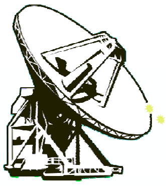

VLBI Delay Model for Radio Source at Finite Distance

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Abstract A relativistic VLBI delay model for radio sources at finite distance was derived. The effect of curved wave front was taken into account up to the second order by solving the delay equation with Halley's method. The precision of the new delay model is of 1 ps for all the Earth-based VLBI observations from Earth satellites to galactic objects. This is an expansion of the consensus model on the region of target radio source from infinite distance to finite one. Correction terms of the effect of curved wave front for consensus model were derived to get 1 ps precision, when the radio source is father than 10 pc.

1. Introduction

Very long baseline interferometry (VLBI) is a powerful tool with the highest angular resolution in space geodesy and astronomy. And it is also quite effective in engineering applications such as spacecraft navigation (Border et al. 1982) and precise tracking of space prove (e.g. Gurvits 2004). Accurate VLBI delay model is essential in VLBI data analysis for astrometry or orbit determination of spacecraft. So called consensus model (Eubanks 1991) is currently used in the VLBI community (McCarthy and Petit 2003) in the world. Although this model is based on plane wave approximation with assumption that the radio source is at infinitely distant away. Whereas the effect of curved wave front is fairly large for radio source at finite distance, especially when the target is in the solar system. JPL/NASA is taking different approach, light time, for computing the delay model for VLBI observation of spacecraft (Moyer 2000). It is based on numerical solution of light time equation. Its disadvantage is loosing its precision as the distance to the radio source increases, since large number of digits are lost at taking difference of the one way ranges for two observation stations. Sovers et al. (1998) derived the delay model for finite distance radio source, although it is not presented in terrestrial time (TT), which is actually measured in observatory, but Barycentric Dynamical Time (TDB). Thus it requires additional coordinate transformation between TDB-frame and TT-frame.

We derived an analytical formula of VLBI delay model expressed in TT for radio source at finite distance in terms of general relativity. This model can be used for ground based VLBI observation for radio source at distance farther than 100 km with better than 1 ps.

2. Finite VLBI Model and Consensus Model

VLBI delay for radio source at finite distance measured in terrestrial time (TT) is expressed by following formula.

$$(\mathrm{TT}_2-\mathrm{TT}_1)_{Finite}=\frac{-\left[1-(1+\gamma)U_E-\dfrac{\mathbf{V}_E^2+2\vec{\mathbf{V}}_E\bullet\vec{\mathbf{w}}_2}{2c^2}\right]\dfrac{\vec{\mathbf{K}}\bullet\vec{\mathbf{b}}}{c}-\dfrac{\vec{\mathbf{V}}_E\bullet\vec{\mathbf{b}}}{c^2}\left(1+\hat{\vec{\mathbf{R}}}_2\bullet\dfrac{\vec{\mathbf{V}}_2}{c}-\dfrac{\vec{\mathbf{K}}\bullet\left(\vec{\mathbf{V}}_E+2\vec{\mathbf{w}}_2\right)}{2c}\right)+\Delta T_{g,21}}{\left(1+\hat{\vec{\mathbf{R}}}_2\bullet\dfrac{\vec{\mathbf{V}}_2}{c}\right)(1+H)},\tag{1}$$

New model

where large capital letter indicates variable in the reference frame of solar system barycenter (TDB-frame), and small Greek letter represents quantity in Terrestrial Reference Frame (TRF). Definitions of each variables are as follows:

TT_i : Terrestrial Time (TT), when the radio signal arrived at the station i. (i=1, 2)

$\vec{\mathbf{b}}$: The baseline vector defined as the difference of the position vector of the two stations in the TRF at the same TT, $\vec{\mathbf{b}}\equiv\vec{\xi}_2(\mathrm{TT}_1)-\vec{\xi}_1(\mathrm{TT}_1)$

U_E : The external gravitational potential at the geocenter at TDB_1.

γ : One of the post-Newtonian parameters. γ=1 for Einstein's theory of general relativity.

$\vec{\mathbf{V}}_E$: The coordinate velocity vector of the geocenter at TDB_1 in the TDB-frame.

$\vec{\mathbf{V}}_2$: The coordinate velocity vector of the station 2 at TDB_1 in the TDB-frame. This is approximated as $\vec{\mathbf{V}}_2\cong\vec{\mathbf{V}}_E+\vec{\mathbf{w}}_2$ with an error of the order of $\left|\vec{\mathbf{V}}_E\right|^2\left|\vec{\mathbf{w}}_2\right|/c^3$.

$\vec{\mathbf{w}}_2$: The coordinate velocity vector of the station 2 in the Geocentric Celestial Coordinate System (GCRS) at TT_1.

c : The speed of light in vacuum.

$\vec{\mathbf{K}}$: The pseudo direction vector directed from the midpoint of the baseline to the radio source.

$$\vec{\mathbf{K}}\equiv\frac{\vec{\mathbf{R}}_1+\vec{\mathbf{R}}_2}{R_1+R_2}\tag{2}$$

$\vec{\mathbf{R}}_i$: The relative position vector of the radio source S at T_0 referred to the station i at T_1 in the TDB-frame (i=1,2). $\vec{\mathbf{R}}_i\equiv\vec{\mathbf{X}}_0(T_0)-\vec{\mathbf{X}}_i(T_1)$

R_i : The magnitude of vector $\vec{\mathbf{R}}_i$ (i=1,2)

$\hat{\vec{\mathbf{R}}}_i$: The unit vector of $\vec{\mathbf{R}}_i$ (i=1,2). $\hat{\vec{\mathbf{R}}}_i\equiv\vec{\mathbf{R}}_i/R_i$

$\vec{\mathbf{R}}_{ij}$: The relative position vector from station j to station i at T in the TDB-frame (i,j=1, 2, E). $\vec{\mathbf{R}}_{i,j}(T)\equiv\vec{\mathbf{X}}_i(T)-\vec{\mathbf{X}}_j(T)$

$\vec{\mathbf{X}}_i$: The position vector of the station i (=1,2) in the TDB-frame. (3)

$\vec{\mathbf{X}}_E$: The position vector of the geocenter in the TDB-frame.

$\Delta T_{g,21}$: The gravitational delay term given by $\Delta T_{g,21}=2\sum_J\frac{GM_J}{c^3}\ln\left|\frac{(R_{0J}+R_{2J}+R_{20})(R_{0J}+R_{1J}-R_{10})}{(R_{0J}+R_{2J}-R_{20})(R_{0J}+R_{1J}+R_{10})}\right|$,

where summation runs for gravitational sources with index J.

H : The correction term of the curved wave front given by Halley's method.

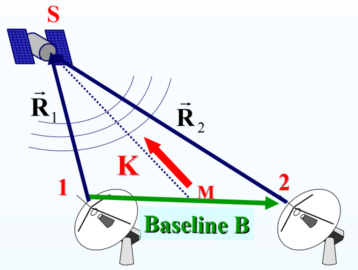

Fig.1 Schematic diagram of pseudo source vector $\vec{\mathbf{K}}$. The vector $\vec{\mathbf{K}}$ is directed from midpoint M of baseline B to the radio source S. And it is neither unit vector nor constant vector.

Correction term of Halley's method H works when the distance to the radio source is comparable with the baseline length. That is given as

$$H=\left|\frac{\vec{\mathbf{V}}_2}{c}\times\hat{\vec{\mathbf{R}}}_2\right|^2\frac{\vec{\mathbf{K}}\bullet\vec{\mathbf{b}}}{2R_2}.\tag{4}$$

The effect of the curved wave front is concentrated in the pseudo source vector $\vec{\mathbf{K}}$ proposed by Fukushima (1994) displayed as equation (2) (see Fig.1).

Vector $\vec{\mathbf{R}}_i$ (i=1,2) used in the equation (2) is defined in TDB-frame at TDB=T_1, and is given by conversion from position vector of station i in TRF at TT_1 as follows:

$$\begin{aligned}\vec{\mathbf{R}}_i&=\vec{\mathbf{X}}_0(T_0)-\vec{\mathbf{X}}_i(T_1)\\&=\vec{\mathbf{X}}_0(T_0)-\vec{\mathbf{X}}_E(T_1)-\vec{\mathbf{R}}_{iE}(T_1)\end{aligned}\tag{5a}$$

and

$$\vec{\mathbf{R}}_{iE}(T_1)=\left(1-\frac{U_E}{c^2}-L_C\right)\vec{\xi}_i(TT_1)-\frac{\vec{\mathbf{V}}_E\bullet\vec{\xi}_i(TT_1)}{2c^2}\vec{\mathbf{V}}_E.\tag{5b}$$

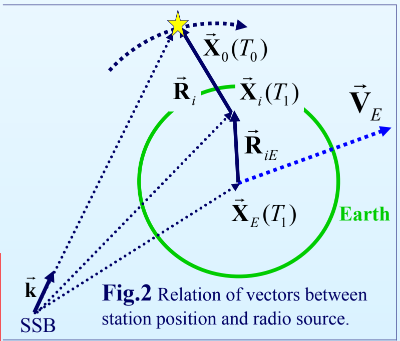

Fig.2 Relation of vectors between station position and radio source.

$\vec{\mathbf{V}}_E,\vec{\mathbf{X}}_E,U_E$, and T_1 can be obtained or computed from planetary ephemeris (e.g. Standish) and time ephemeris (e.g. Irwin and Fukushima 1999) for given TT_1. T_0 is given by solution of light time equation with given (predicted) orbit of the radio source. The light time equation is given as

$$T_1-T_0=\frac{\left|\vec{\mathbf{X}}_0(T_0)-\vec{\mathbf{X}}_E(T_1)-\vec{\mathbf{R}}_{1E}(T_1)\right|}{c}+\Delta T_{g,01}\tag{6}$$

where $\Delta T_{g,01}$ is delay due to gravitational ray path bending.

3. Effect of curved wave front

--Comparison with Consensus Model --.

Our new VLBI delay model has the same level of precision of delay (better than 1 ps) for radio sources at more than 100 km away from observer. Thus it can be though as an expansion of consensus model on the region of radio source from infinite to finite distance. The consensus model is displayed in equation (7) as reference for comparison.

$$(\mathrm{TT}_2-\mathrm{TT}_1)_{IERS}=\frac{-\left[1-(1+\gamma)U_E-\dfrac{\mathbf{V}_E^2+2\vec{\mathbf{V}}_E\bullet\vec{\mathbf{w}}_2}{2c^2}\right]\dfrac{\vec{\mathbf{k}}\bullet\vec{\mathbf{b}}}{c}-\dfrac{\vec{\mathbf{V}}_E\bullet\vec{\mathbf{b}}}{c^2}\left(1+\vec{\mathbf{k}}\bullet\dfrac{\vec{\mathbf{V}}_E}{2c}\right)+\Delta T_{g,21}}{\left(1+\vec{\mathbf{k}}\bullet\dfrac{\vec{\mathbf{V}}_E+\vec{\mathbf{w}}_2}{c}\right)},\tag{7}$$

Consensus model

The most significant effect of curved wave front appears as annual parallax. When distance to the radio source is larger than 10 pc, difference between the new model and the consensus model can be approximated with enough precision (< 1ps) as

$$c\left(\Delta TT_{Finite}-\Delta TT_{IERS}\right)=\vec{\mathbf{b}}\bullet\vec{\mathbf{p}}_M\left(1-\frac{\vec{\mathbf{k}}\bullet\vec{\mathbf{V}}_2}{c}\right)-\vec{\mathbf{k}}\bullet\vec{\mathbf{b}}\left(\vec{\mathbf{p}}_M\bullet\frac{\vec{\mathbf{V}}_2}{c}\right)+O(b\varepsilon^2),\tag{8}$$

where we call $\vec{\mathbf{p}}_M$ as parallax vector for the midpoint of baseline defined by

$$\vec{\mathbf{p}}_M=\vec{\varepsilon}_M-\left(\vec{\varepsilon}_M\bullet\vec{\mathbf{k}}\right)\vec{\mathbf{k}},\quad\text{and}\quad\vec{\varepsilon}_M=\left(\vec{\mathbf{X}}_1+\vec{\mathbf{X}}_2\right)/2R,\tag{9}$$

Where R is distance to the radio source from SSB. It is notable that the effect of curved wave front can be taken into account with enough precision (< 1ps) by including a few correction terms of equation (8) with the consensus model, when radio source distance is larger than 10 pc.

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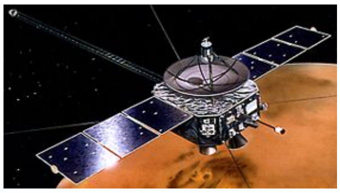

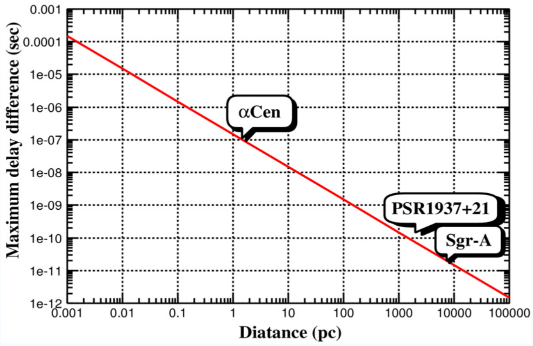

Fig.3 Difference between the new model and the consensus model on Kashima-Algonquin (9000km) baseline is plotted as a function of the distance to radio source. This plot was made for virtual radio source at the north pole (δ =90deg.). Although, we confirmed that no significant difference by dependence on the direction of radio source.

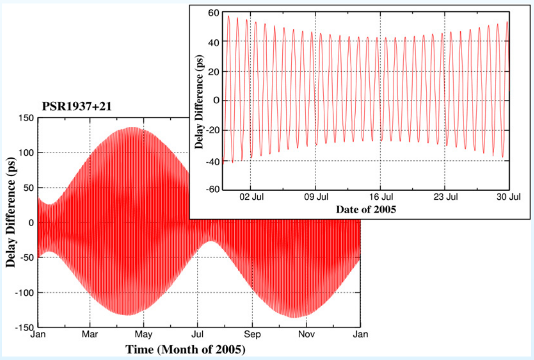

Fig.4 Difference between the new model and the consensus model on Kashima-Algonquin (9000 km) baseline for millisecond pulsar PSR1937+21 (2.1 kpc) in 2005. Magnified plot in July 2005 is superimposed at upper right corner to show clearly the daily variation of the delay.

Maximum difference between the new model and consensus model is plotted for the case of Kashima-Algonquin (900km) baseline as a function of the distance to the radio source in Fig. 3, where unit source vector $\vec{k}$ in the consensus model is directed from the solar system barycenter (SSB) to the radio source by definition of ICRS. The difference between the new model and the consensus model is observed as mixture of annual parallax and daily variation due to the Earth spin (Fig. 4). This amount reaches to larger than 100 ps for the case of millisecond pulsar PSR1937+21 (2.1kpc), which is known as candidate for frame tie between ICRF and dynamical reference frame (e.g. Bartel et al. 1996). The magnitude of 100 ps variation is detectable in VLBI observations. And the daily variation is usually used for estimation of shift of radio source coordinates due to annual parallax. It may be interesting to use equation (8) for direct estimation of distance to the radio source R with VLBI delay data set over different seasons, instead of estimating the shift of radio source coordinates at every epoch of observations. Partial derivative of delay with respect to a parameter $p = a_0 / R$, where R is distance to the radio source from SSB and a_0 is astronomical unit, is expressed as

$$\frac{\partial\tau}{\partial p} = \frac{\vec{X}_M}{a_0}\bullet\vec{b}\left(1-\vec{k}\bullet\frac{\vec{V}_2}{c}\right) - \vec{k}\bullet\vec{b}\left[\frac{\vec{X}_M}{a_0}\bullet\frac{\vec{V}_2}{c} + \left(\frac{\vec{X}_M}{a_0}\bullet\vec{k}\right)\left(1-2\vec{k}\bullet\frac{\vec{V}_2}{c}\right)\right] \qquad (10)$$

4. Target in the Solar System

When radio source is in the solar system, using the ICRF definition of source unit vector $\vec{k}$ directed from SSB to the radio source is not practical, since its direction is quite different from that from observer to the target. In this section, we take geocentric unit vector (Fig.5) as the $\vec{k}$ vector in the consensus model for comparison with the our model.

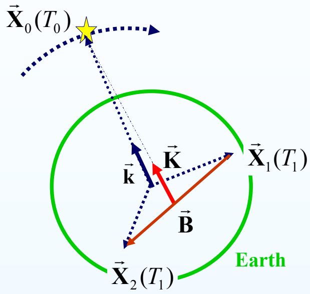

Fig.5 Geocentric unit vector directed to the radio source was used as the $\vec{k}$ vector in the consensus model for the comparison. The pseudo source vector $\vec{K}$ is defined with the midpoint of virtual baseline in TDB-frame $\vec{X}_2(T_1) - \vec{X}_1(T_1)$.

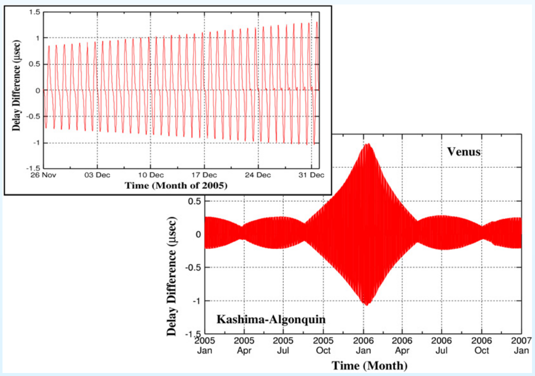

Fig.6 Difference between the new model and the consensus model for observation of Venus with Kashima-Algonquin (9000 km) baseline is plotted for two years from 2005. Geocentric unit vector was used as source vector in consensus model. Magnified plot of the difference in December 2005 is superimposed at upper left corner.

Fig. 6 shows the difference between the new model and the consensus model. Note that the magnitude of the difference is order of μsec. Thus innocent use of the consensus model for the target in the solar system might cause serious probrem.

4. Conclusion

- We derived a new VLBI delay model for radio source at finite distance (eqn. (1)).
- The precision of the formula is better than 1 ps for any radio sources more than 100 km distant away from observer on ground based VLBI observation.
- This new model can be thought as an expansion of the consensus model for radio source region from infinite to finite distance.
- The effect of curved wave front is detectable for radio sources in our galaxy. And taking into account the effect of curved wave front is essential for VLBI observation of radio source in the solar system.
- When the distance to the radio source is more than 10 pc, a few additional correction terms (eqn. (8)) to the consensus model is enough to get 1 ps precision.
- Direct least square estimation of parallax parameter $p = a_0 / R$ from observed delay data set with the eqn. (8) may be useful. And it might give better precision for estimation of the distance.

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