

Constrained simultaneous algebraic reconstruction technique (CSART) – a new and simple algorithm for ionospheric tomography



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SART



- Simultaneous algebraic reconstruction technique (SART) has been developed for medical applications
- Soon adopted for other (geo) scientific applications
- Easy to implement
- No matrix inversions
- Iterative process
- Several improvements concerning convergence speed (ART derivatives)
- But cells that are not crossed by rays are usually removed from estimation process → **CSART**

SART (ctd.)



Andersen & Kak (1984)

$$x_j^{(k+1)} = x_j^{(k)} + \frac{\omega}{A_{\oplus,j}} \sum_{i=1}^M \frac{A_{i,j}}{A_{i,\oplus}} (b_i - \bar{b}_i(x^{(k)}))$$

Iterator

**this ensures
convergence**

$$A_{i,j} \geq 0$$

Path-length
in cell

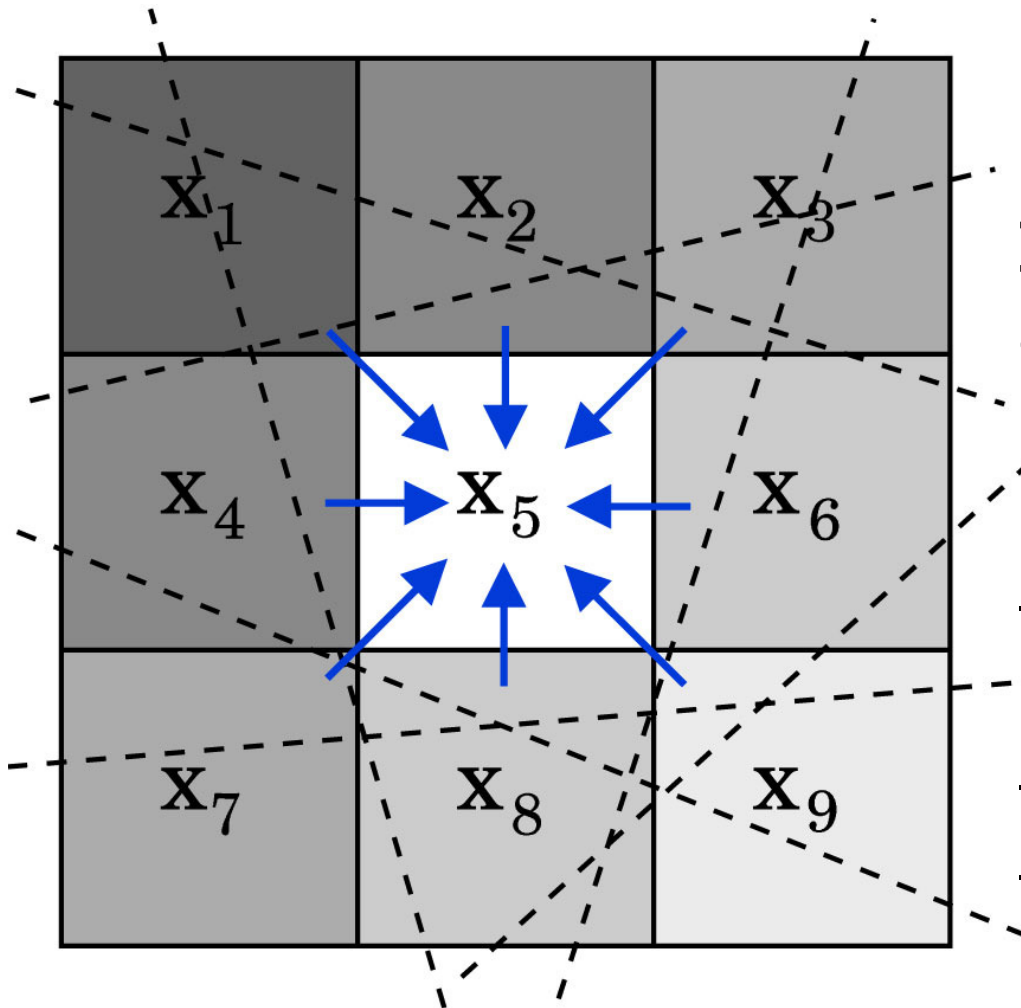
$$A_{i,\oplus} = \sum_{j=1}^N A_{i,j} \quad \text{for } i = 1, 2, \dots, M$$

Cell: Tot. length
of all crossing

$$A_{\oplus,j} = \sum_{i=1}^M A_{i,j} \quad \text{for } j = 1, 2, \dots, N$$

Ray: Tot. length

C-SART – Basic idea



Introduction of artificial observations (“constraints”)

$$x_i - x_j = 0$$

- information transfer from surrounding cells
- 8 obs./cell (2D)
- 26 obs./cell (3D)

But: needs modification of SART since $A_{ij} > 0$ is violated !

CSART – mathematical prerequisites



$$x_j^{(k+1)} = x_j^{(k)} + \frac{\omega}{A_{\oplus,j}} \sum_{i=1}^M \frac{A_{i,j}}{A_{i,\oplus}} (b_i - \bar{b}_i(x^{(k)}))$$

Iterator
(unchanged)

Censor & Elfving (2002) have shown convergence for

$$A_{i,\oplus} = \sum_{j=1}^N |A_{i,j}| \quad \text{for } i = 1, 2, \dots, M$$

Note: only absolute operator is introduced

$$A_{\oplus,j} = \sum_{i=1}^M |A_{i,j}| \quad \text{for } j = 1, 2, \dots, N$$

GNSS tomography – test case



- Artificial 2D electron density field “donut”
- 100 x 100 cells
- 14 GNSS ground receiver
- 2 LEOs
- 6 GNSS satellites
- data from 3 epochs
- white noise with SNR 20 added
- DCBs are estimated too
- 288 obs. vs. 100022 unknowns
- no initial guess, starting from zero values at all cells

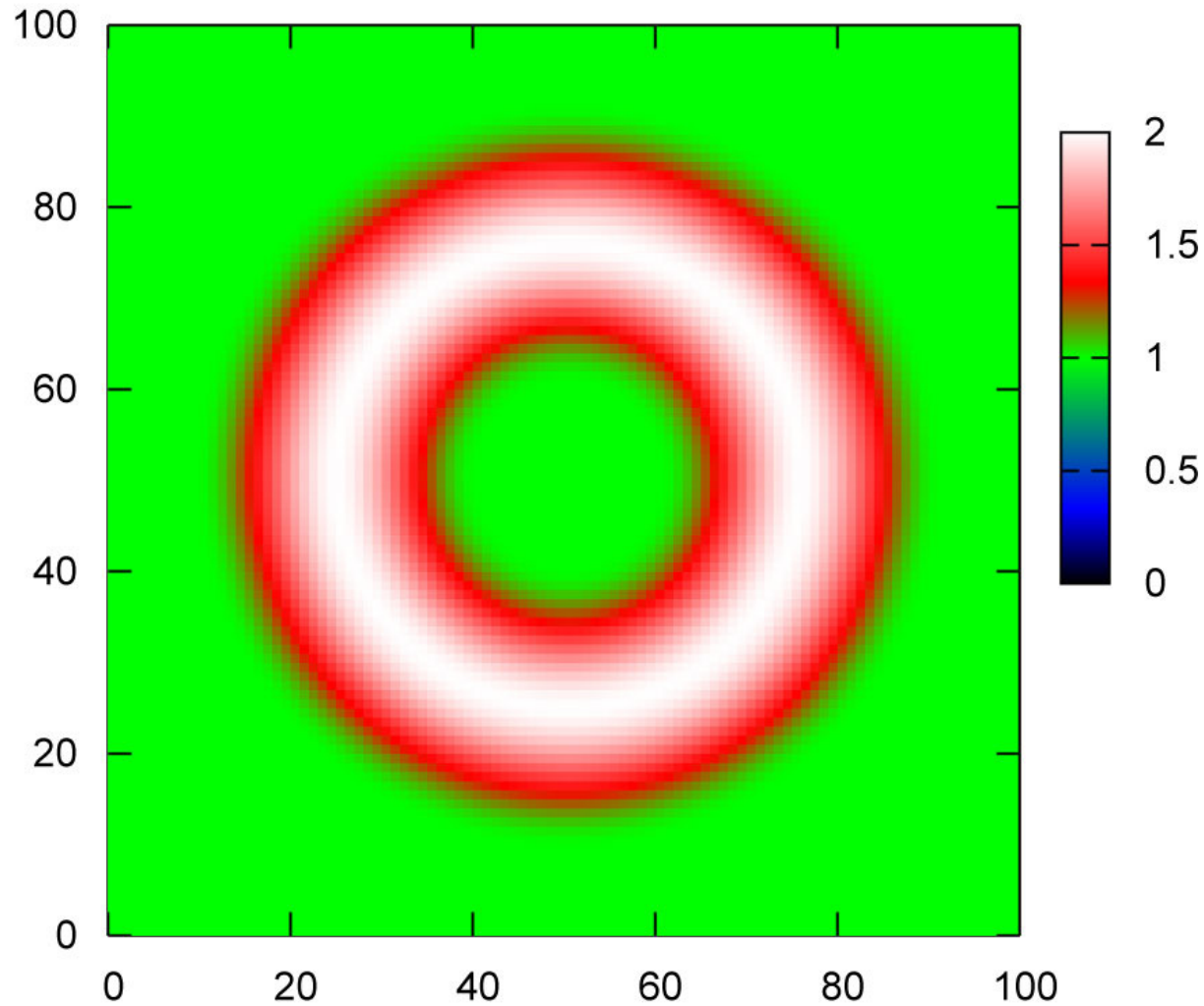
Assumptions:

- Cycle slips corrected
- slant TEC already code leveled
- no ray-paths outside the model

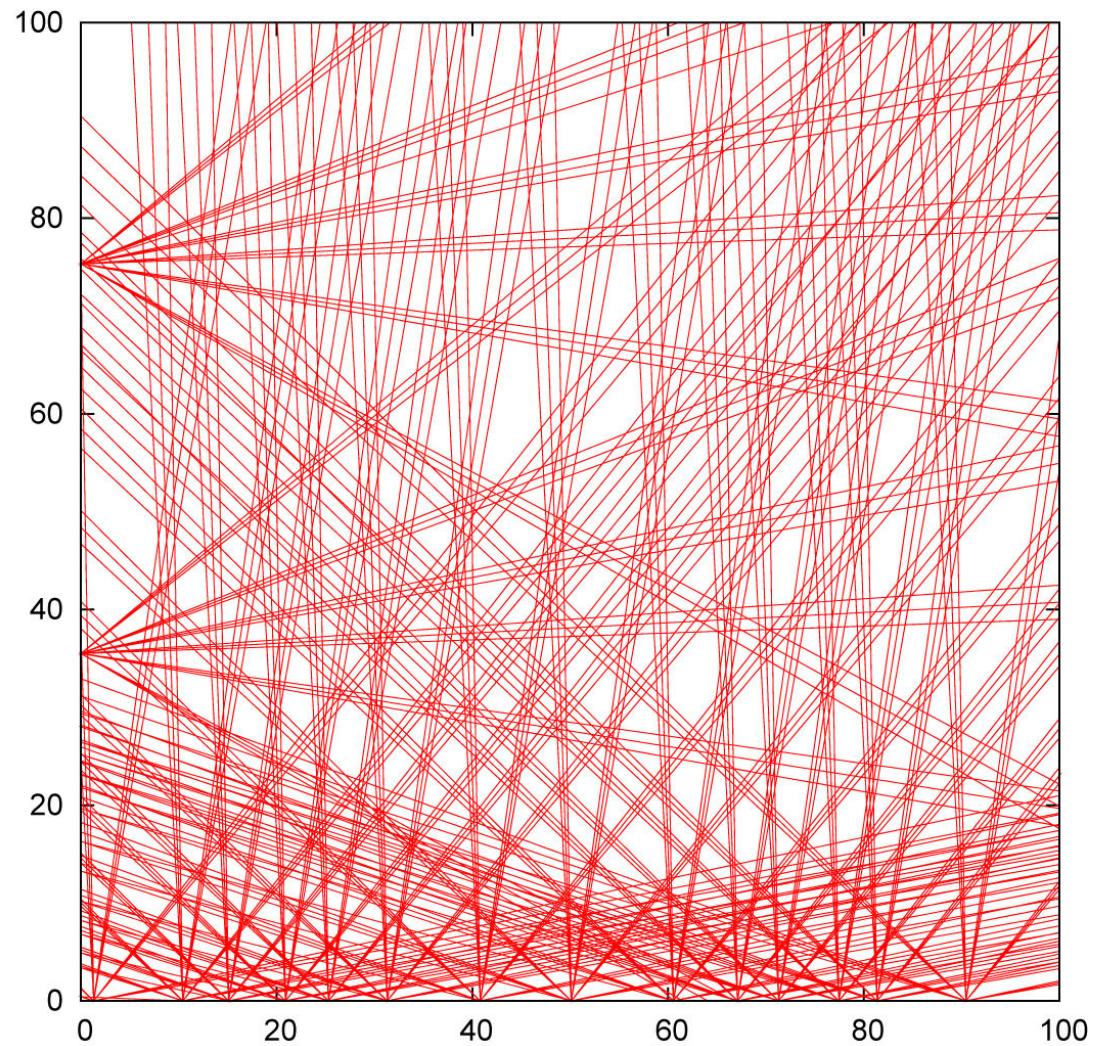
GNSS tomography – test case (ctd.)



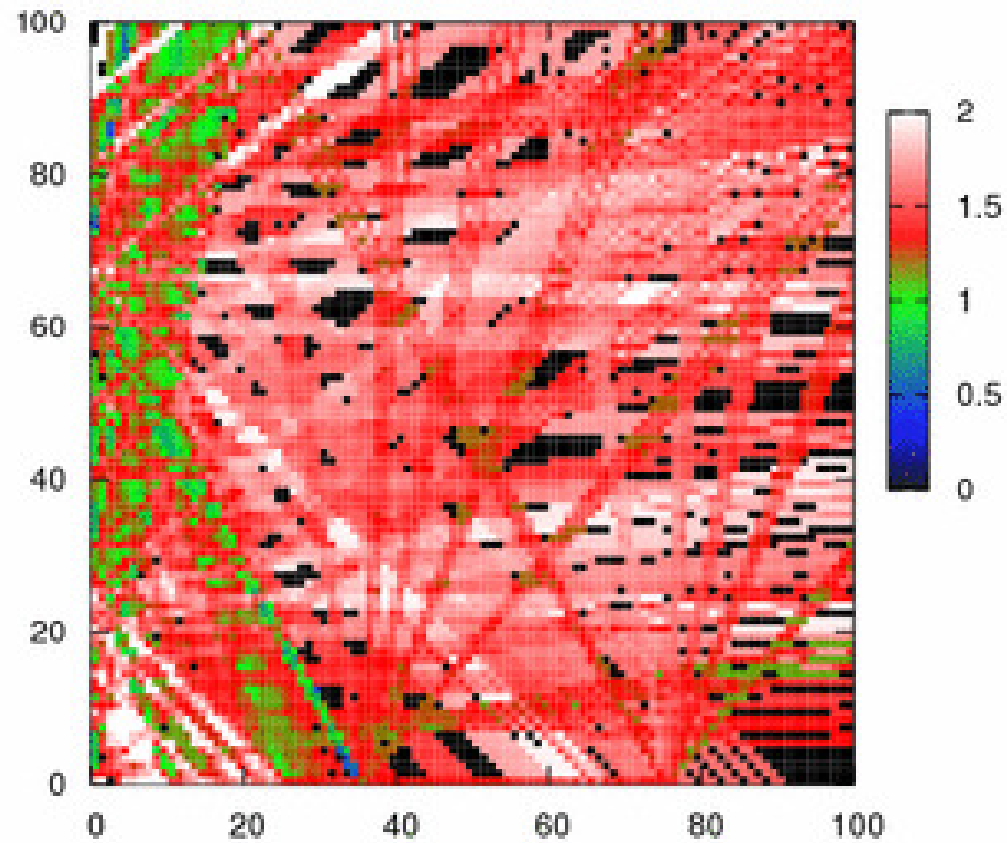
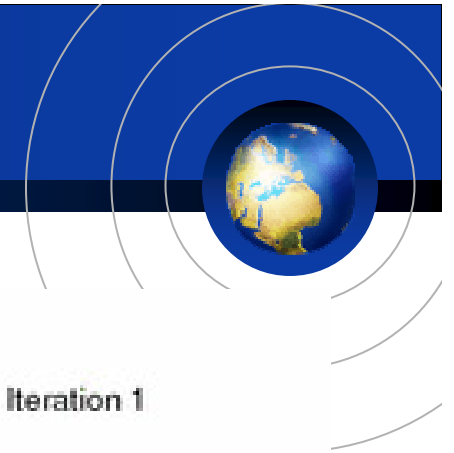
Test data set [arb. units of electr./m³]



GNSS tomography – test case (ctd.)

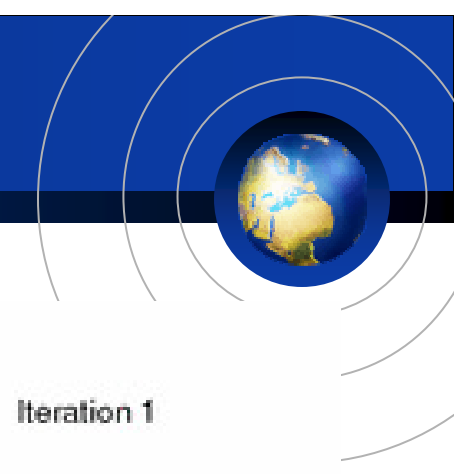


Reconstruction with SART

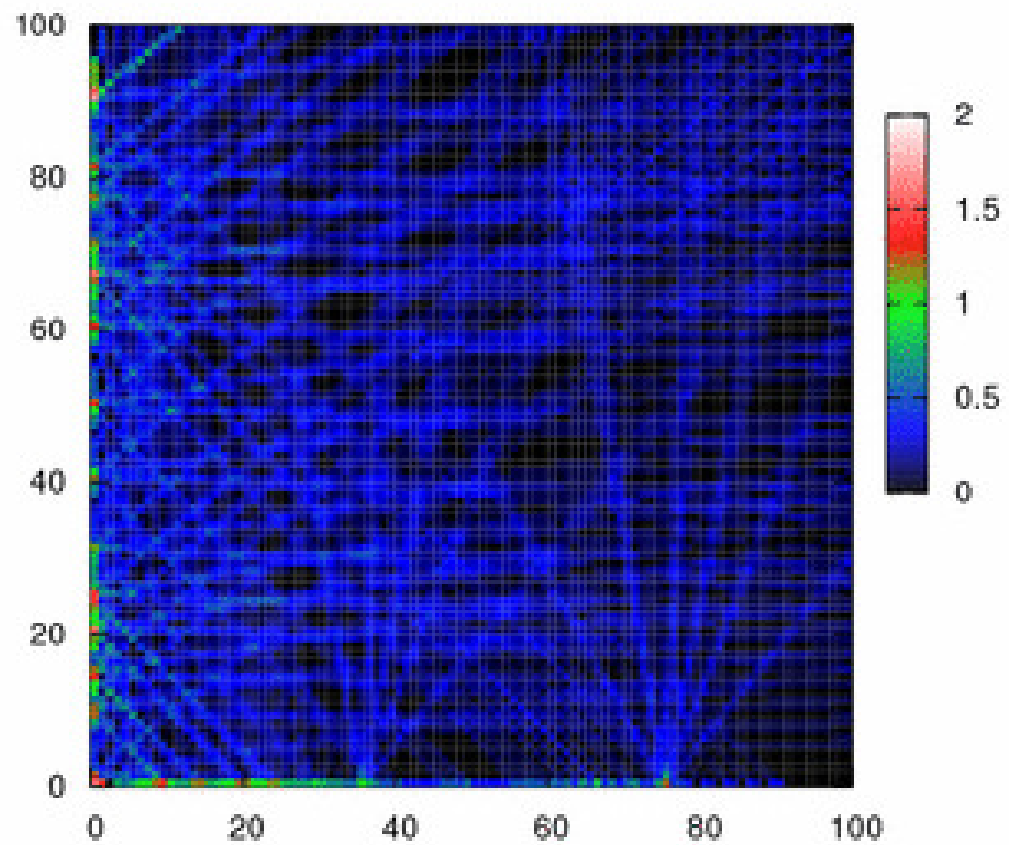


(Note: time scale of animation is logarithmic)

Reconstruction with CSART



Iteration 1

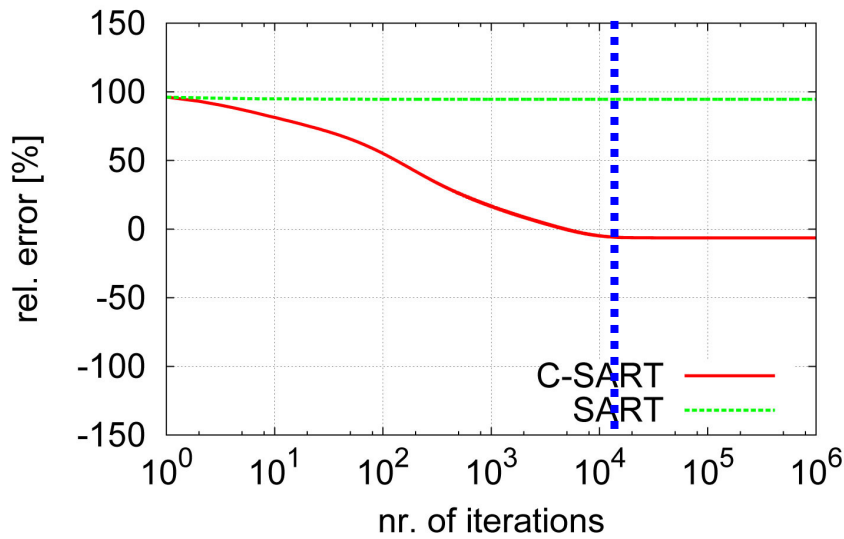


(Note: time scale of animation is logarithmic)

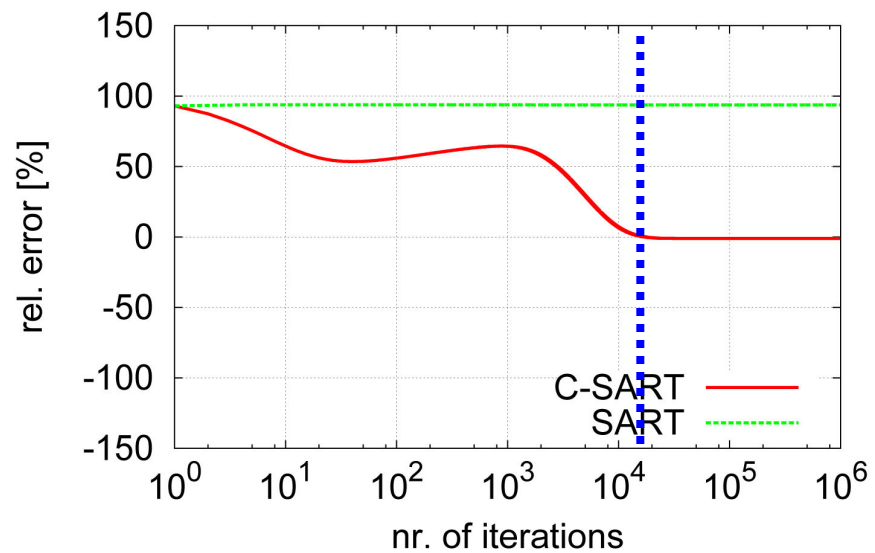
Estimated DCBs – examples



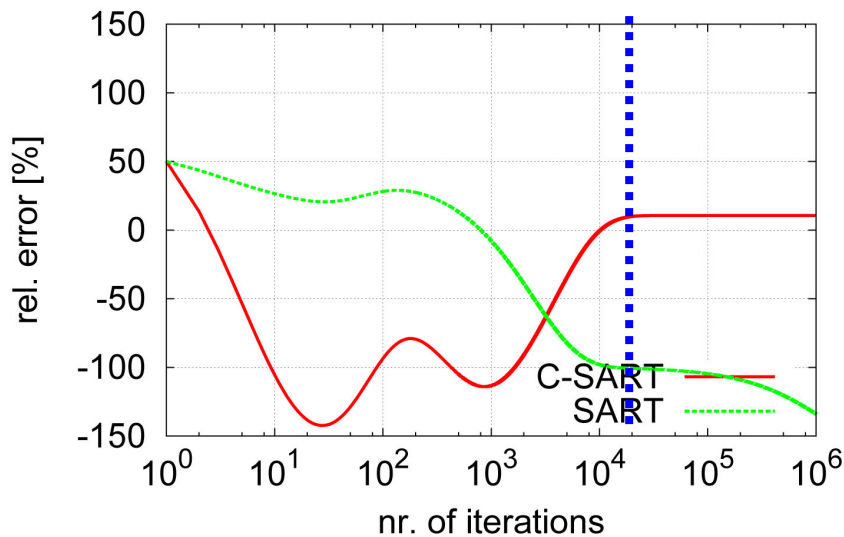
REC 5



LEO REC 2



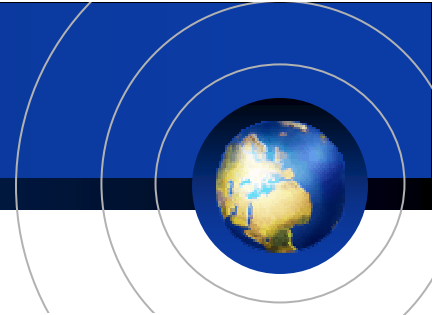
SV 5



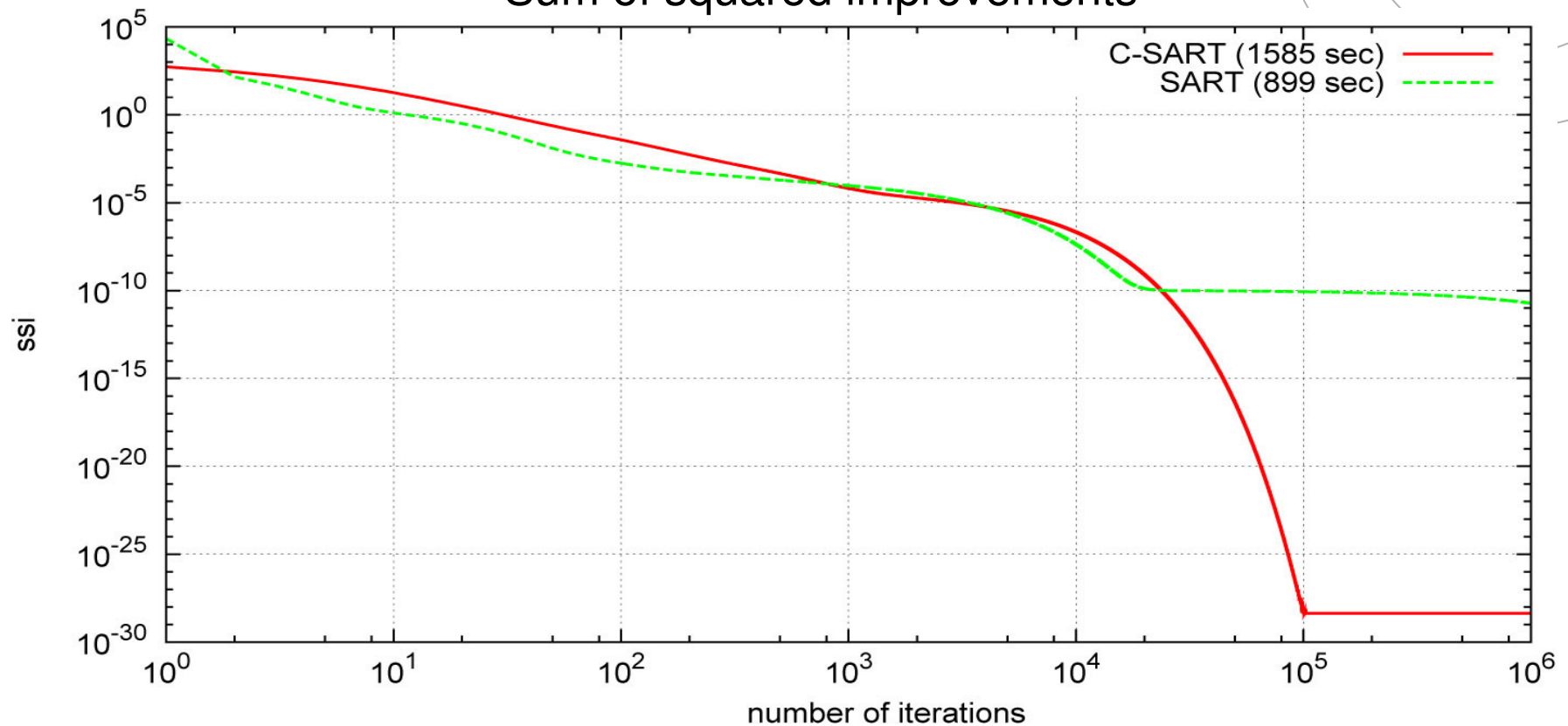
SART – no convergence

C-SART – converged after about 10,000 iterations

Convergence



Sum of squared improvements



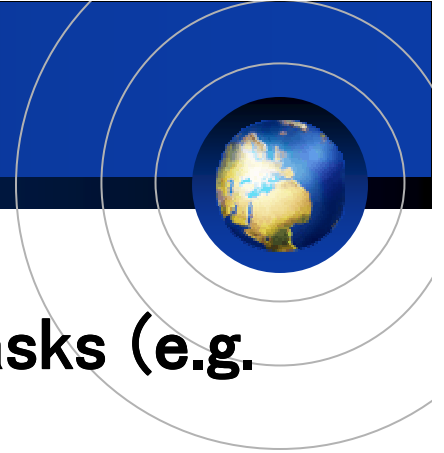
- SART does not converge, even after 1,000,000 iterations
- C-SART converges after about 100,000 iteration (159 sec)

Performance summary



	SART	CSART
Avg. abs. err (el. density)	52 %	13 %
Avg. abs. err. (DCBs)	> 100 %	5 %
Convergence	-----	100,000 iter.
Time for 10⁶ iter	900 sec	1585 sec
DCB convergence	-----	10,000 iter.

Outlook



- CSART can be applied to other scientific tasks (e.g. seismology, boundary value problems, ...)
- The constraint operator can be chosen to consider physical conditions in the form of finite differences
- Good a-priori DCB values are much more important than ionosphere start model
- If the initial DCB are only 10% off in the “donut” example iteration time is reduced to 5 seconds.
- Started working on an expression for the computation of a “standard deviation equivalent” in order to judge the results by their “formal error”

Outlook (ctd.)



- Actually routines are coded in C++
- Recoding in ASSEMBLER under progress, expected speedup factor 10–30x
- Support of multi-core processors via OpenMP™
- Online cycle-slip detection and phase-smoothing under development

Thereafter

- Tests using the Japanese GEONET & 3D voxel structure
- Verification of results by ionosonde measurements

Acknowledgements



Thank you for your attention !



行政法人 日本学術振興会
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NiCT  **Kashima**