Constrained simultaneous algebraic reconstruction technique (CSART) – a new and simple algorithm for ionospheric tomography

Hobiger T., Kondo T., Koyama Y.

Space-Time Standards Group

Kashima Space Research Center

National Institute of Information and Communications Technology (NICT), Japan

SART

- Simultaneous algebraic reconstruction technique (SART) has been developed for medical applications
- Soon adopted for other (geo) scientific applications
- Easy to implement
- No matrix inversions
- Iterative process
- Several improvements concerning convergence speed (ART derivatives)
- But cells that are not crossed by rays are usually removed from estimation process → CSART

SART (ctd.)

Andersen & Kak (1984)

$$x_j^{(k+1)} = x_j^{(k)} + \frac{\omega}{A_{\oplus,j}} \sum_{i=1}^{M} \frac{A_{i,j}}{A_{i,\oplus}} \left(b_i - \bar{b}_i(x^{(k)})\right)$$

Iterator

this ensures convergence

$$A_{i,j} \geq 0$$

Path-length in cell

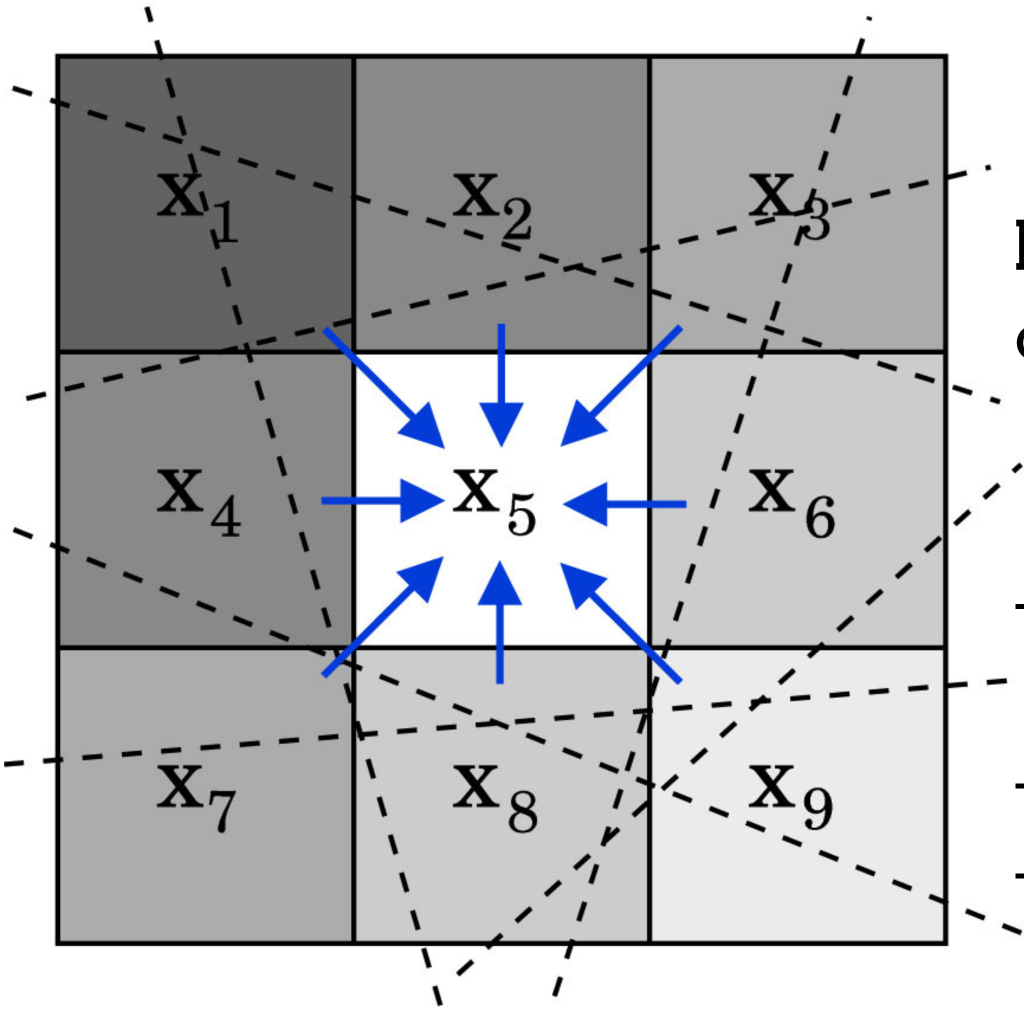
$$A_{i,\oplus} = \sum_{j=1}^{N} A_{i,j} \quad \text{for } i = 1, 2, \ldots, M$$

Cell: Tot. length of all crossing

$$A_{\oplus,j} = \sum_{i=1}^{M} A_{i,j} \quad \text{for } j = 1, 2, \ldots, N$$

Ray: Tot. length

C-SART – Basic idea



Introduction of artificial observations ("constraints")

$$x_\gamma - x_j = 0$$

- information transfer from surrounding cells
- 8 obs/cell (2D)
- 26 obs./cell (3D)

But: needs modification of SART since $A_{i,j} > 0$ is violated !

CSART – mathematical prerequisites

$$x_j^{(k+1)} = x_j^{(k)} + \frac{\omega}{A_{\oplus,j}} \sum_{i=1}^{M} \frac{A_{i,j}}{A_{i,\oplus}} \left(b_i - \bar{b}_i(x^{(k)})\right)$$

Iterator (unchanged)

Censor & Elfving (2002) have shown convergence for

$$A_{i,\oplus} = \sum_{j=1}^{N} |A_{i,j}| \quad \text{for } i = 1, 2, \ldots, M$$

Note: only absolute operator is introduced

$$A_{\oplus,j} = \sum_{i=1}^{M} |A_{i,j}| \quad \text{for } j = 1, 2, \ldots, N$$

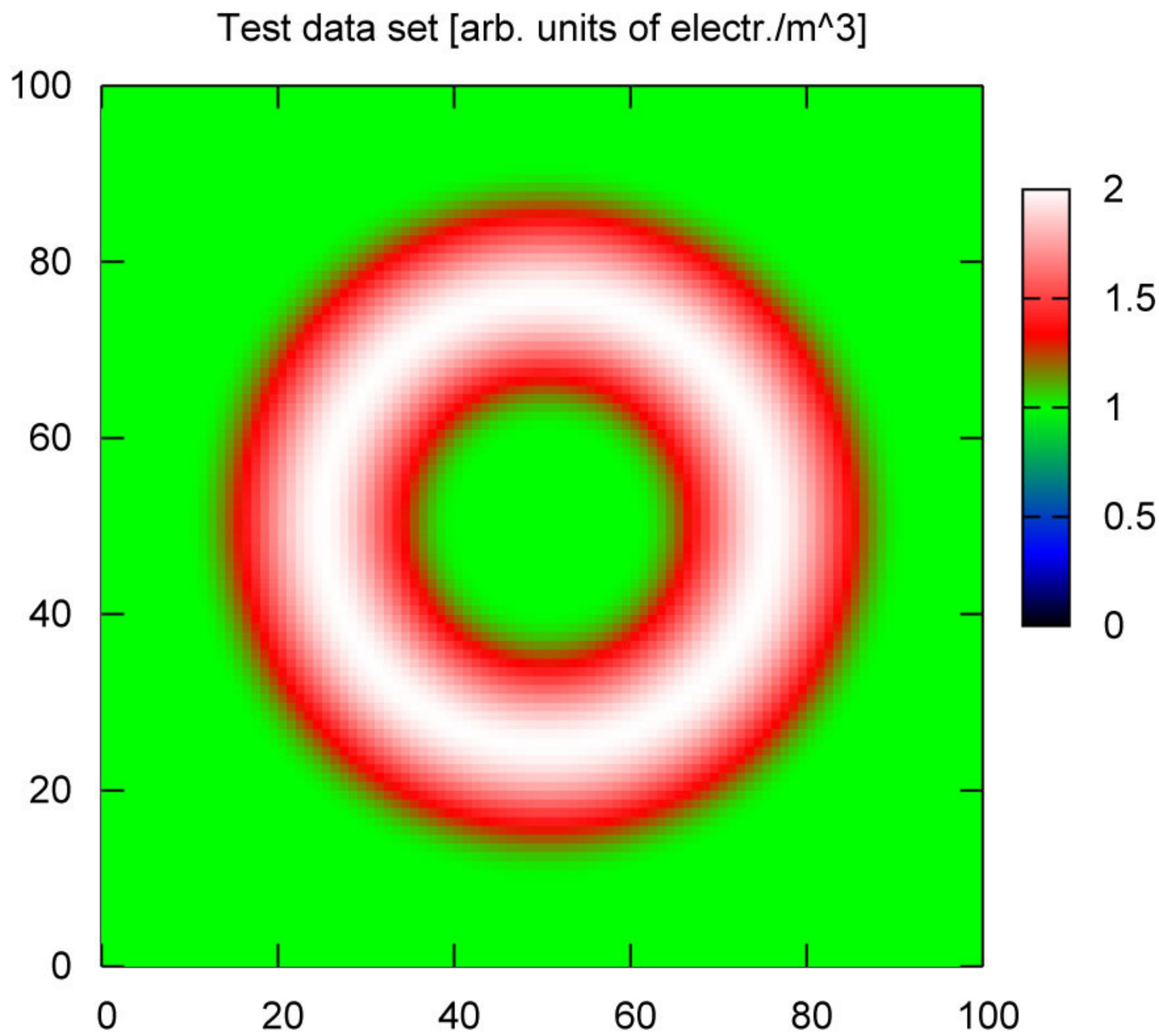
GNSS tomography – test case

- Artificial 2D electron density field “donut”
- 100 x 100 cells
- 14 GNNS ground receiver
- 2 LEOs
- 6 GNSS satellites
- data from 3 epochs
- white noise with SNR 20 added
- DCBs are estimated too
- 288 obs. vs. 100022 unknowns

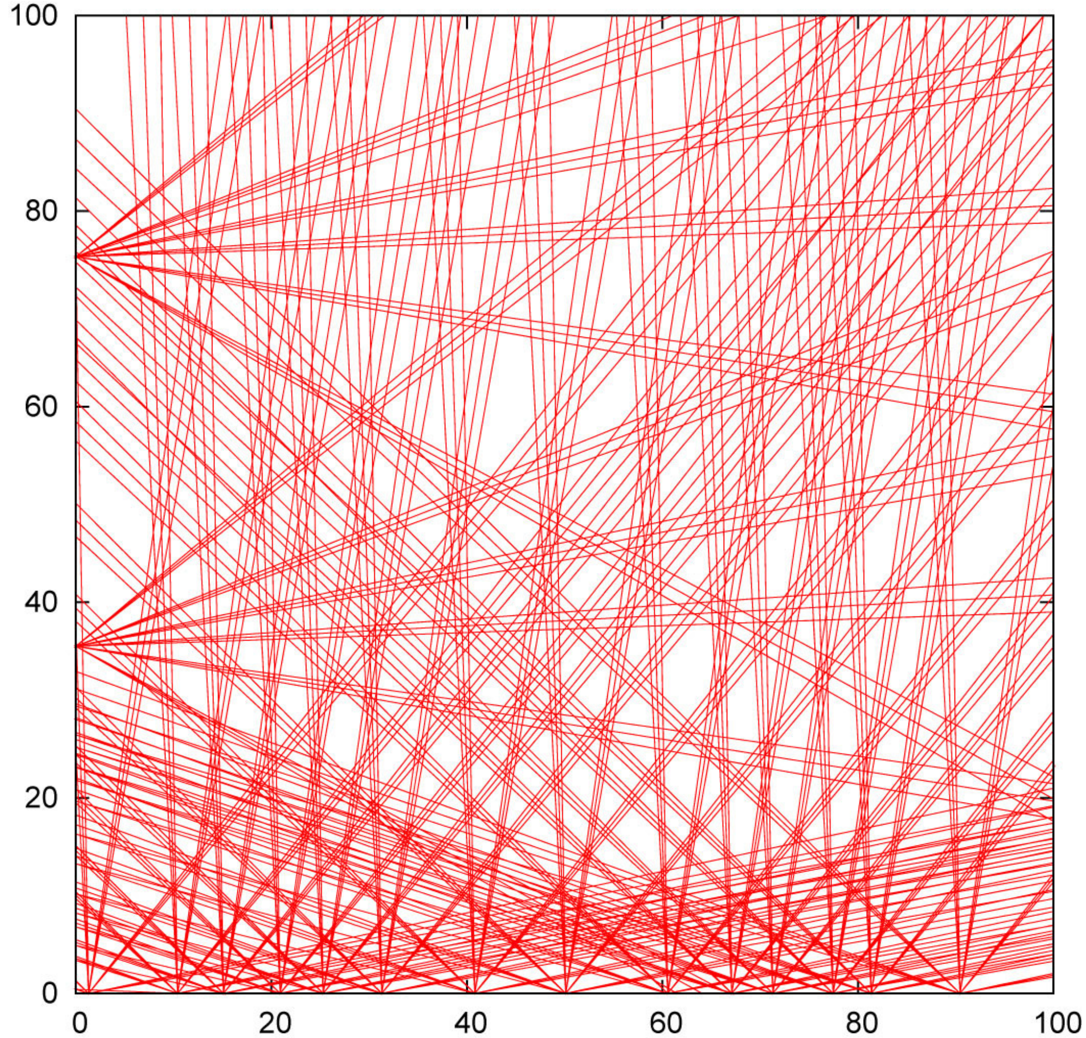
Assumptions:

- Cycle slips corrected
- slant TEC already code leveled
- no ray-paths outside the model

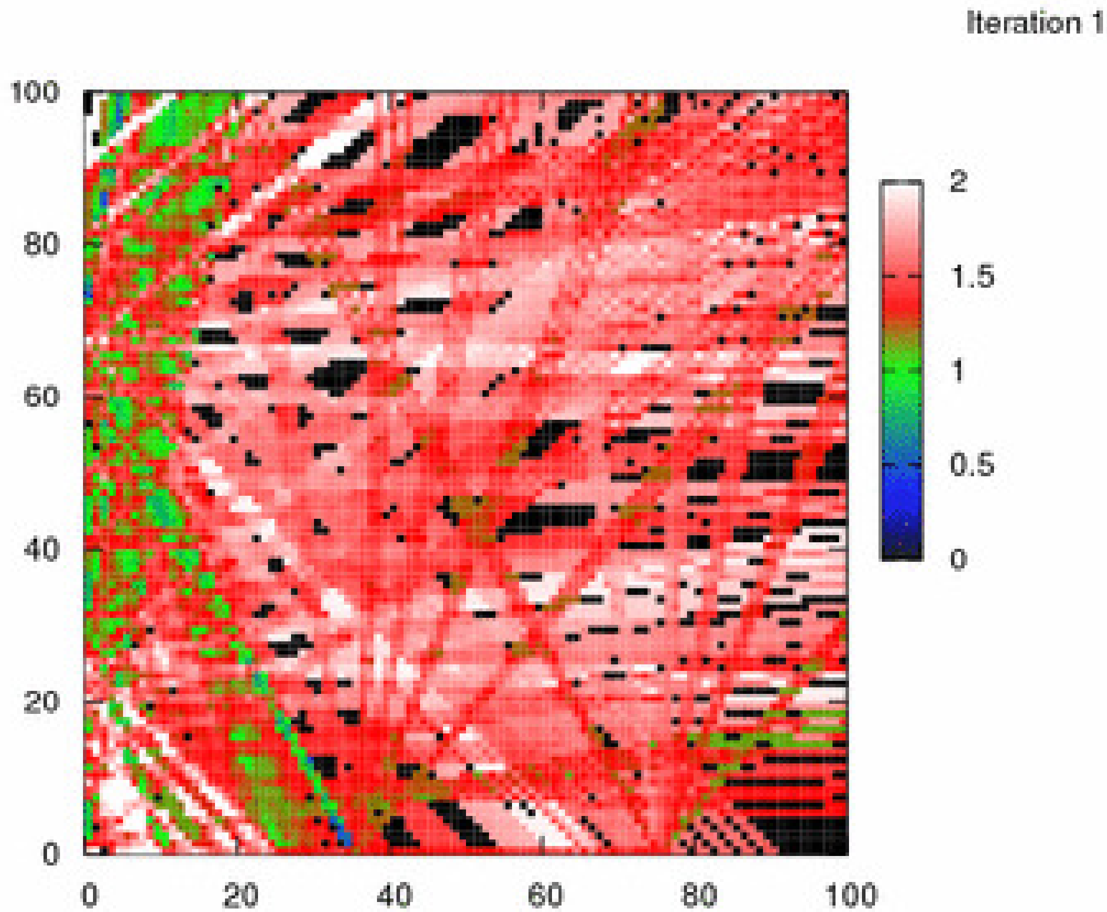
GNSS tomography – test case (ctd.)



GNSS tomography – test case (ctd.)

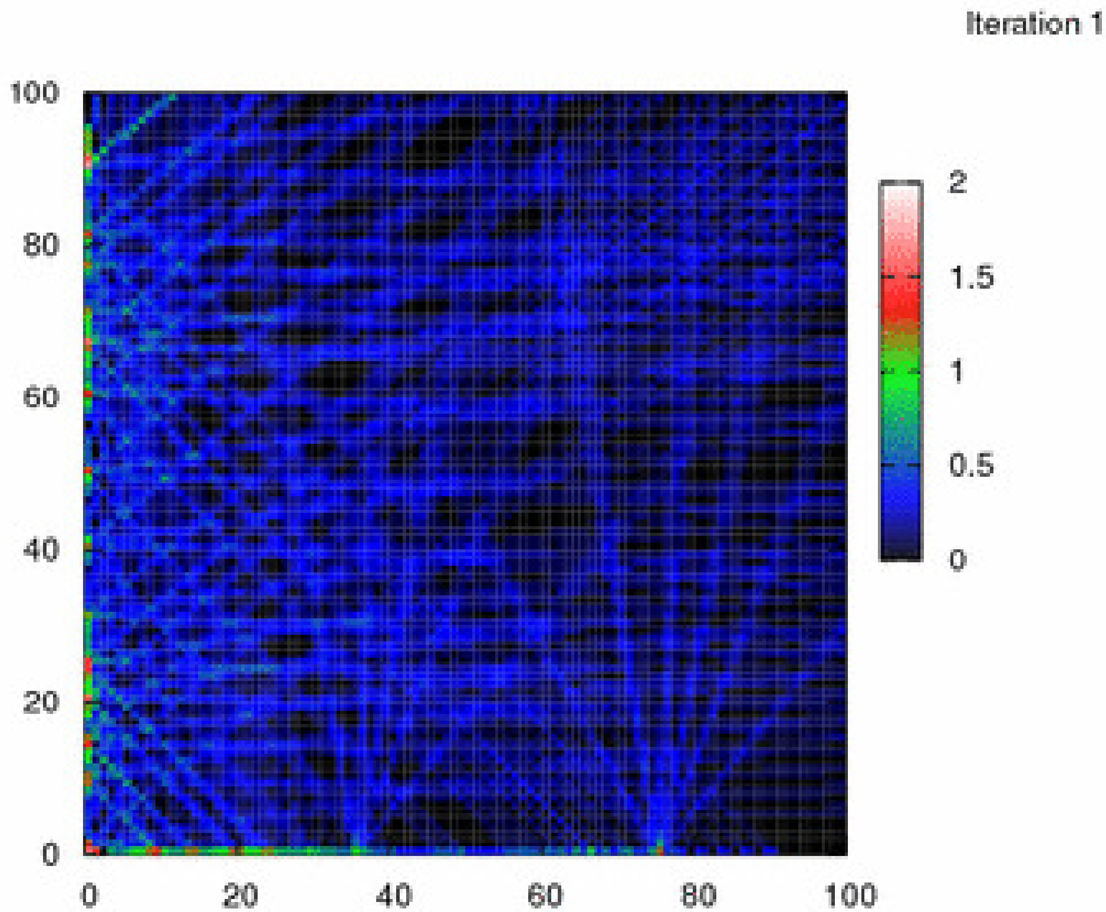


Reconstruction with SART



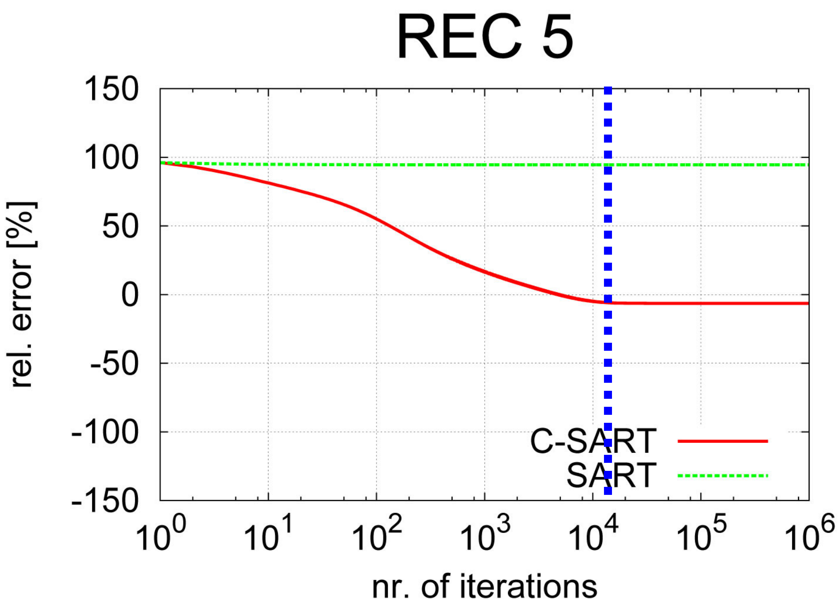
(Note: time scale of animation is logarithmic)

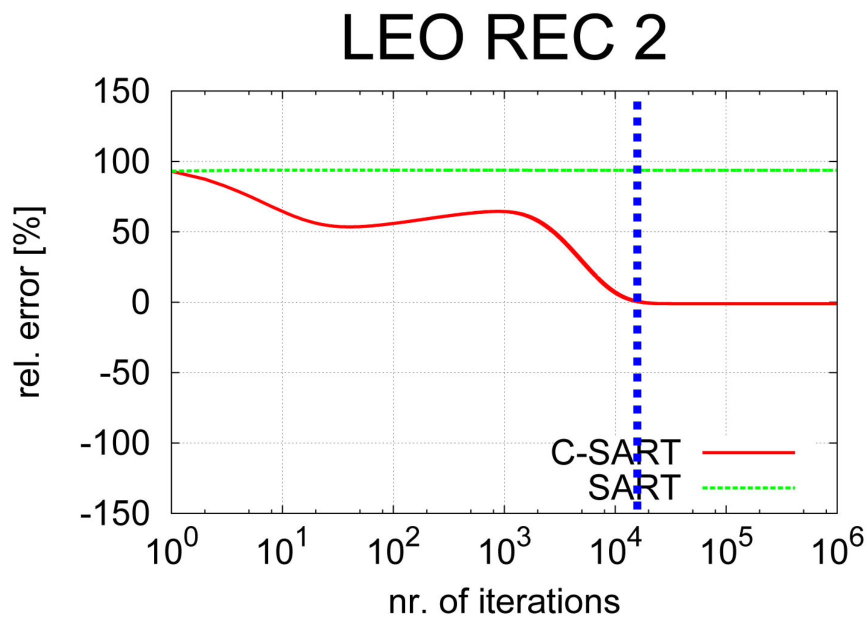
Reconstruction with CSART

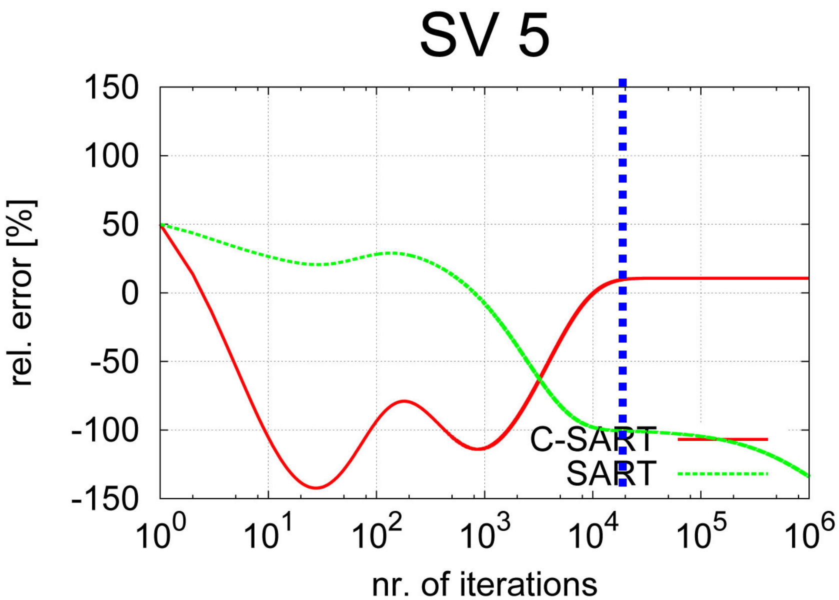


(Note: time scale of animation is logarithmic)

Estimated DCBs – examples



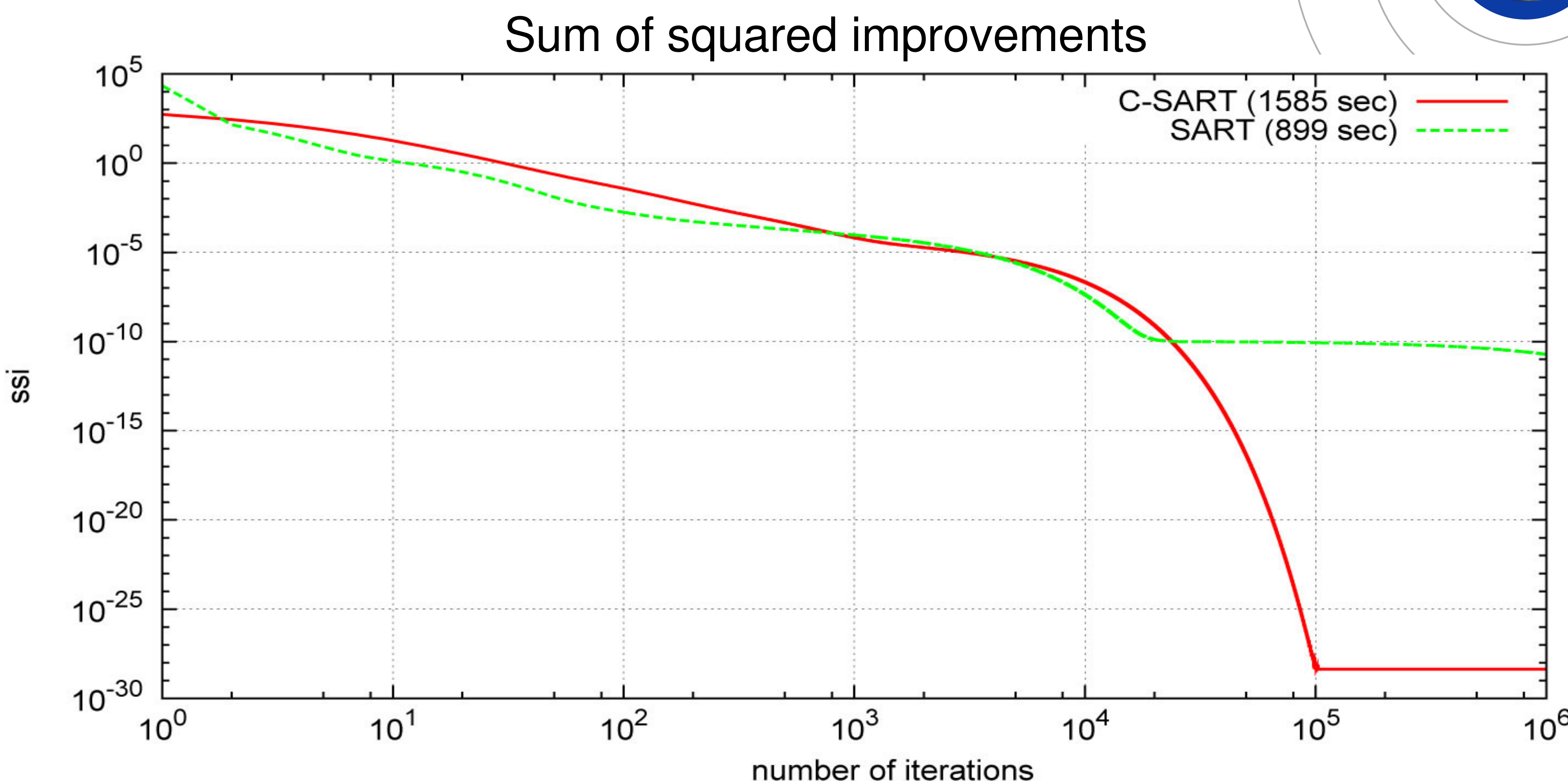




SART – no convergence

CSART – converged after about 10,000 iterations

Convergence



- SART does not converge, even after 1,000,000 iterations
- C–SART converges after about 100,000 iteration (159 sec)

Performance summary

	SART	CSART
Avg. abs. err (el. density)	52 %	13 %
Avg. abs. err. (DCBs)	＞100 %	5 %
Convergence	----	100,000 iter.
Time for 10^6 iter	900 sec	1585 sec
DCB convergence	----	10,000 iter.

Outlook

- Actually routines are coded in C++
- Recoding in ASSEMBLER under progress, expected speedup factor 10-30x
- Support of multi-core processors via OpenMP™
- Online cycle-slip detection and phase-smoothing under development

<u>Thereafter</u>

- Tests using the Japanese GEONET & 3D voxel structure
- Verification of results by ionosonde measurements

Acknowledgements

Thank you for your attention !

立行政法人日本学術振興会

an Society for the Promotion of Science

NiCT Kashima