VLBI Delay Model for a Radio Source at Finite Distance

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Abstract

A new VLBI Delay model for radio source at finite distance is presented. The geometrical effect of curved wavefront is fully considered with pseudo source vector defined by $\vec{K} = (\vec{R}_{01} + \vec{R}_{02})/(R_{01} + R_{02})$. Our VLBI delay model gives delay in the scale of terrestrial time (TT) with baseline vector of the TT-scale. Since the new delay model is in the similar form with the current standard VLBI delay model (consensus model), implementation to current VLBI analysis software, such as CALC/SOLVE and OCCAM will be relatively easy. Our VLBI delay model is regarded as expansion of the consensus model from infinite to finite region in terms of distance to radio source. Accuracy of the new delay model is better than 1 ps in ground based VLBI observation of radio source. And applicable range is any the radio sources at altitude of 100 km or more. Analytical correction terms to adapt the consensus model to finite-distance radio source is also given under the condition that the distance to radio source is farther than 10 pc. This may be useful for analysis of parallax for galactic radio sources.

1. Introduction

Standard VLBI delay model, which is so called consensus model [1, 2], is designed to compute accurate time difference of signal arrival between two stations on the earth for radio source at infinite distance. However the effect of curved wave front have to be taken into account for radio source at finite distance. For radio sources closer than 200 kpc, the effect of curved wavefront will exceed 1 ps on VLBI observation with 12000 km baseline [7]. Pulsars, maser sources and all radio sources in our galaxy are included. Radio source in the solar system such as planets and space proves are important target of VLBI observation, however intolerable error will be caused if the effect of curved wavefront is not considered. Moyer [3] provides delay model by computing difference of light time for two legs from the spacecraft to observation stations. This is a straight forward approach and it has been used in deep space missions by JPL/NASA in practice. Although this model is fairly different from current VLBI delay model, and implementation to VLBI analysis software is not so simple. VLBI-like approaches, in which the difference of light time is computed analytically, were proposed by several authors [4, 5, 6]. Fukushima proposed iterative scheme to give VLBI delay for lunar project[5]. Klioner gives analytical formula for radio source in the solar system [4]. These models, However, does not give relation between baseline and time delay observable in TT-frame where practical delay measurements are performed, but TDB-frame or Barycentric Celestial Reference System (BCRS), which are not directly accessible. BCRS is defined in the resolution B1.3 of IAU general Assembly in 2000 (see appendix of reference [2]). TDB-frame is a kind of barycentric celestial reference system differs from BCRS only by a scaling factor L_B. Its time coordinates is regarded as Barycentric Dynamical Time (TDB). TT-frame is a non-rotation celestial reference system differing from Geocentric Celestial Reference System (GCRS) by constant scale L_G. The GCRS is also defined in IAU resolution B1.3 as a non-rotation local flat coordinate system near the earth, in which scale of spatial scale is defined as consistent with Geocentric Coordinate Time (TCG). And spatial scale of TT-frame is consistent with Terrestrial time (TT). Current major VLBI analysis software such as CALC/SOLVE and OCCAM deal with delay data and station coordinates in TT-frame. The International Terrestrial Reference System (ITRS) is defined in GCRS, although realization of ITRS such as ITRF2000 is actually given in TT-frame. Due to these reasons, our new VLBI delay model is intended to give the relation between baseline vector and delay observable in TT-frame. This paper presents formula of our new VLBI delay model. See paper [7] for detail of the derivation of the formula and comparison with other models.

2. The New VLBI Delay Model

Formula of the new VLBI delay model is given as

$$(TT_2 - TT_1)_{\text{Finite}} = \left\{ - \left[1 - 2\frac{W_{\text{E}}}{c^2} - \frac{\vec{V}_{\text{E}}^2 + 2\vec{V}_{\text{E}} \cdot \vec{v}_2}{2c^2} \right] \frac{\vec{K} \cdot \vec{b}}{c} - \frac{\vec{V}_{\text{E}} \cdot \vec{b}}{c^2} \left[1 + \hat{\vec{R}}_2 \cdot \frac{\vec{V}_2}{c} - \frac{(\vec{V}_{\text{E}} + 2\vec{V}_2) \cdot \vec{K}}{2c} \right] + \Delta T_{g,21} \right\} \Big/ \left[(1 + \hat{\vec{R}}_2 \cdot \frac{\vec{V}_2}{c})(1 + H) \right], \quad (1)$$

where suffix 0,1,2, and E indicate radio source, observation station 1, 2, and geocenter, respectively. W_E is Gravitational potential given by $W_E = \sum_{J \neq E} GM_J / |\vec{X}_E - \vec{X}_J|$. $\vec{X}_i$ is position vector of $i = (0, 1, 2, E)$ in the TDB-frame. TDB-frame is chosen as barycentric celestial reference system in our derivation, because the position and velocity of objects in the solar system is given by planetary ephemeris such as JPL ephemeris DE405. $\vec{V}_i$, $(i = 2, E)$ is coordinate velocity of i with respect to the solar system barycenter (SSB). $\vec{v}_2$ is geocentric station vector of station 2. $\vec{b}$ is a baseline vector given by transformation from the coordinates in ITRF2000 to celestial reference frame (see chapter 5 of [2]).$\vec{K}$ is a pseudo source vector introduce by Fukushima [5], by which geometrical effect of curved wavefront is expressed. That is given by

$$\vec{K} \stackrel{\text{def}}{=} \frac{\vec{R}_1 + \vec{R}_2}{R_1 + R_2}, \text{ where } \vec{R}_i = \vec{X}_i(T_1) - \vec{X}_0(T_0), \text{ and } R_i = |\vec{R}_i|, \quad (2)$$

where i=1,2. Position vector $\vec{X}_0(T)$ should be given by predicted orbit as function of time in TDB-frame. Position vector in TDB-frame $\vec{X}_i$ (i=1,2) is given with geocentric station coordinates in TT-frame $(\vec{\xi}_i, TT_1)$ as

$$\vec{X}_i(T_1) = \vec{X}_{\text{E}}(T_1) + \left(1 - \frac{W_{\text{E}}}{c^2} - L_C\right) \vec{\xi}_i(TT_1) - \left(\frac{\vec{V}_{\text{E}} \cdot \vec{\xi}_i(TT_1)}{2c^2}\right) \vec{V}_{\text{E}}, \quad (3)$$

where $\vec{X}_E, \vec{X}_J, \vec{V}_2$, and W_E are given from planetary ephemeris. Scaling factor $L_C = 1.48082686741 \times 10^{-8} \pm 2 \times 10^{-17}$ [8]. $\hat{\vec{R}}_2$ is a unit vector given by $\hat{\vec{R}}_2 = \vec{R}_2 / R_2$ The epoch, when the signal departed from radio source 0, is denoted T_0 in TDB and arrival time to station 1 is T_1. Observed delay data are time tagged with UTC. Here arrival time to station 1 is denoted as UTC_1. Then corresponding TT_1 is computed from UTC_1 by

$$TT_1 = (\text{TT} - \text{TAI}) + (\text{TAI} - \text{UTC}) + UTC_1, \quad (4)$$

where (TT-TAI) is 32.184 for historical reason. (TAI-UTC) is 32 sec in 2005 and is 33 sec from 0h UTC on 1st January 2006. Then T_1 in TDB is computed by using time ephemeris $\Delta T_{\oplus}(T_{eph})$ [8] as

$$T_1 = TT_1 + \Delta T_{\oplus}(TT_1) - \Delta T_{\oplus}(TT_0) + \frac{V_E \cdot \vec{\xi}_1}{c^2}, \tag{5}$$

where TT_0 corresponds to 0h UT on January 1st 1977. $\vec{\xi}_1$ is geocentric vector of station 1 in TT-frame. The epoch of signal emission from radio source in TDB: T_0 is obtained from solution of light time equation

$$T_0 = T_1 - \frac{\left|\vec{X}_0(T_0) - \vec{X}_1(T_1)\right|}{c} - 2\sum_J \frac{GM_J}{c^2} \ln \frac{R_{1J} + R_{0J} + R_{01}}{R_{1J} + R_{0J} - R_{01}}, \Delta T_{g,01}. \tag{6}$$

The last term is gravitational effect in the ray path from radio source 0 to observations station 1. In this term, the position of graviting body J must be evaluated at the epoch of closest approach of the photon to the graviting body. The light time equation (6) may be solved by numerical iteration such as Newton-Raphson method and the solution converges very rapidly. The term H in the denominator of equation (1) is the correction term with Halley's method [7].

$$H = \left|\frac{\vec{V}_2}{c} \times \widehat{\vec{R}}_2\right|^2 \frac{\vec{K} \cdot \vec{b}}{2R_2}. \tag{7}$$

The gravitational effect $\Delta T_{g,21}$ is composed from several terms as discussed by Klioner [4]: Post-Newtonian ΔT_{pN}, effect in the field of moving body ΔT_M, influence of quadruple field ΔT_Q, rotation of the bodies ΔT_R, and the post-post-Newtonian effects ΔT_{ppN}.

$$\Delta T_{g,21} = \Delta T_{pN} + \Delta T_M + \Delta T_R + \Delta T_Q + \Delta T_{ppN}. \tag{8}$$

Post-Newtonian term (ΔT_{pN}) is the most significant and it must be included at least for the Sun, moon and major planets (Jupiter, Saturn, Venus, Mars, and the Earth). This term is given by

$$\Delta T_{pN} = 2\sum_J \frac{GM_J}{c^3} \ln \left(\frac{R_{2J} + R_{0J} + R_{20}}{R_{2J} + R_{0J} - R_{20}}\right)\left(\frac{R_{1J} + R_{0J} - R_{10}}{R_{1J} + R_{0J} + R_{10}}\right). \tag{9}$$

According to Klioner [4], the post-post-Newtonian term of the Sun becomes a few hundreds of ps in case grazing ray and several ps even the source direction is 1 deg. away from the Sun. The term ΔT_Q reaches a few tens ps when the ray passes through the rim of the Jupiter or Saturn. The term ΔT_M of Jupiter and ΔT_R of the sun reaches 0.5 ps when ray passes the rim of those gravitating bodies. Refer to literature [9, 4] for formula of each gravitational effects.

3. Adapting Consensus Model for Finite-Distance Radio Source

Effect of curved wavefront larger than 1 ps is caused up to 200 kpc of distance with earth diameter baseline. And full consideration this effect is indispensable for observation of radio source in the solar system. However, when the distance to the radio source from the earth is larger than

10 pc, effect of curved wavefront can be approximated with a few correction terms with parallax vector $\vec{p}_M$ and $\vec{p}_2$ as

$$\Delta\tau_{\text{Finite}} - \Delta\tau_{\text{IERS}} = (\Delta T_{g,12} - \Delta t_g) + \frac{\vec{b}\cdot\vec{p}_M}{c}\left(1 - \vec{k}\cdot\frac{\vec{V}_2}{c}\right) - \frac{\vec{k}\cdot\vec{b}}{c}\left(\vec{p}_2\cdot\frac{\vec{V}_2}{c} - H\right) + O(b\varepsilon^2), \quad (10)$$

where $\vec{p}_M = \vec{\varepsilon}_M - (\vec{\varepsilon}_M\cdot\vec{k})\vec{k}$, $\vec{p}_2 = \vec{\varepsilon}_2 - (\vec{\varepsilon}_2\cdot\vec{k})\vec{k}$. $\vec{\varepsilon}_i = \vec{X}_i/R$ (i=1,2), and $\vec{\varepsilon}_M = (\vec{\varepsilon}_1 + \vec{\varepsilon}_2)/2$. See paper [7] for detail of derivation of the approximation. The right hand side of equation (10) can be used as correction terms to consensus model for adapting it to finite distance radio source. Galactic radio source such as pulsars and maser sources are candidate of such target. Parallax measurement has been traditionally made by mapping source positions on the celestial sphere with data observed in multiple seasons. Then parallax parameter is estimated by fitting their apparent motion with model. This technique has been used used since the era of classical optical measurement. And the same technique is still being used in modern VLBI, in which delay observable is directly available. Equation (10) suggest possibility of parallax parameter estimation by least square technique with directly using delay data-set of multiple seasons. See paper [7] for partial derivative of delay with respect to parallax parameter. The equation (10) gives daily variation with 40 ps amplitude in the case of PSR1937+21 (3.6kpc), for example. This approach may enhance the precision of parallax measurement. Possible difficulty in this approach may be delay resolution to detect the variation and clock discontinuity among VLBI sessions at different epochs.

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