Integer least-squares adjustment for VLBI

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Introduction

- Classical least-squares adjustments does not distinguish between float and integer numbers $\rightarrow$ loss of useful information
- Many applications need to estimate integer unknowns (e.g. ambiguities)
- Teunissen (1996) presented the Least-squares Ambiguity De-correlation Algorithm (LAMBDA) for GPS applications
- LAMBDA solves a mixed (floats+integer) least-squares problem under consideration of variance-covariance information $\rightarrow$ “most likely solution” (L2 minimization criteria)
- Until now several modifications of LAMBDA which speed up the algorithm

Problem statement

$$o - c = \underline{Bb} + \underline{Aa} + \underline{\varepsilon}$$

$$\boxed{b \in R^p \quad \dim(B) = m \times p}$$

$$\boxed{a \in Z^n \quad \dim(A) = m \times n}$$

measurement noise

Estimation criteria: $$\min \left\| (o - c) - Bb - Aa \right\|^2_{Q^{-1}_{obs}}$$

LAMBDA – Step 1: “Float solution”

$$\underbrace{\begin{pmatrix} B^T Q^{-1} B & B^T Q^{-1} A \\ A^T Q^{-1} B & A^T Q^{-1} A \end{pmatrix}}_{N} \cdot \underbrace{\begin{pmatrix} b \\ a \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} B^T Q^{-1}(o-c) \\ A^T Q^{-1}(o-c) \end{pmatrix}}_{r}$$

“Float solution”, identical to classical least-squares

$$\begin{pmatrix} \hat{b} \\ \hat{a} \end{pmatrix} = N^{-1} r$$

$$N^{-1} = \begin{pmatrix} Q_{\hat{b}} & Q_{\hat{b}\hat{a}} \\ Q_{\hat{a}\hat{b}} & \boxed{Q_{\hat{a}}} \end{pmatrix}$$

Variance-covariance information of the integer unknowns

LAMBDA – Step 2: Integer ambiguity estimation

"Z-transformation" for **de-correlation**

$$\hat{z} = Z^T \hat{a} \qquad Q_{\hat{z}} = Z^T Q_{\hat{a}} Z$$

Search over all grid points inside the **hyper-ellipsoid** to find most likely integer values of z

$$\left(\hat{z} - z\right)^T Q_{\hat{z}}^{-1} \left(\hat{z} - z\right) \leq \chi^2$$

"Inverse Z-transformation" yields **integer unknowns**

$$a = \left(Z^T\right)^{-1} z$$

LAMBDA – Step 3: “Fixed solution”

Correction of the float estimates

$$b = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} \left(\hat{a} - a \right)$$

Thus: obtained estimates represents the **least-squares** solution under the consideration that the **vector a belongs to the integer vector space**.

Application to VLBI

- Next generation VLBI systems currently under development, coordinated by IVS (VLBI2010)
- Current specification include
  - 4 broad-bands (1GHz BW)
  - center freqs. @ 2.5, 5.25, 7.0 and 11.25 GHz
  - calibration of instrumental delays
  - broad-band delay + phase / band are expected to act as observables for geodetic analysis
- Overall accuracy goal: 1mm (3D station position error) <u>→ can only be achieved when phase delay is utilized</u>

VLBI 2010 observables

$$\tau^{*}_{i,j}(t_k) = \boxed{\tau_i(t_k)} + \boxed{\frac{E_i(t_k)}{f_j^{2}}} + \boxed{\frac{\partial\tau_i(t_k)}{\partial\alpha}\Delta\alpha + \frac{\partial\tau_i(t_k)}{\partial\delta}\Delta\delta} + \tau^{\varepsilon}_{i,j}(t_k)$$

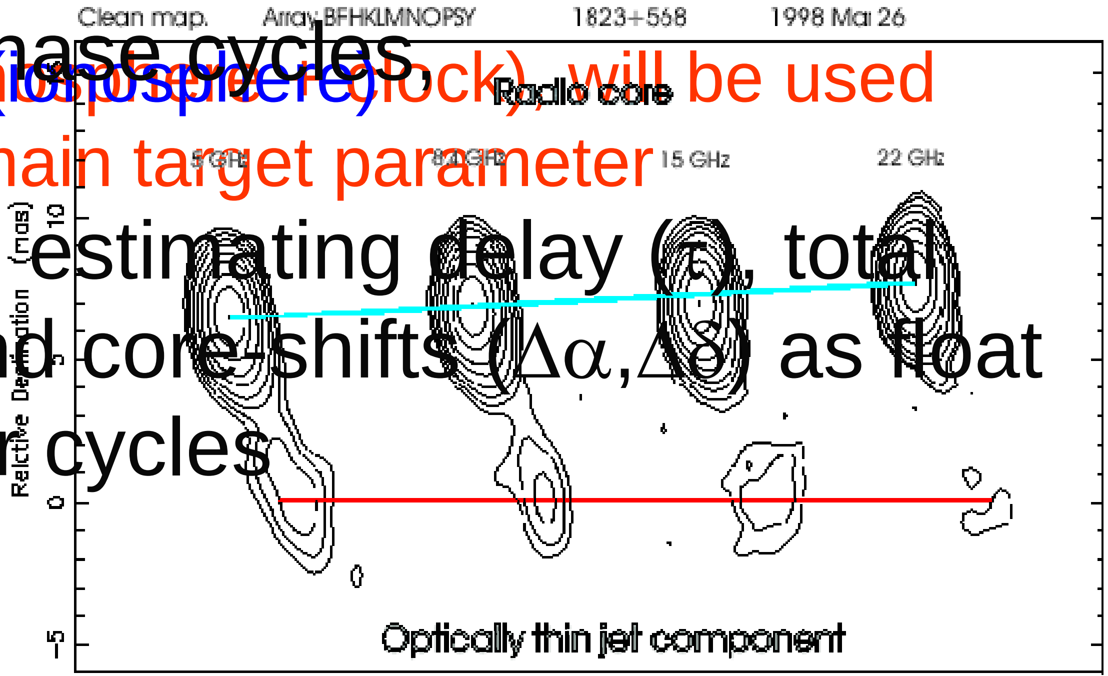
$$\phi^{*}_{i,j}(t_k) = \boxed{f\tau_i(t_k)} - \boxed{\frac{E_i(t_k)}{f_j}} + \boxed{f\frac{\partial\tau_i(t_k)}{\partial\alpha}\Delta\alpha + f\frac{\partial\tau_i(t_k)}{\partial\delta}\Delta\delta} + \boxed{N_{i,j}(t_k)} + \phi^{\varepsilon}_{i,j}(t_k)$$

Number of unknown phase cycles, $N=2\pi M,\ M\in\mathbb{Z}$
→ LAMBDA algorithm, estimating delay (τ), total electron content (E) and core-shifts (Δα,Δδ) as float unknowns AND integer cycles

Delay (geometric + atmosphere + clock), will be used for geodetic analysis, main target parameter

Dispersive delay (ionosphere)

Frequency dependent radio core Position (“core shift”), partial derivatives can be computed a-priori from source maps



Solution strategy

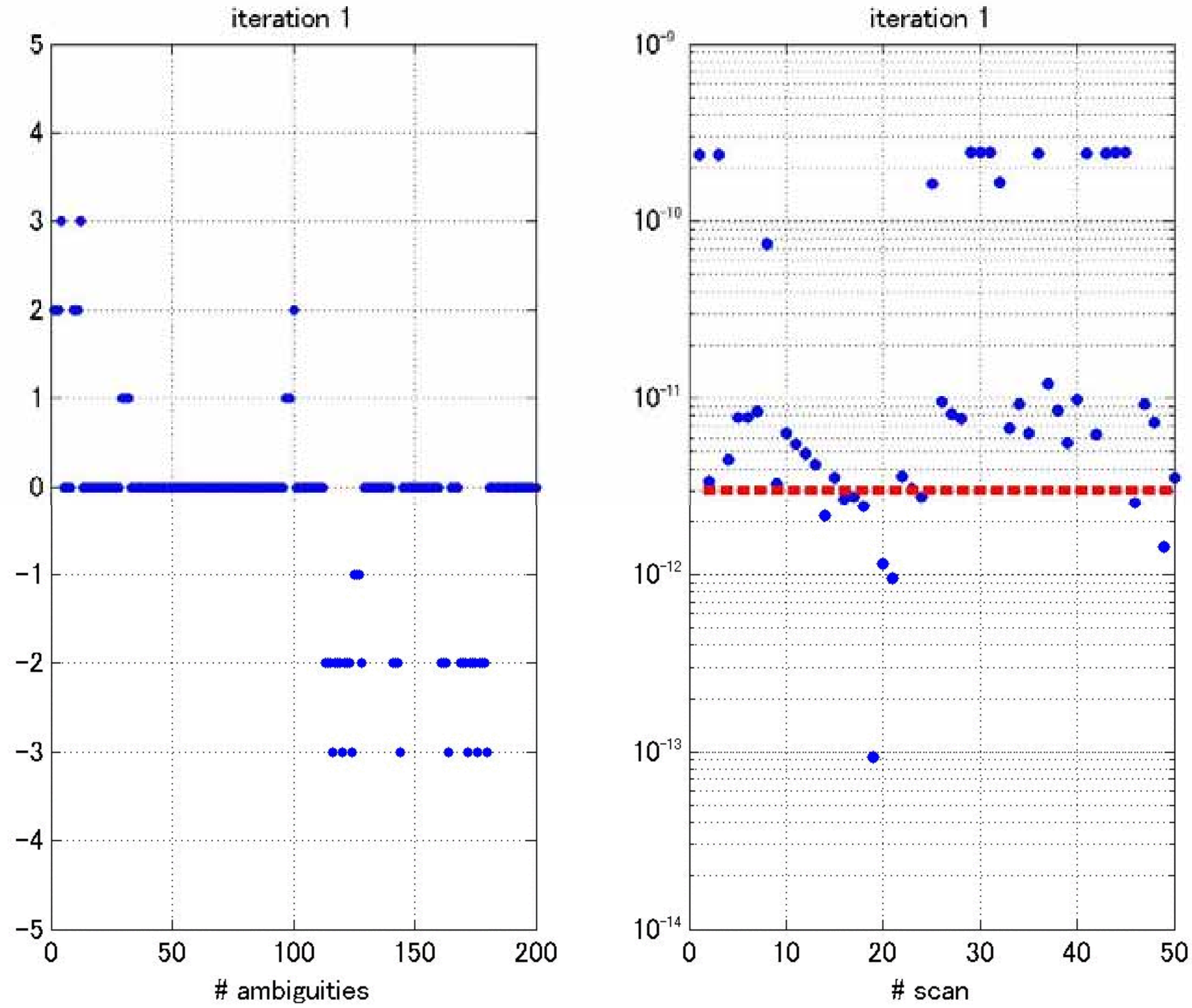
- Because core-shifts are highly correlated with delay and ionosphere an iterative strategy is needed

1. Initial guess of core-shift values from classical LSQ (inversion via SVD) using delays only
2. Correct obs. for core-shift effects → makes scans independent from each other
3. Estimate delay, ionosphere and ambiguities for each scan with LAMBDA
4. Compute improvement of core-shift values using the residuals of all scans

Simulations

- VLBI2010 specs
  - 4 broad-bands (1GHz BW)
  - center freqs. @ 2.5, 5.25, 7.0 and 11.25 GHz
- Picked one source from 16 station schedule (233 scans)
- Source structure effects ($\partial\tau/\partial\alpha$, $\partial\tau/\partial\delta$) computed by P.Charlot (Obs. Bordeaux)
- Ionospheric contribution from global maps
- SNR/band $\rightarrow$ random noise for each scan

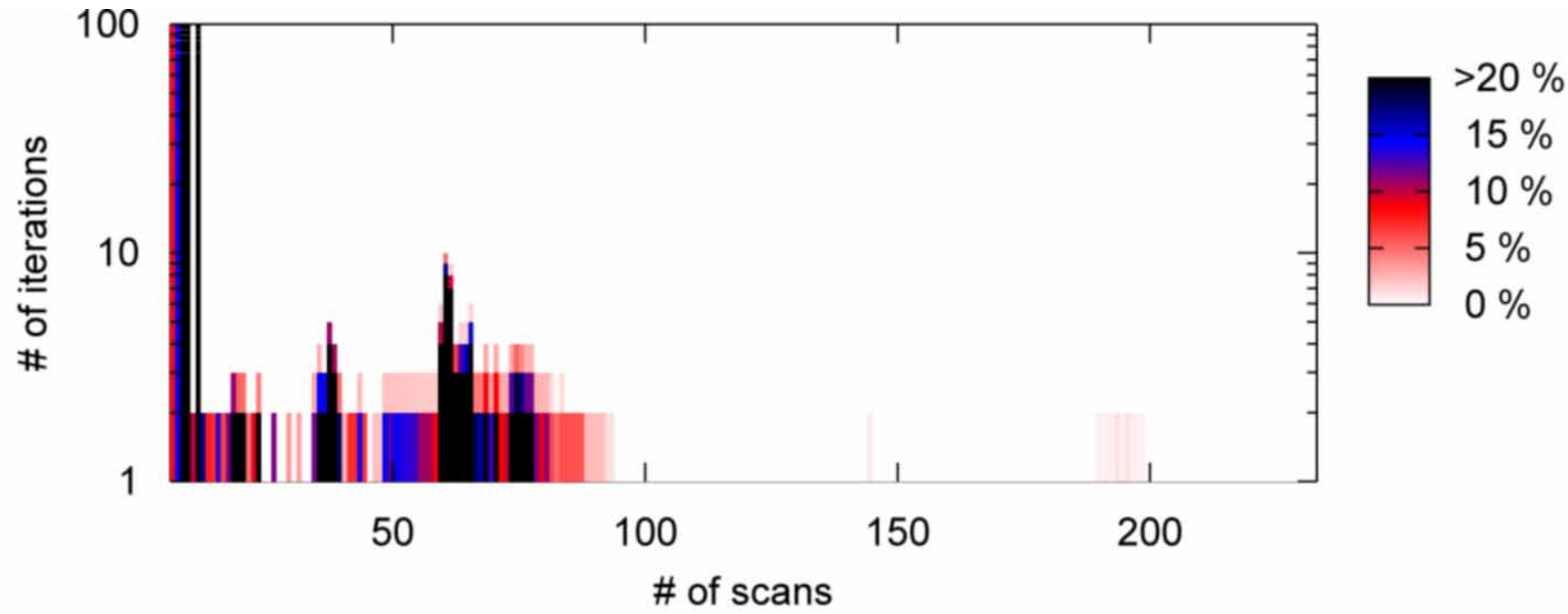
Performance (subset of scans)



Settings
- 50 scans
- one source
- SNR/band=14
- 4 bands
- 15 iterations

Performance (ctd.)

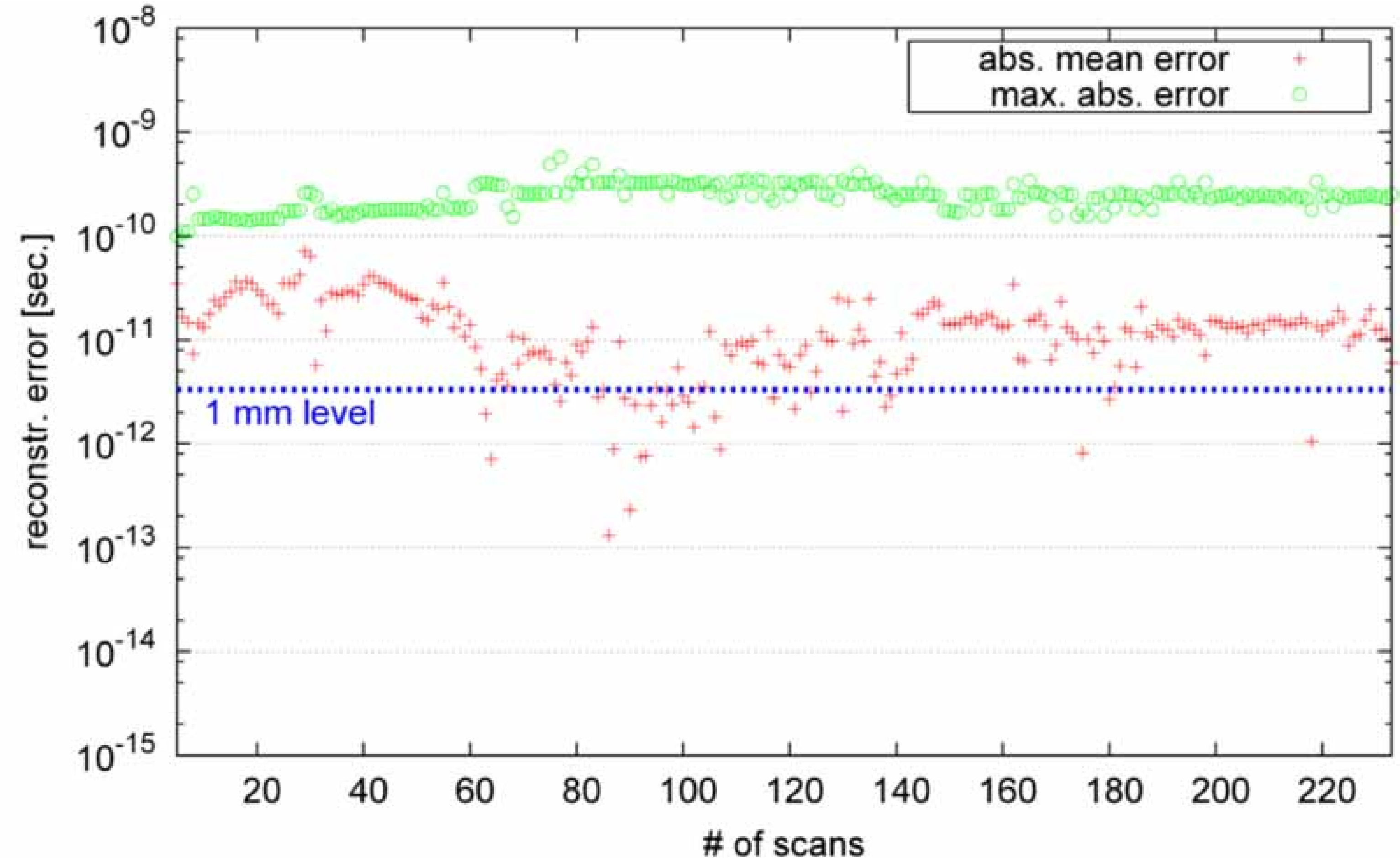
Percentage of unresolved ambiguities



(SNR/band=14)

Comparison to an integer rounding strategy

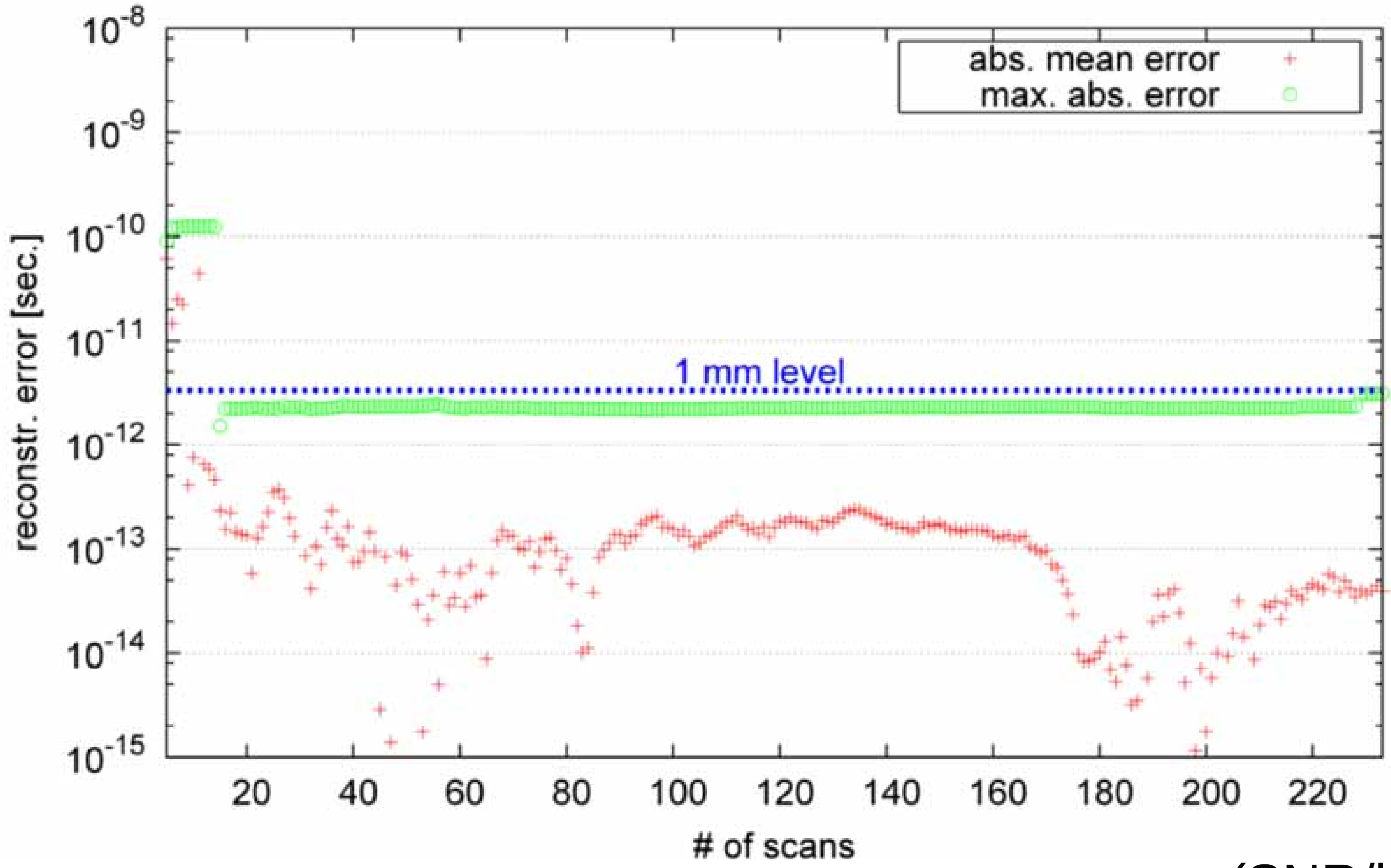
Error of reconstructed delay after 100 iterations



(SNR/band=14)

Performance (ctd.)

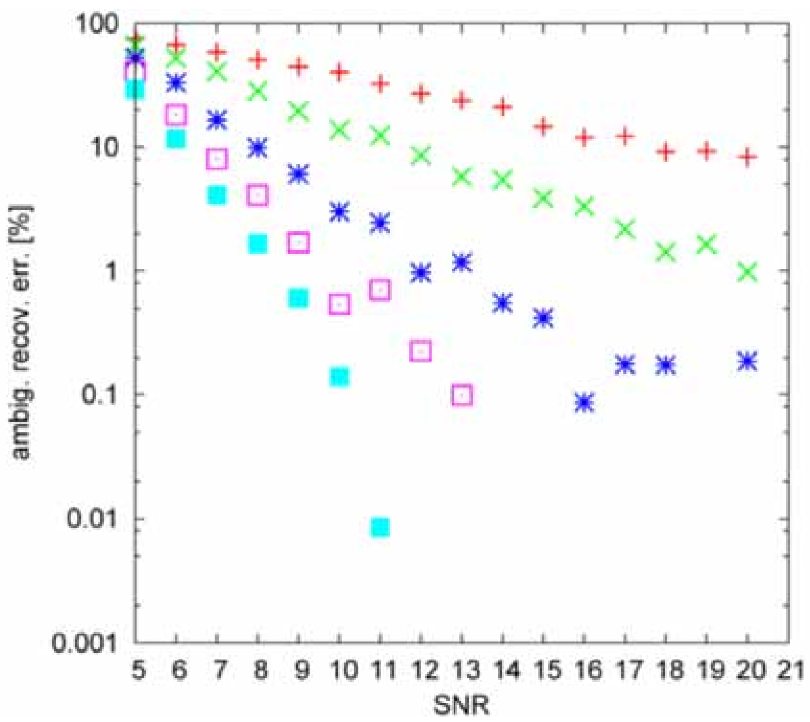
Error of reconstructed delay after 100 iterations



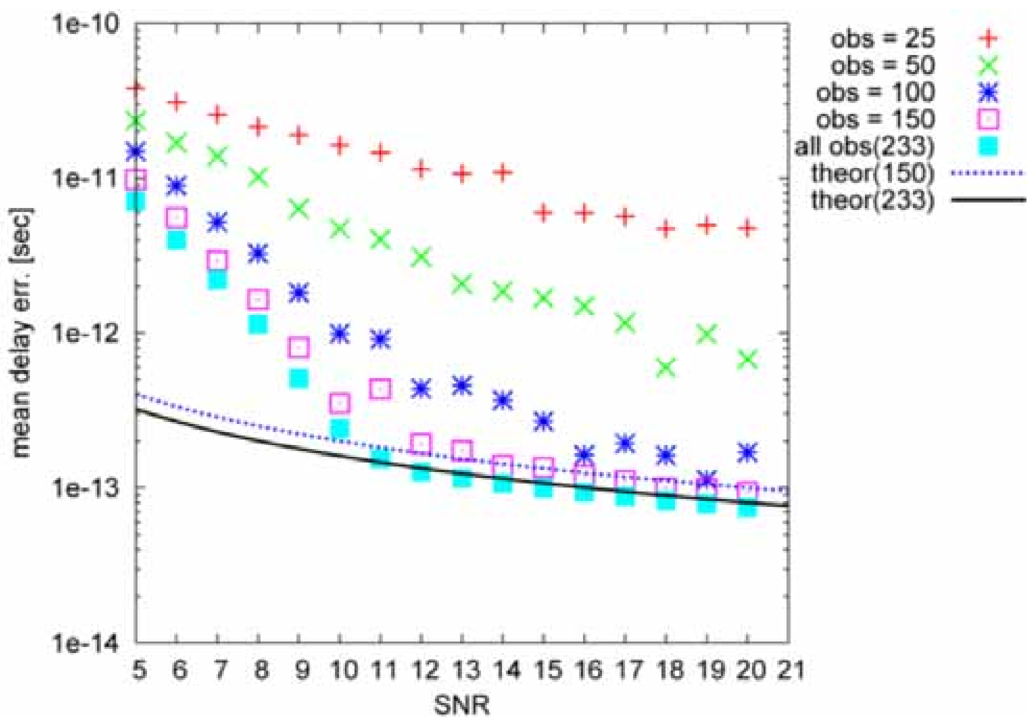
(SNR/band=14)

Performance for different SNR

% of unres. ambig.



mean abs. delay error



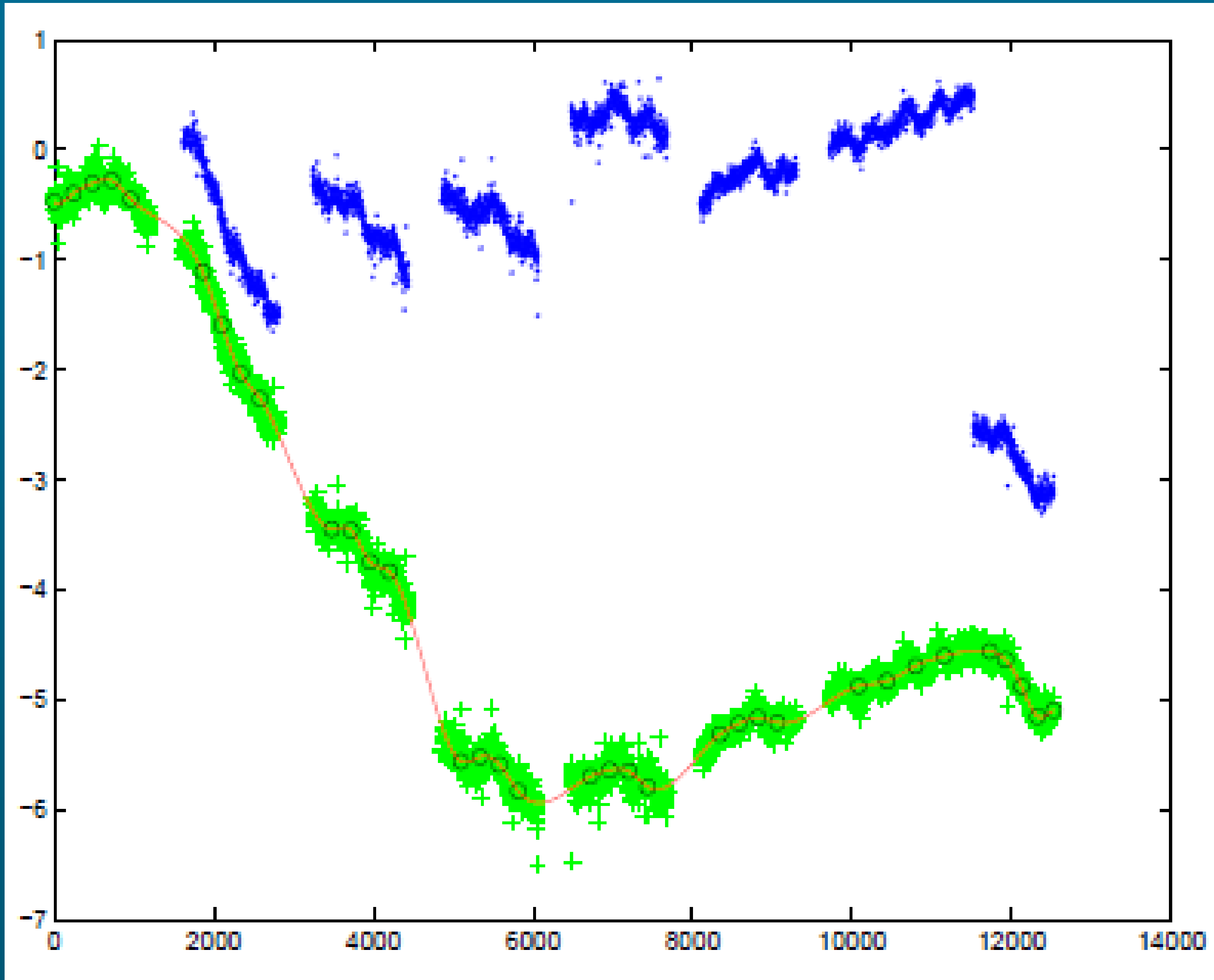
Another possible VLBI application:

Connection of fringe phases from consecutive scans

Idea

- Model the time-variation of the fringe phase by a continuous spline functions (→ smoothest connection)
- Estimate phase jumps as integer together with spline parameters
- Apply closure conditions in multi-baseline experiments
- Solve for all unknowns in one consistent adjustment step

Example with real data



Conclusions

- Integer least-squares adjustment allows to consider the number space each unknown belongs properly → straightforward approach
- Easy to implement, just two more steps after classical least-squares estimation
- Clear benefit for estimation of VLBI ambiguities → lowers SNR requirements
- Useful for other applications (e.g. connection of fringe phases)
- BUT: Computation time increases exponentially by the number of integer unknowns, more than one hour on standard PC for n>100

Thank you for your attention !