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VLBI Delay Model for Radio Source at Finite Distance

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Abstract

A relativistic VLBI delay model for radio sources at finite distance was derived. The new delay formula is similar with the standard VLBI delay model 'consensus model' (Petit & McCarthy 2003). And it gives delay in time scale of TT so that it is easily implemented in VLBI analysis software, such as CALC/SOLVE and OCCAM. The precision of the new delay model is better than 1 ps in any Earth-based VLBI observations from Earth satellites to galactic objects. This is regarded as an expansion of the consensus model on the region of target radio source from infinite distance to finite one. Correction terms for adaptation of consensus model to finite distance radio source were derived in 1 ps precision, when the radio source is father than 10 pc. This may be useful for direct estimation of parallax of galactic radio source from VLBI delay data,

1. Introduction

VLBI is not only a powerful tool in space geodesy and astronomy, but also it is quite effective in engineering applications such as spacecraft navigation (Border et al. 1982) and precise tracking of space prove (e.g. Gurvits 2004). Accurate VLBI delay model is essential in VLBI data analysis for astrometry. So called consensus model (Eubanks 1991) is currently used in the world VLBI community as a convection (McCarthy and Petit 2003). Although this model is based on plane wave approximation with assumption that the radio source is at infinitely distant away. Whereas the effect of curved wave front is fairly large for radio source at finite distance, especially when the target is in the solar system.

We derived an analytical formula of VLBI delay model expressed in TT for radio source at finite distance in terms of general relativity. <u>This model has precision of 1 ps for ground-based VLBI observation for radio source at distance farther than 100 km.</u>

2. Finite VLBI Model

Resolution B1.3 of the XXIV[th] IAU General Assembly gives definition of Barycentric Celestial Reference System (BCRS) for the space-time coordinates system in the solar system and Geocentric Celestial Reference System (GCRS) for that in vicinity of the earth. Time-coordinate of BCRS is called TCB, whose mean rate is different from Terrestrial Time (TT) by factor $1\text{-}L_B$=<dTT/dTCB>. Although, in practice, coordinates of spacecrafts and planets are not presented in BCRS yet. In fact, spacecraft coordinates in deep space are frequently given in the reference frame of JPL ephemeris, which is regarded as dynamical realization of ICRF and recommended as IERS standard (Petit and McCarthy 2003). The time argument of JPL ephemeris (Teph) is automatically adjusted to be the same rate with TT in the process of generating the ephemeris (Standish 1998). Thus this is very consistent with the definition of TDB. Here we define TDB-frame, which is a barycentric celestial reference frame differs from BCRS by a constant scaling factor $1\text{-}L_B$ for practical use. And JPL ephemeris may be regarded as realization of TDB-frame. Derivation of the VLBI delay model for finite distance radio source (Finite-VLBI-model) is performed on this frame. See appendix of papers (Sekido & Fukushima 2006) for detail of derivation. <u>As the same with consensus model, the time interval of the new VLBI delay model is given in TT, which is the time scale measured by atomic time standard on the earth. Hence it can be easily implemented in the current VLBI analysis software such as CALC/SOLVE, OCCAM, and other VLBI analysis software.</u> The form of the Finite-VLBI-model is given as follows:

$$(\mathrm{TT}_2-\mathrm{TT}_1)_{Finite}=\frac{-\left[1-2U_E-\dfrac{\mathbf{V}_E^2+2\vec{\mathbf{V}}_E\bullet\vec{\mathbf{w}}_2}{2c^2}\right]\dfrac{\vec{\mathbf{K}}\bullet\vec{\mathbf{b}}}{c}-\dfrac{\vec{\mathbf{V}}_E\bullet\vec{\mathbf{b}}}{c^2}\left(1+\hat{\vec{\mathbf{R}}}_2\bullet\dfrac{\vec{\mathbf{V}}_2}{c}-\dfrac{\vec{\mathbf{K}}\bullet\left(\vec{\mathbf{V}}_E+2\vec{\mathbf{w}}_2\right)}{2c}\right)+\Delta T_{g,21}}{\left(1+\hat{\vec{\mathbf{R}}}_2\bullet\dfrac{\vec{\mathbf{V}}_2}{c}\right)(1+H)}, \quad (1)$$

New model

where we represents delay in TT with quantity in the TDB-frame and that in TT-frame. TT-frame is defined as a geocentric celestial reference frame differs from the GCRS by constant factor $1\text{-}L_G$. Events of signal arrivals to the two stations (i=1,2) are denoted as $(TT_i,\vec{\xi}_i)$ in TT-frame and $(TDB_i,\vec{\mathbf{X}}_i)$ in TDB-frame, respectively. Definitions of each variables are as follows:

TT_i : Terrestrial Time (TT), when the radio signal arrived at the station *i*. (*i*=1, 2)

$\vec{\mathbf{b}}$: The baseline vector defined in the TT-frame. $\vec{\mathbf{b}}\equiv\vec{\xi}_2(\mathrm{TT}_1)-\vec{\xi}_1(\mathrm{TT}_1)$

U_E : The external gravitational potential at the geocenter.

$\vec{\mathbf{V}}_E$: The coordinate velocity vector of the geocenter at TDB_1 in the TDB-frame.

$\vec{\mathbf{V}}_2$: The coordinate velocity vector of the station 2 at TDB_1 in the TDB-frame. This may be approximated as $\vec{\mathbf{V}}_2\cong\vec{\mathbf{V}}_E+\vec{\mathbf{w}}_2$ within a delay error less than 4.e-14 sec.

$\vec{\mathbf{w}}_2$: The coordinate velocity vector of the station 2 in the TT-frame at TT_1.

c : The speed of light in vacuum.

$\vec{\mathbf{K}}$: The pseudo direction vector directed from the midpoint of the baseline to the radio source in the TDB-frame.

$$\vec{\mathbf{K}}\equiv\frac{\vec{\mathbf{R}}_1+\vec{\mathbf{R}}_2}{R_1+R_2} \quad (2)$$

$\vec{\mathbf{R}}_i$: The relative position vector of the radio source S at T_0 from the station *i* at T_1 in the TDB-frame (*i*=1,2). $\vec{\mathbf{R}}_i\equiv\vec{\mathbf{X}}_0(T_0)-\vec{\mathbf{X}}_i(T_1)$

R_i : The magnitude of vector (*i*=1,2)

$\hat{\vec{\mathbf{R}}}_i$: The unit vector in the direction of $\vec{\mathbf{R}}_i$ (*i*=1,2). $\hat{\vec{\mathbf{R}}}_i\equiv\vec{\mathbf{R}}_i/R_i$

$\vec{\mathbf{R}}_{ij}$: The relative position vector from station *j* to station *i* at T in the TDB-frame (*i,j*=1, 2, E).

$$\vec{\mathbf{R}}_{i,j}(T)\equiv\vec{\mathbf{X}}_i(T)-\vec{\mathbf{X}}_j(T) \quad (3)$$

$\vec{\mathbf{X}}_i$: The position vector of the station *i* (=1,2) in the TDB-frame.

$\vec{\mathbf{X}}_E$: The position vector of the geocenter in the TDB-frame.

$\Delta T_{g,21}$: The gravitational delay term given by

$$\Delta T_{g,21}=2\sum_J\frac{GM_J}{c^3}\ln\left|\frac{(R_{0J}+R_{2J}+R_{20})(R_{0J}+R_{1J}-R_{10})}{(R_{0J}+R_{2J}-R_{20})(R_{0J}+R_{1J}+R_{10})}\right|,$$

where summation index *J* runs for gravitational sources.

H : The correction term by Halley's method for the second order effect of station motion during the delay time between two stations.

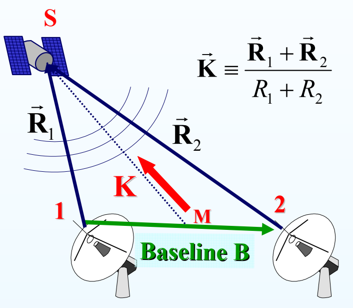

Fig.1 Schematic diagram of pseudo source vector $\vec{\mathbf{K}}$. The vector $\vec{\mathbf{K}}$ is directed from midpoint M of baseline B to the radio source S. And it is neither unit vector nor constant vector.

The correction term of Halley's method *H* works when the distance to the radio source is comparable with the baseline length. That is given by

$$H=\left|\frac{\vec{\mathbf{V}}_2}{c}\times\hat{\vec{\mathbf{R}}}_2\right|^2\frac{\vec{\mathbf{K}}\bullet\vec{\mathbf{b}}}{2R_2}. \quad (4)$$

The effect of the curved wave front is represented with the pseudo source vector $\vec{\mathbf{K}}$ proposed by Fukushima (1994), which is displayed in equation (2) (see Fig.1).

3. Adaptation of consensus model to observation of finite distance radio source.

The most significant effect of curved wave front appears as annual parallax. When distance to the radio source is larger than 10 pc, difference between the new model and the consensus model can be approximated with enough precision (< 1ps) as

$$c\left(\Delta TT_{Finite}-\Delta TT_{IERS}\right)=\vec{\mathbf{b}}\bullet\vec{\mathbf{p}}_M\left(1-\frac{\vec{\mathbf{k}}\bullet\vec{\mathbf{V}}_2}{c}\right)-\vec{\mathbf{k}}\bullet\vec{\mathbf{b}}\left(\frac{\vec{\mathbf{p}}_2\bullet\vec{\mathbf{V}}_2}{c}+H\right)+O(b\varepsilon^2), \quad (5)$$

where we call $\vec{\mathbf{p}}_M$ and $\vec{\mathbf{p}}_2$ as parallax vector for the midpoint of baseline and for the station 2, respectively defined by

$$\vec{\mathbf{p}}_M=\vec{\varepsilon}_M-\left(\vec{\varepsilon}_M\bullet\vec{\mathbf{k}}\right)\vec{\mathbf{k}},\quad\text{and}\quad\vec{\varepsilon}_M=\left(\vec{\mathbf{X}}_1+\vec{\mathbf{X}}_2\right)/2R, \quad (6)$$
$$\vec{\mathbf{p}}_2=\vec{\varepsilon}_2-\left(\vec{\varepsilon}_2\bullet\vec{\mathbf{k}}\right)\vec{\mathbf{k}},\quad\text{and}\quad\vec{\varepsilon}_2=\vec{\mathbf{X}}_2/2R,$$

where R is distance to the radio source from SSB. It is notable that the consensus model may be adapted for observation of finite distance (> 10pc) with enough precision (< 1ps) by including a few correction terms of equation (5). This parameterization of parallax in VLBI delay may be enables new method to estimate the parallax of the radio source from global analysis of delay data, whereas currently parallax is analyzed by variation of radio source coordinates mapped on the celestial sphere by observations in different seasons.

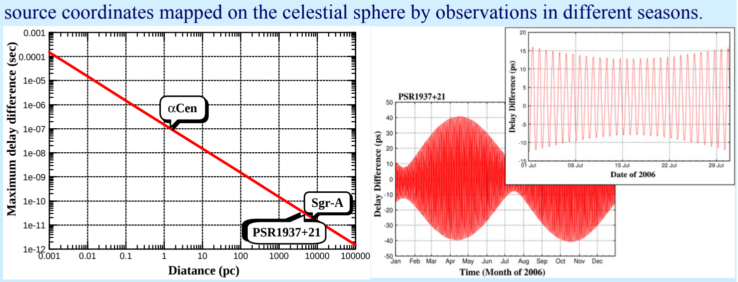

Fig.2 Maximum difference between the new delay model and the consensus model is evaluated in the case of Kashima-Algonquin baseline as function of the distance to the radio source (left). Time variation of that difference for observation of PSR1937+21 (distance=3.6kpc) is demonstrated as an example (right). The main contribution to the difference comes from annual parallax. The effect of curved wave front will be detectable for galactic radio sources.

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