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Relativistic VLBI Delay Model for Finite Distance Radio Source

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Abstract. When very long baseline interferometry (VLBI) is applied to observation of finite distance radio source at close distance from observer, the curvature of the the wave front cannot be approximated as plane wave, the current standard VLBI model ('Consensus model') does not have enough accuracy in this case and an alternative precise VLBI delay model for finite distance radio source corresponding to the 'Consensus model' is required in observation of radio source in the solar system (e.g. planets, asteroids, and spacecraft). We derived a formula of relativistic VLBI delay model for finite distance radio source by taking into account coordinate transformation and based on linearized Parameterized Post Newtonian metric. This model is valid in order of several pico seconds accuracy when the radio source is at distance beyond 10^9 m.

Key words. Very Long Baseline Interferometry – Theory of Relativity – Finite Distance – Spacecraft Navigation

1 Introduction

An accurate delay model is required in VLBI data processing and analysis, especially for precise astrometry and geodesy. Based on plane wave approximation, VLBI observation models including relativistic effects are discussed from 1980s to beginning of 1990s. Ultimately, T. M. Eubanks has summarized those models and proposed a 'consensus model'(Eubanks, 1991). The 'Consensus model' is widely used in VLBI community over the world as a standard VLBI model (McCarthy and Petit, 2003, chap. 11)[1].

However, this standard VLBI model does not have enough accuracy, when radio source is closer than 30 light years (e.g. planets, asteroids, and spacecraft in the solar system), or in observation of relatively closer radio source (a hundred light year away) with space VLBI (Ground - satellite or stellite - satellite) baseline. Because curvature of the wave front cannot be ignored in those observations, an alternative VLBI delay model is required for finite distance radio sources.

Sovers and Jacobs (1996) discussed on curvature effect of finite distance radio source. Fukushima (1994) introduced an useful expression of VLBI delay model for finite distance radio source and discussed on partial derivatives. However, an analytical formula of relativistic VLBI delay model corresponding to the 'Consensus model' was not on their papers. Moyer (2000) has developed formulation of radiometric observation data for spacecraft navigation with based on light time equation (light-time approach). Solving light time equation by numerical procedure is straightforward and precise enough, however it need iteration of computation to solve light time equation in the analysis software. We took VLBI-like approach rather than the light-time approach because we wish to find an alternative formula of consensus model so that it can be easily implemented in current standard a priori computation software CALC[2] by modification. In this paper, we derived an analytical formula of VLBI delay model for finite distance radio source based on approach of Hellings (1986), Shahid-Saless and Hellings (1991), and Fukushima (1994). And our target was order of pico seconds accuracy with ground based baseline for the radio source in the solar system.

2 Delay in the rest frame of Solar System Barycenter

Variables of large capital indicate quantity in the rest frame of solar system barycenter (hereafter referred as FCB) and small ones denote those of geocentric reference frame (hereafter referred as FCG). Then Barycentric Coordinate Time (TCB) is represented by T and Geocentric Coordinate Time (TCG) is represented by t. Suffix 0,1,2 represent radio source and two VLBI observation stations, respectively. Position vector of i^{th} station in the

[1] http://maia.usno.navy.mil/conv2000.html

[2] http://gemini.gsfc.nasa.gov/solve/solve.shtml

FCB is expressed by $\boldsymbol{X}_i$. Radio signal is supposed to be emitted at T_0 from the radio source and to arrive at i^{th} observation station at T_i. Relative position vector and its magnitude are denoted as follows:

$$\mathbf{R}_{ij} = \boldsymbol{X}_i - \boldsymbol{X}_j, \quad R_{ij} = |\mathbf{R}_{ij}|$$

Let us define a pseudo baseline vector $\mathbf{B}^*$ and pseudo source vector $\mathbf{K}^*$ as

$$\begin{aligned} \mathbf{B}^* &= \boldsymbol{X}_2(T_2) - \boldsymbol{X}_1(T_1) = \boldsymbol{R}_{20}(T_2) - \boldsymbol{R}_{10}(T_1) \\ \mathbf{K}^* &= \frac{\mathbf{R}_{02}(T_2) + \mathbf{R}_{01}(T_1)}{R_{01}(T_1) + R_{02}(T_2)}. \end{aligned} \tag{1}$$

Then the time interval between the arrival of the signal at two stations is given as (Fukushima, 1994):

$$\begin{aligned} c(T_2 - T_1) &= R_{02}(T_2) - R_{01}(T_1) + c\Delta t_g \\ &= -\mathbf{K}^* \cdot \mathbf{B}^* + c\Delta t_g, \end{aligned} \tag{2}$$

where Δt_g indicates difference of delay due to gravitational refraction of light between two VLBI stations. Predicted coordinates of radio source are supposed to be given a priori as function of time. The coordinates of radio source $\boldsymbol{X}_0(T_0)$ is given from predicted orbit of the radio source at time epoch T_0. The time T_0, when a signal is departed from radio source and arrived to station $\boldsymbol{X}_1$ at T_1, is obtained by a solution of light time equation

$$\begin{aligned} c(T_1 - T_0) &= |\mathbf{X}_1(T_1) - \mathbf{X}_0(T_0)| \\ &\quad + (1+\gamma)\sum_J \frac{GM_J}{c^2} \ln\left|\frac{R_{0\mathrm{J}} + R_{1\mathrm{J}} + R_{01}}{R_{0\mathrm{J}} + R_{1\mathrm{J}} - R_{01}}\right| \end{aligned} \tag{3}$$

Let baseline vector and pseudo source vector be defined at the same epoch T_1 in TCB respectively, $\boldsymbol{B}$ and $\boldsymbol{K}$. We think about to express the delay time of equation (2) with $\boldsymbol{B}$ and $\boldsymbol{K}$. Since the time interval $T_2 - T_1$ is small (43 msec for earth diameter), contribution from acceleration during this time interval (less than 6 micro meter) can be eliminated and $\mathbf{X}_2(T_1)$ is expressed as

$$\mathbf{X}_2(T_2) = \mathbf{V}_2(T_2 - T_1) + \boldsymbol{X}_2(T_1). \tag{4}$$

Then vector $\mathbf{R}_{02}$ and its magnitude is expressed as follows:

$$\mathbf{R}_{02}(T_2) = \mathbf{R}_{02}(T_1) - \mathbf{V}_2(T_2 - T_1) \tag{5}$$

$$\begin{aligned} R_{02}(T_2) &= |\mathbf{R}_{02}(T_1) - \mathbf{V}_2(T_2 - T_1)| \\ &\cong R_{02}(T_1) - \widehat{\mathbf{R}}_{02} \cdot \mathbf{V}_2(T_2 - T_1) \\ &\quad + \frac{[V_2(T_2 - T_1)]^2 - [\widehat{\mathbf{R}}_{02} \cdot \mathbf{V}_2(T_2 - T_1)]^2}{2R_{02}(T_1)} \end{aligned} \tag{6}$$

where, $\widehat{\mathbf{R}}_{ij} = \mathbf{R}_{ij}/R_{ij}$. Square root of the equation was approximated up to second order of $V_2(T_2 - T_1)/R_{02}$. In ground based VLBI observation of radio source at distance beyond 10^9 m, the contribution from the second order term is less than 1.5 mm. Thus we will keep remain

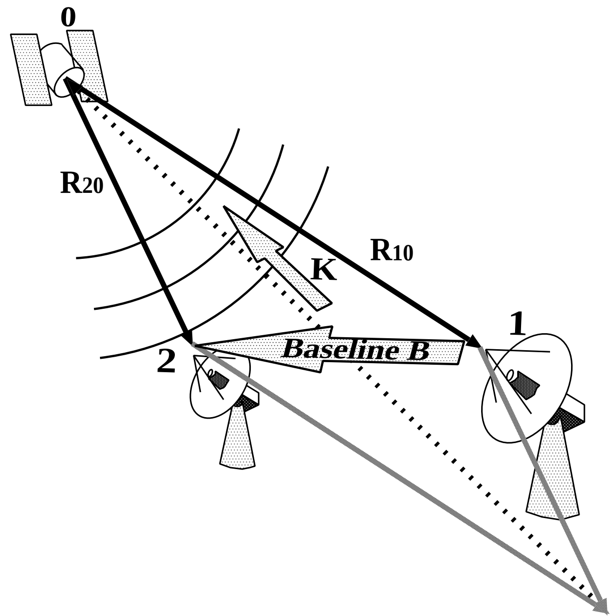

Fig. 1. Schematic diagram of pseudo source vector $\mathbf{K}$. The vector $\mathbf{K}$ is on the diagonal line of the parallelogram composed from vector $\mathbf{R}_{10}$ and $\mathbf{R}_{20}$.

up to the first order term, and substituting this into equation (2) becomes

$$\begin{aligned} &c(T_2 - T_1) \\ &\quad = R_{02}(T_1) - R_{01}(T_1) - \widehat{\mathbf{R}}_{02} \cdot \mathbf{V}_2(T_2 - T_1) + c\Delta t_g \\ &\quad = -\mathbf{K} \cdot \mathbf{B} - \widehat{\mathbf{R}}_{02} \cdot \mathbf{V}_2(T_2 - T_1) + c\Delta t_g. \end{aligned} \tag{7}$$

Therefore, VLBI delay for finite distance radio source expressed in TCB is given by

$$T_2 - T_1 = \frac{1}{1+\beta_{02}}\left(-\frac{\mathbf{K} \cdot \mathbf{B}}{c} + \Delta t_g\right) \tag{8}$$

where,

$$\beta_{02} = \widehat{\mathbf{R}}_{02} \cdot \frac{\mathbf{V}_2}{c} \tag{9}$$

$$\mathbf{B} = \boldsymbol{X}_2(T_1) - \boldsymbol{X}_1(T_1) \tag{10}$$

$$\mathbf{K} = \frac{\mathbf{R}_{02}(T_1) + \mathbf{R}_{01}(T_1)}{R_{01}(T_1) + R_{02}(T_1)} \tag{11}$$

Attention have to be paid on the vector $\mathbf{K}$, which should be called as pseudo source vector because it is neither unit vector nor constant vector differently from the ordinary source vector. The geometrical image of this vector is sketched in figure 1. The direction of the $\mathbf{K}$ vector is on the diagonal line of the parallelogram composed from two vectors $\mathbf{R}_{01}$ and $\mathbf{R}_{02}$, thus the direction and amplitude of the pseudo source vector depend on time and baseline. Behavior of $\mathbf{K}$ vector and difference from ordinary source vector is discussed in section 4.

3 Transformation from Barycentric Coordinates to Geocentric Coordinates

General relativity tells that coordinate transformation in gravity field belongs to a group of general coordinate transformations, which keep infinitesimal line element of events $(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu$ invariant. Here we use a metric tensor $g_{\mu\nu}$ of post-Galilean (linearized post-Newtonian) approximation by a following approach of Hellings(1986)

$$\begin{aligned} g_{00} &= 1 - 2\phi + O(c^{-4}) \\ g_{0k} &= O(c^{-3}) \\ g_{mn} &= -\delta_{mn}(1 + 2\gamma\phi) + O(c^{-4}), \end{aligned} \tag{12}$$

where $\phi = \sum_p \frac{GM_p}{|\mathbf{X}-\mathbf{X}_p|c^2}$ is summation of gravitational potential at $\mathbf{X}$ produced by p^{th} body. γ is one of Parameterized Post Newtonian parameters, and is unity in general relativity. An approximation up to the second order of c^{-1} is enough for our target accuracy.

Infinitesimal coordinate transformation between FCB $(T, \mathbf{X})$ and a FCG coordinates nearby earth $(t, \mathbf{x})$ is given by Hellings (1986) as

$$\begin{aligned} dt &= (1 - U + \frac{V_e^2}{2c^2})dT - (1 + \gamma U + \frac{V_e^2}{2c^2})\frac{\mathbf{V}_e \cdot d\mathbf{X}}{c^2} \\ d\mathbf{x} &= (1 + \gamma U)(d\mathbf{X} + \frac{d\mathbf{X} \cdot \mathbf{V}_e}{2c^2}\mathbf{V}_e) - (1 - U + \frac{V_e^2}{2c^2})\mathbf{V}_e dT. \end{aligned} \tag{13}$$

And its inverse transformation is

$$\begin{aligned} dT &= (1 + U + \frac{V_e^2}{2c^2})dt + (1 - \gamma U + \frac{V_e^2}{2c^2})\frac{\mathbf{V}_e \cdot d\mathbf{x}}{c^2} \\ d\mathbf{X} &= (1 - \gamma U)(d\mathbf{x} + \frac{d\mathbf{x} \cdot \mathbf{V}_e}{2c^2}\mathbf{V}_e) + (1 + U + \frac{V_e^2}{2c^2})\mathbf{V}_e dt. \end{aligned} \tag{14}$$

Baseline vector of equation (10) is defined in simultaneity in TCB ($dT = 0$), however, this is not simultaneous in TCG. Corresponding events are $(T_1, \mathbf{X}_1) \Leftrightarrow (t_1, \mathbf{x}_1)$ and $(T_1, \mathbf{X}_2) \Leftrightarrow (t_1^*, \mathbf{x}_2)$. Time difference $t_1^* - t_1$ is derived from the first equation of (13) by substituting $d\mathbf{X} = \mathbf{X}_2(T_1) - \mathbf{X}_1(T_1) = \mathbf{B}$ and a condition $dT = 0$. In addition, difference between $\mathbf{B}$ and $\mathbf{b}$ is order of $B \cdot (V_e/c)^2$, so taking up to second order of V_e/c gives

$$\begin{aligned} t_1^* - t_1 &= -(1 + \gamma U + \frac{V_e^2}{2c^2})\frac{\mathbf{V}_e \cdot \mathbf{B}}{c^2} \\ &\cong -\frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2} \end{aligned} \tag{15}$$

In integration of the second equation of (14) from $(T_1, \mathbf{X}_1)$ to $(T_1, \mathbf{X}_2)$, d$\mathbf{x}$ at right hand side of equation is substituted by $\mathbf{x}_2(t_1^*) - \mathbf{x}_1(t_1) \cong \mathbf{b} + \mathbf{w}_2(t_1^* - t_1)$, where $\mathbf{w}_2$ is velocity vector of station 2 defined in the FCG. Thus by using these and the relation (15), baseline vector defined by equation (10) is expressed as follows at the order of $(V_e/c)^2$:

$$\begin{aligned} \mathbf{B} &= \mathbf{X}_2(T_1) - \mathbf{X}_1(T_1) \\ &= (1 - \gamma U)(\mathbf{b} - \frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}\mathbf{w}_2) \\ &\quad +(1 - \gamma U)\frac{[\mathbf{b} - \frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}\mathbf{w}_2] \cdot \mathbf{V}_e}{2c^2}\mathbf{V}_e \\ &\quad -(1 + U + \frac{V_e^2}{2c^2})\frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}\mathbf{V}_e \\ &\cong (1 - \gamma U)\mathbf{b} - \frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}(\frac{\mathbf{V}_e}{2} + \mathbf{w}_2) \end{aligned} \tag{16}$$

The relation of the time interval $T_2 - T_1$ and $t_2 - t_1$ is derived form the first equation of (14) as follows :

$$\begin{aligned} T_2 - T_1 &= (1 + U + \frac{V_e^2}{2c^2})(t_2 - t_1) \\ &\quad +(1 - \gamma U + \frac{V_e^2}{2c^2})\frac{\mathbf{V}_e \cdot [\mathbf{x}_2(t_2) - \mathbf{x}_1(t_1)]}{c^2} \\ &= (1 + U + \frac{V_e^2}{2c^2} + \frac{\mathbf{V}_e \cdot \mathbf{w}_2}{c^2})(t_2 - t_1) \\ &\quad +(1 - \gamma U + \frac{V_e^2}{2c^2})\frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}, \end{aligned} \tag{17}$$

where, a relation $\mathbf{x}_2(t_2) = \mathbf{x}_2(t_1) + \mathbf{w}_2(t_2 - t_1)$ was used.

Substituting the expression of the baseline vector (equation (16)) and the relation of the time interval (equation (17)) to equation (8), we get the relation of the time interval $t_2 - t_1$ and baseline vector $\mathbf{b}$ as

$$\begin{aligned} (t_2 - t_1) &= (1 + \beta_{02})^{-1} \\ &\quad \left\{ \Delta t_g - \frac{\mathbf{K} \cdot \mathbf{b}}{c}\left[1 - (1 + \gamma)U - \frac{V_e^2 + 2\mathbf{V}_e \cdot \mathbf{w}_2}{2c^2}\right] \right. \\ &\quad \left. -\frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}\left(1 + \beta_{02} - \frac{\mathbf{K} \cdot (\mathbf{V}_e + 2\mathbf{w}_2)}{2c}\right) \right\} \end{aligned} \tag{18}$$

This formula relate quantity t and $\mathbf{x}$, which are quantities in geocentric reference frame. Time interval of signal arrival at VLBI stations is, however, based on time scale of terrestrial time (TT), which is defined as idealized atomic time scale on the geoid (Seidelmann and Fukushima, 1992). The clock rate of TT is slightly different from TCG, and they are related with $TT = (1 - L_G)TCG$, where $L_G = 6.969290134 \times 10^{-10}$ (McCarthy and Petit, 2003, chap. 1). By using time scale conversion from TCG (t) to TT (τ): $\tau = (1 - L_G)t$ and corresponding spatial scale conversion: $\boldsymbol{\xi} = (1 - L_G)\mathbf{x}$ to keep speed of light constant, VLBI delay measured by a clock on the geoid is found to be

$$\begin{aligned} \tau_2 - \tau_1 &= (1 + \beta_{02})^{-1} \\ &\quad \left\{ \Delta t_g - \frac{\mathbf{K} \cdot \mathbf{b}}{c}\left[1 - (1 + \gamma)U - \frac{V_e^2 + 2\mathbf{V}_e \cdot \mathbf{w}_2}{2c^2}\right] \right. \\ &\quad \left. -\frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2}\left(1 + \beta_{02} - \frac{\mathbf{K} \cdot (\mathbf{V}_e + 2\mathbf{w}_2)}{2c}\right) \right\}, \end{aligned} \tag{19}$$

where baseline vector on the geoid $\mathbf{b} = \boldsymbol{\xi}_2(\tau_1) - \boldsymbol{\xi}_1(\tau_2)$ was re-defined. Used variables are

$$\begin{aligned}
\beta_{02} &= \widehat{\mathbf{R}}_{02} \cdot \frac{\mathbf{V}_2}{c} \\
\widehat{\mathbf{R}}_{02} &= \frac{\mathbf{R}_{02}}{R_{02}} \\
\mathbf{K} &= \frac{\mathbf{R}_{02}(T_1) + \mathbf{R}_{01}(T_1)}{R_{02}(T_1) + R_{01}(T_1)}.
\end{aligned}$$

and gravitational delay Δt_g is expressed

$$\Delta t_g = \sum_J \frac{GM_J}{c^3} \ln \left(\frac{R_{2J} - \widehat{\mathbf{R}}_{02} \cdot \mathbf{R}_{2J}}{R_{0J} - \widehat{\mathbf{R}}_{02} \cdot \mathbf{R}_{0J}} \cdot \frac{R_{0J} - \widehat{\mathbf{R}}_{01} \cdot \mathbf{R}_{0J}}{R_{1J} - \widehat{\mathbf{R}}_{01} \cdot \mathbf{R}_{1J}} \right) \tag{20}$$

where, G is Newtonian gravitational constant, M_J is mass of the gravitation source J and $\mathbf{R}_{ij} = \mathbf{X}_i - \mathbf{X}_j$, $R_{ij} = |\mathbf{R}_{ij}|$, and $\widehat{\mathbf{R}}_{ij} = \mathbf{R}_{ij}/R_{ij}$. Each position epoch of gravitating body must be chosen when distance becomes minimum between the body and the photon (radio wave) to be measured (Sovers and Jacobs, 1996).

4 Comparison with the 'Consensus Model'

The 'Consensus Model' (Eubanks, 1991) is given as

$$\begin{aligned}
\tau_2 - \tau_1 = &\left[1 + \frac{\mathbf{K}_s \cdot (\mathbf{V}_e + \mathbf{w}_2)}{c} \right]^{-1} \\
&\left\{ \Delta t_g - \frac{\mathbf{K}_s \cdot \mathbf{b}}{c} \left[1 - (1+\gamma)U - \frac{V_e^2 + 2\mathbf{V}_e \cdot \mathbf{w}_2}{2c^2} \right] \right. \\
&\left. - \frac{\mathbf{V}_e \cdot \mathbf{b}}{c^2} \left(1 + \frac{\mathbf{V}_e \cdot \mathbf{K}_s}{2c} \right) \right\},
\end{aligned} \tag{21}$$

where $\mathbf{K}_s$ is unit source vector defined in rest frame of solar system barycenter.

The finite VLBI delay model (equation (19)) and the consensus model is quite similar in the form. The difference from normal VLBI delay model is, however, significant as displayed in Figure 2. The figure shows the difference of delay and rate between finite VLBI model and consensus model when spacecraft NOZOMI[3] is observed with Kashima-Algonquin baseline, as an example. It is remarkable that delay rate of finite VLBI model deviates in the order of 10^{-9}(sec/sec) from the normal VLBI model, and it makes difficult to detect fringes of the interferometer without appropriate correction.

The unique characteristic of the delay of the finite VLBI delay model is mainly originated from the properties of pseudo source vector $\mathbf{K}$. Differently from ordinary source vector $\mathbf{K}_s$, the pseudo source vector $\mathbf{K}$ changes its direction and magnitude with time and baseline. Examples of behavior of $\mathbf{K}$ vector is displayed in Figure 3 in comparison with geocentric unit vector to the source $\mathbf{K}_0 \equiv \mathbf{R}_{0g}/R_{0g}$. The upper panel shows that the deviation of direction of the $\mathbf{K}$ vector from the geocentric vector $\mathbf{K}_0$ becomes more significant as baseline is shorter. Since the spacecraft is moving, the track of the vectors are not closed curves in 24 hours. The lower pannel indicates that magnitude of the vector $\mathbf{K}$ differs from unity greatly as baseline becomes longer.

[3] Spacecraft NOZOMI was launched for Mars exploration by Institute of Space and Astronautical Sciences of Japan.

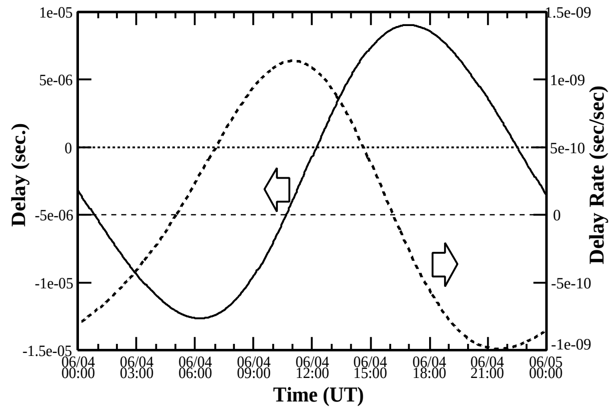

Fig. 2. An example of difference of delay and delay rate between finite VLBI delay model and normal VLBI delay model when spacecraft NOZOMI is observed with Kashima-Algonquin (9000 km) baseline. Consensus delay and rate model, where geocentric unit vector to the radio source was used as source vector, were subtracted from finite VLBI model. Solid line indicates delay scaled with left vertical axis. Dashed line is delay rate scaled with right vertical axis. Geocentric distance to the NOZOMI was $4. \times 10^9$ m at this time.

As it is obvious from the definition, that the pseudo source vector $\mathbf{K}$ (equation (11)) and the unit vector $\widehat{\mathbf{R}}_{02}$ converge to the ordinary source vector $\mathbf{K}_s$ as distance to the radio source increases.

$$\lim_{R_{01}, R_{02} \to \infty} \mathbf{K}(R_{01}, R_{02}) = \mathbf{K}_s \tag{22}$$

$$\lim_{R_{02} \to \infty} \widehat{\mathbf{R}}_{02} = \mathbf{K}_s \tag{23}$$

By using $\mathbf{V}_2 \cong \mathbf{V}_e + \mathbf{w}_2$ and equation (23), we can find

$$\lim_{R_{02} \to \infty} \beta_{02} = \mathbf{K}_s \cdot \frac{\mathbf{V}_2}{c} \cong \mathbf{K}_s \cdot \frac{\mathbf{V}_e + \mathbf{w}_2}{c}. \tag{24}$$

Then the last term of equation (19) approach to that of equation (21) at the limit of infinite distance ($R_{i0} \to \infty$). Therefore it can be said that the finite distance VLBI model include the 'consensus model' as a special case that the distance to the radio source is infinity. This finite VLBI model has an accuracy of several pico seconds when a radio source at beyond 10^9 m away from observer is observed with ground-based baselines.

Solving light time equation (3) for two legs of signal propagation paths as described by Moyer (2000) (light-time approach) is another straight forward way to give accurate delay and delay rate of interferometer observation of finite distance radio source. And it is superior especially at shorter radio source distance. On the other hand, the formula of VLBI delay for finite distance radio source, which we introduced in this paper (VLBI-like approach), has higher affinities with ordinally VLBI

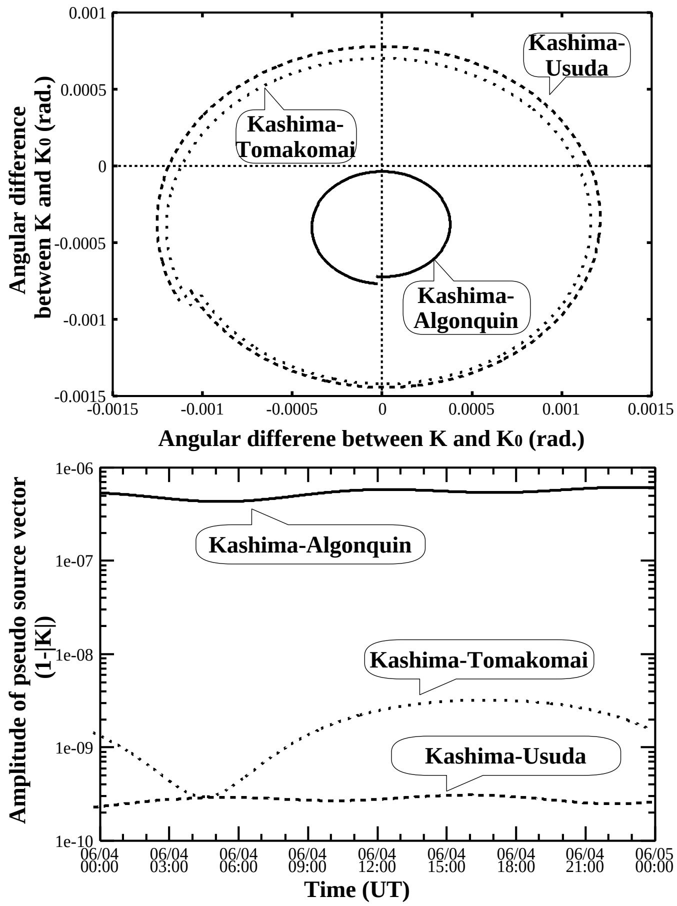

Fig. 3. Behavior of pseudo source vector $\mathbf{K}$ in comparison with geocentric unit source vector $\mathbf{K}_0$. Upper panel shows changes of the direction of $\mathbf{K}$ vector during 24 hours. The origin of the plot is the direction of $\mathbf{K}_0$ vector. Lower panel shows deviation of the magnitude of $\mathbf{K}$ vector from unity. Three lines on both two panels represent Kashima-Algonquin (9109 km), Kashima-Tomakomai (750 km), and Kashima-Usuda (208 km) baselines. Predicted orbit of NOZOMI on 4th-5th June 2003 was used for computation of the vectors here. Geocentric distance to the NOZOMI was $4. \times 10^9$ m in this period.

model and VLBI analysis software. Then it is more easier for implementation into VLBI analysis packages currently used. And inherently, our VLBI-like approach is more suitable for distant radio source (beyond 10^9 m), because larger part of light time solutions for two legs cancel out as the distance increase in case of light-time approach.

5 Summary

An analytical formula for a relativistic VLBI delay model of finite distance radio source is derived based on linearized post Newtonian metric. This formula has precision of less than five pico seconds for the radio source beyond 10^9 m from observer.

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References

Eubanks, T. M. (1991) A Consensus Model for Relativistic Effects in Geodetic VLBI. Proc. of the USNO workshop on Relativistic Models for Use in Space Geodesy: 60-82

Fukushima, T. (1994) Lunar VLBI observation model. A&A 291: 320-323

Hellings, R. W. (1986) RELATIVISTIC EFFECTS IN ASTRONOMICAL TIMING MEASUREMENTS. AJ, 91: 650-659

McCarthy, D. D. and Petit, G. (2003), IERS Conventions 2003, IERS Technical Note No. 32.

Moyer, T. D. (2000) Formulation for Observed and Computed Values of Deep Space Network Data Types for Navigation, JPL Monograph 2 (JPL Publication 00-7).

Seidelmann, P. K. and Fukushima, T. (1992) Why new time scale?. A&A 265: 833-838

Sovers, O. J. & Jacobs C. S. (1996) Observation Model and Parameter Partials for the JPL VLBI Parameter Estimation Software "MODEST"-1996". JPL Publication 83-39, Rev. 6: 6–8

Shahid-Saless, B. & Hellings, R. W. (1991) A Picosecond Accuracy Relativistic VLBI Model via Fermi Normal Coordinates. GRL 18: 1139-1142