBI stations. Predicted coordinates of radio source is supposed to be given a priori as function of time. The coordinates of radio source $\vec{\mathbf{X}}_0(T_0)$, where a signal is departed at T_0 and arrived to station $\vec{\mathbf{X}}_1$ at T_1, is obtained by a solution of light time equation.

Let baseline vector and pseudo source vector defined at the same epoch T_1 in TCB be respectively $\vec{\mathbf{B}}$ and $\vec{\mathbf{K}}$. Since the time interval T_2-T_1 is small (43 msec for earth diameter), $\vec{\mathbf{X}}_2(T_1)$ can be expressed by eliminating higher order terms than velocity as:

$$\vec{\mathbf{X}}_2(T_2) = \vec{\mathbf{V}}_2(T_2 - T_1) + \vec{\mathbf{X}}_2(T_1) \qquad (4)$$

$$\vec{\mathbf{R}}_{02}(T_2) = \vec{\mathbf{R}}_{02}(T_1) - \vec{\mathbf{V}}_2(T_2 - T_1) \qquad (5)$$

$$R_{02}(T_2) = \left|\vec{\mathbf{R}}_{02}(T_1) - \vec{\mathbf{V}}_2(T_2 - T_1)\right|$$
$$\cong R_{02}(T_1) - \hat{\mathbf{R}}_{02} \bullet \vec{\mathbf{V}}_2(T_2 - T_1)$$
$$+ \frac{\left[V_2(T_2 - T_1)\right]^2 - \left[\hat{\mathbf{R}}_{02} \bullet \vec{\mathbf{V}}_2(T_2 - T_1)\right]^2}{2R_{02}(T_1)}, \qquad (6)$$

where $\hat{\mathbf{R}}_{ij} = \vec{\mathbf{R}}_{ij}/R_{ij}$ .Square root of the equation was approximated up to second order of $V_2(T_2 - T_1)/R_{02}$. In ground based VLBI observation of radio source at distance beyond 10^9 m, the contribution from the second order term is less than 1.5 mm. Thus we will keep remain up to the second term of equation (6). Substituting this into equation (2) becomes

$$c(T_2 - T_1) = R_{02}(T_1) - R_{01}(T_1) - \hat{\mathbf{R}}_{02} \bullet \vec{\mathbf{V}}(T_2 - T_1) + c\Delta t_g$$
$$= -\vec{\mathbf{K}} \bullet \vec{\mathbf{B}} - \hat{\mathbf{R}}_{02} \bullet \vec{\mathbf{V}}(T_2 - T_1) + c\Delta t_g \qquad (7)$$

Therefore, VLBI delay for finite distance radio source measured in TCB is

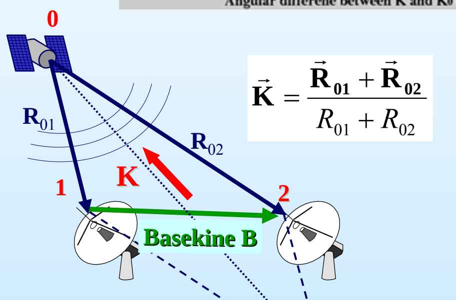
$$\boxed{T_2 - T_1 = \frac{1}{1 + \beta_{02}}\left(-\frac{\vec{\mathbf{K}} \bullet \vec{\mathbf{B}}}{c} + \Delta t_g\right),} \qquad (8)$$

where,

$$\vec{\mathbf{B}} = \vec{\mathbf{X}}_2(T_1) - \vec{\mathbf{X}}_1(T_1), \quad \vec{\mathbf{K}} = \frac{\vec{\mathbf{R}}_{02}(T_1) - \vec{\mathbf{R}}_{01}(T_1)}{R_{02}(T_1) - R_{01}(T_1)}, \qquad (9)$$

and

$$\beta_{02} = \hat{\mathbf{R}}_{02} \bullet \frac{\vec{\mathbf{V}}_2}{c} \qquad (10)$$



$$\vec{\mathbf{K}} = \frac{\vec{\mathbf{R}}_{01} + \vec{\mathbf{R}}_{02}}{R_{01} + R_{02}}$$

Fig.1 Schematic diagram of pseudo source vector $\vec{\mathbf{K}}$. The vector $\vec{\mathbf{K}}$ is on the diagonal line of parallelogram composed from $\vec{\mathbf{R}}_{10}$ and $\vec{\mathbf{R}}_{20}$ And it is neither unit vector nor constant vector.

2. Coordinate Transformation

We follow the approach of Hellings (1986) and Shahid-Saless & Hellings (1991). Linearized post-Newtonian metric was used for relativistic transformation from the delay expression in coordinates of Solar system barycenter to that in geocentric coordinates.

$$g_{00} = 1 - 2\varphi + O(c^{-4})$$
$$g_{0k} = O(c^{-3}) \qquad (11)$$
$$g_{mn} = -\delta_{mn}(1 + 2\gamma\varphi) + O(c^{-4})$$

where ϕ is gravitational potential, and γ is of of PPN parameter, which is unity in case of general relativity. Based on this metric infinitesimal coordinates transformation between FCB (T, X) and FCG coordinates nearby earth (t, x) is given as follows (Hellings, 1986) :

$$dt = \left(1 - U + \frac{V_e^2}{2c^2}\right)dT - \left(1 + \gamma U + \frac{V_e^2}{2c^2}\right)\frac{\vec{\mathbf{V}}_e \bullet d\vec{\mathbf{X}}}{c^2}$$
$$d\vec{\mathbf{x}} = (1 + \gamma U)\left(d\vec{\mathbf{X}} + \frac{\vec{\mathbf{V}}_e \bullet d\vec{\mathbf{X}}}{2c^2}\vec{\mathbf{V}}_e\right) - \left(1 - U + \frac{V_e^2}{2c^2}\right)\vec{\mathbf{V}}_e dT \qquad (12)$$

By using these relation of coordinates between the FCB and the FCG, the expression VLBI delay in TT (Terrestrial Time) is derived through that TCG. Detail of derivation procedure is described in proceeding paper. The form is

$$\boxed{\begin{aligned}\tau_2 - \tau_1 = {} & \left(1 + \beta_{02}\right)^{-1} \\ & \left\{\Delta t_g - \frac{\vec{\mathbf{K}} \bullet \vec{\mathbf{b}}}{c}\left[1 - (1+\gamma)U - \frac{V_e^2 + 2\vec{\mathbf{V}}_e \bullet \vec{\mathbf{w}}_2}{2c^2}\right]\right. \\ & \left. - \frac{\vec{\mathbf{V}}_e \bullet \vec{\mathbf{b}}}{c^2}\left(1 + \beta_{02} - \frac{\vec{\mathbf{K}} \bullet \left(\vec{\mathbf{V}}_e + 2\vec{\mathbf{w}}_2\right)}{2c}\right)\right\}\end{aligned}} \qquad (13)$$

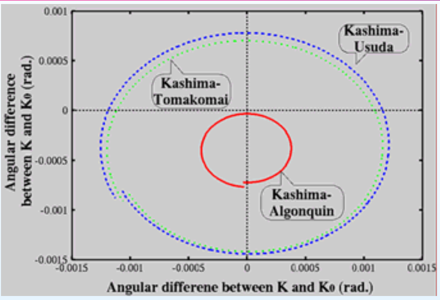
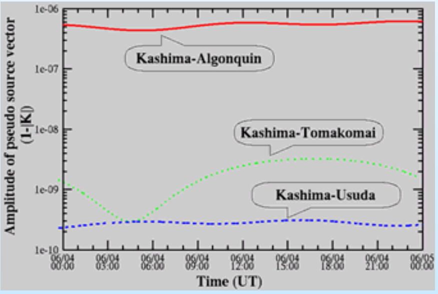


Fig.3 Behavior of pseudo source vector $\vec{\mathbf{K}}$ in comparison with geocentric unit source vector $\vec{\mathbf{K}}_0$. Left panel shows changes of the direction of vector during 24 hours. The origin of the plot is the direction of $\vec{\mathbf{K}}_0$ vector. Lower panel shows deviation of the magnitude of $\vec{\mathbf{K}}$ vector from unity. Three lines on both two panels represent Kashima-Algonquin (9109 km), Kashima-Tomakomai (750 km), and Kashima-Usuda (208 km) baselines. Predicted orbit of NOZOMI on 4th-5th June 2003 was used for computation of the vectors here. Geocentric distance to the NOZOMI was 4×10^9 m at this time.



3. Comparison with the consensus model.

The formula of consensus model (Eubanks, 1991) is

$$\tau_2 - \tau_1 = \left(1 + \frac{\vec{\mathbf{K}}_0 \bullet (\vec{\mathbf{V}}_e + \vec{\mathbf{w}}_2)}{c}\right)^{-1}$$
$$\left\{\Delta t_g - \frac{\vec{\mathbf{K}}_0 \bullet \vec{\mathbf{b}}}{c}\left[1 - (1+\gamma)U - \frac{V_e^2 + 2\vec{\mathbf{V}}_e \bullet \vec{\mathbf{w}}_2}{2c^2}\right]\right.$$
$$\left. - \frac{\vec{\mathbf{V}}_e \bullet \vec{\mathbf{b}}}{c^2}\left(1 + \frac{\vec{\mathbf{K}}_0 \bullet \vec{\mathbf{V}}_e}{2c}\right)\right\}, \qquad (14)$$

Where, $\mathbf{K}_0$ is unit vector to the radio source in FCB. The form of (13) and (14) are similar each other and it can be shown that they are identical when radio source is at infinite distance. However, they differ significantly when they are applied to finite distance radio source as displayed in Fig.2. The figure indicates that even detecting fringe is difficult without using finite distance VLBI model, especially on intercontinental baseline. The main cause of difference comes from the pseudo source vector **K,** which is neither unit vector nor constant vector as defined by equation (9). **B**ehavior of the **K** vector in comparison with geocentric unit source vector $\mathbf{K}_0$ is drawn in Fig. 3.

The finite distance VLBI model was implemented in our software by modification of CALC Ver.9. And it has been used for correlation processing and analysis of VLBI observations data for spacecraft NOZOMI (Ichikawa et al. 2003).

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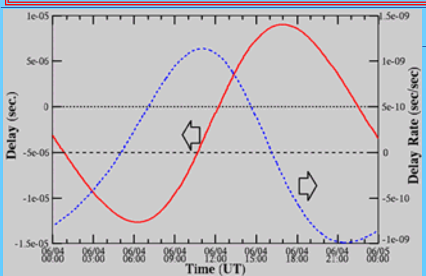

Fig.2 Difference of delay and rate of finite VLBI delay model from the consensus model when spacecraft NOZOMI is observed with Kashima-Algonquin (9000 km) baseline. Delay and rate of consensus model were already subtracted, where geocentric unit vector to the radio source was used as source vector. Solid line indicates delay scaled with left vertical axis. Dashed line is delay rate scaled with right vertical axis. Geocentric distance to the NOZOMI was 4×10^9 m at this time.