P4.12

Accuracy of Wind Fields in Convective and Stratiform Echoes Observed by a Bistatic Doppler Radar Network

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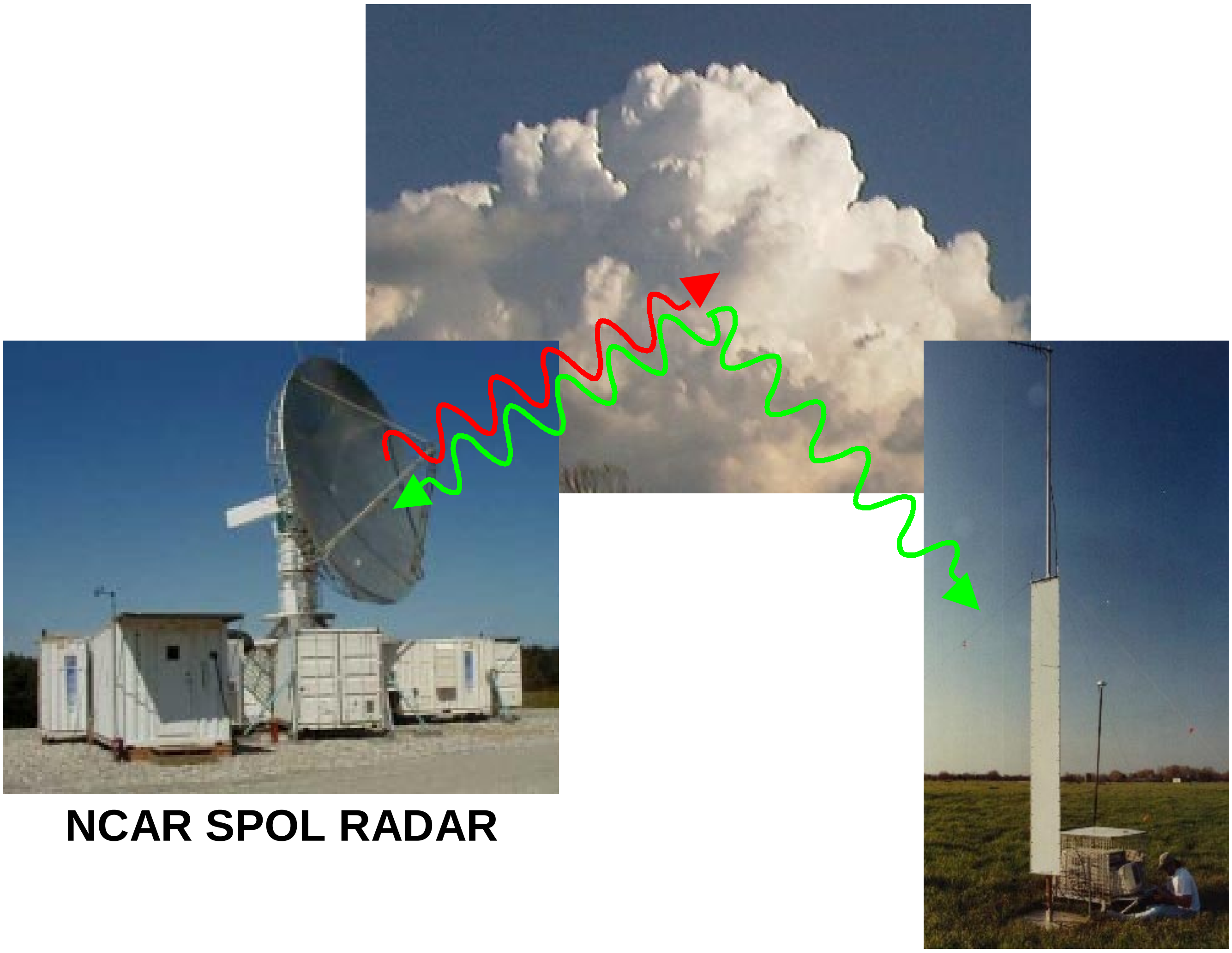


NCAR SPOL RADAR

BISTATIC ANTENNA

Problems

The mathematical accuracy of the bistatic network has not been yet examined. Most of the previous studies dealt with small area analysis.

Objectives

The accuracy of wind fields derived from a bistatic Doppler network is investigated based on a mathematical examination, and is evaluated using actual observation data. Also, elimination of sidelobe contamination and a practical composite method for bistatic multiple-Doppler radar network will be discussed.

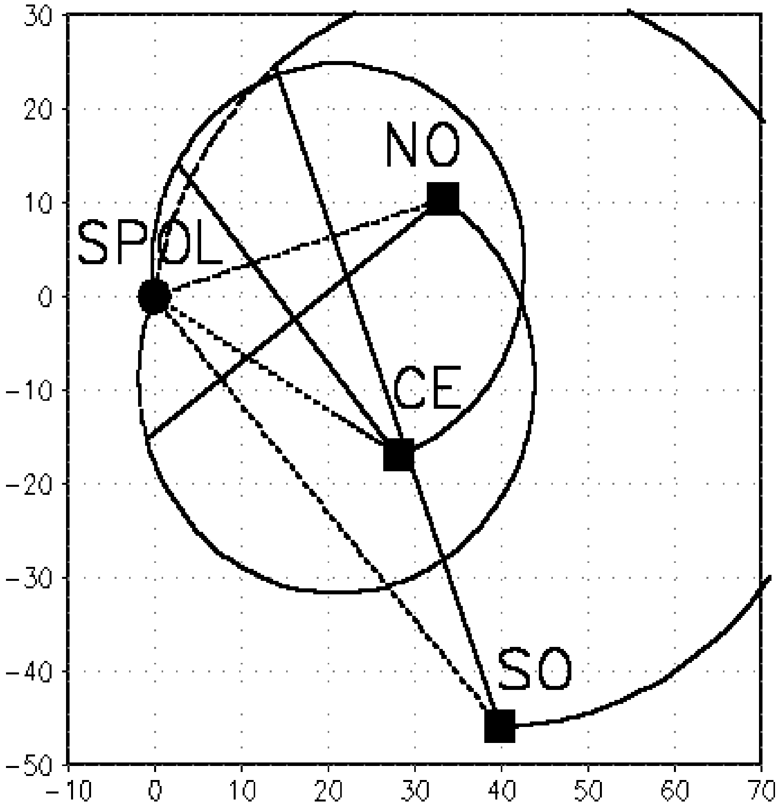
Data

observed in Kansas during CASES-97

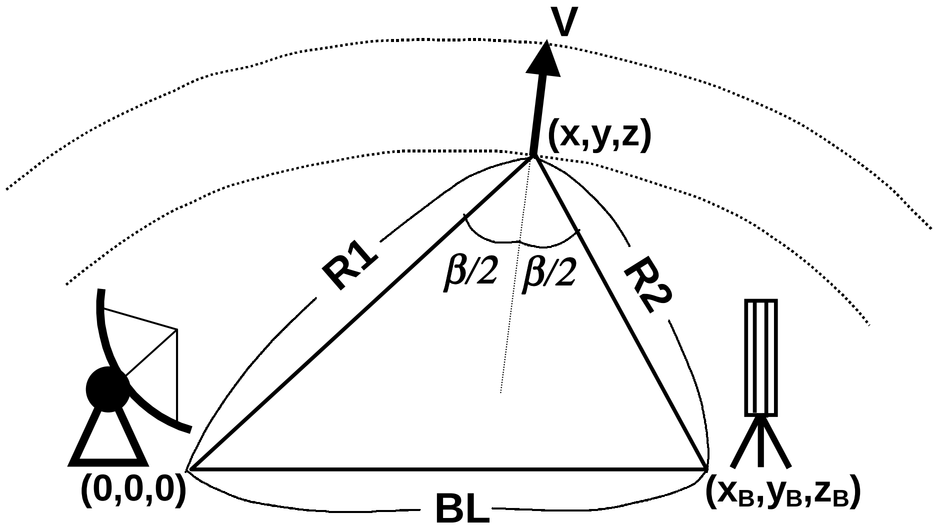
05/19/97, 0300-1120GMT
convective echo,
SPOL and one bistatic site (NO) only

05/30/97, 0200-0300GMT
stratiform echo, SPOL and three bistatic sites (NO, CE, SO)

EL angles:
0.5, 1.0, 1.4, 2.4, 3.4 deg



Bistatic Doppler velocity



The Doppler velocity measured by a bistatic receiver corresponds to the time differential of the path length *R(=R1+R2)*. Since the *R* is a function of (x, y, z, t), the amplitude of the bistatic Doppler velocity is expressed by V= $\partial R/\partial t = |\nabla R|\, dR/dt$, and the direction of the velocity vector is expressed by ∇R.

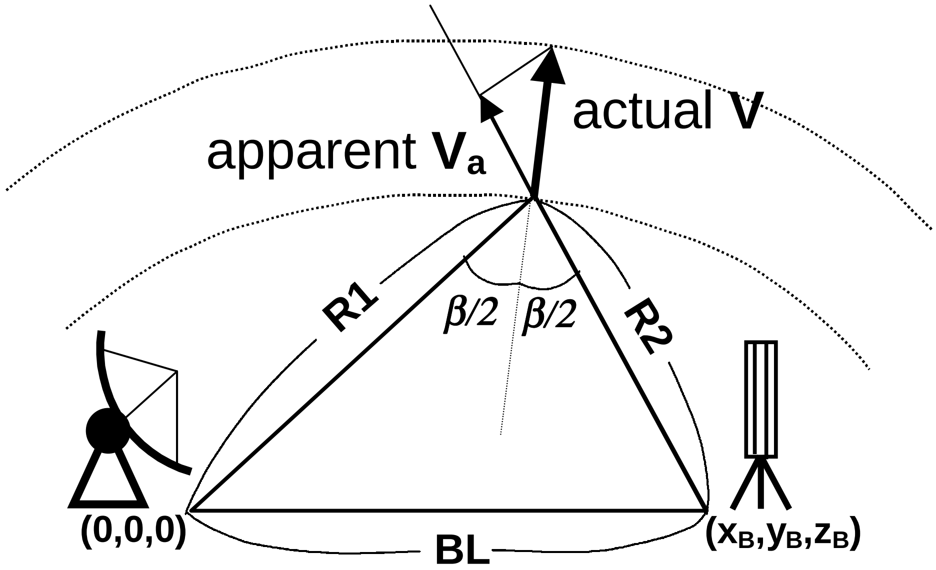
$$\begin{aligned}\nabla R &= \frac{\partial R}{\partial x}\boldsymbol{i} + \frac{\partial R}{\partial y}\boldsymbol{j} + \frac{\partial R}{\partial z}\boldsymbol{k} \\ &= \left\{\frac{x}{R1} + \frac{x - x_B}{R2}\right\}\boldsymbol{i} + \left\{\frac{y}{R1} + \frac{y - y_B}{R2}\right\}\boldsymbol{j} + \left\{\frac{z}{R1} + \frac{z - z_B}{R2}\right\}\boldsymbol{k} \\ &= \left(\sin(a)\cos(e) + \sin(a_B)\cos(e_B)\right)\boldsymbol{i} + \left(\cos(a)\cos(e) + \cos(a_B)\cos(e_B)\right)\boldsymbol{j} + \left(\sin(e) + \sin(e_B)\right)\boldsymbol{k}\end{aligned}$$

$$\begin{aligned}|\nabla R| &= \sqrt{\left(\frac{\partial R}{\partial x}\right)^2 + \left(\frac{\partial R}{\partial y}\right)^2 + \left(\frac{\partial R}{\partial z}\right)^2} = \sqrt{1 + 1 + \frac{R1^2 + R2^2 - BL^2}{R1 \cdot R2}} \\ &= 2\cos(\beta/2)\end{aligned}$$

The three-dimensional orthogonal components of the bistatic Doppler velocity vector are expressed by $\nabla R / |\nabla R|$. When the bistatic velocity vector is expressed by azimuth and elevation angles (a_V and e_V), the relationship between the angles and the components of $\nabla R / |\nabla R|$ are represented by

$$\left.\begin{aligned}\sin(a_V)\cos(e_V) &= \frac{\sin(a)\cos(e) + \sin(a_B)\cos(e_B)}{2\cos(\beta/2)} \\ \cos(a_V)\cos(e_V) &= \frac{\cos(a)\cos(e) + \cos(a_B)\cos(e_B)}{2\cos(\beta/2)} \\ \sin(e_V) &= \frac{\sin(e) + \sin(e_B)}{2\cos(\beta/2)}\end{aligned}\right\}$$

Measured bistatic Doppler velocity



Since the bistatic receiver does not know the scattering angle β beforehand, the bistatic receiver can detect only apparent velocity (Va), which is a projection of the actual Doppler velocity vector. In other words, the Doppler velocity data is measured, as if a transmitter were at the same position of the bistatic receiver, that is $\beta = 0$. To obtain the actual Doppler velocity, an velocity expansion factor $(\cos(\beta/2))^{-1}$ should be multiplied by the apparent velocity as follows.

$$V = V_a \ / \cos(\beta / 2)$$

$(\cos(\beta/2)^{-1}$: velocity expansion factor

At the same time, the actual Nyquist velocity (unambiguous velocity) and variance of mean Doppler velocity are also expanded as follows.

$$Nyquist velocity: \quad V_{\max} = V_{a\ \max} / \cos(\beta / 2)$$

$$Variance of\ mean Doppler velocity: \quad \sigma = \sigma_a \ / \cos(\beta / 2)$$

Dual-Doppler velocity synthesis

To obtain the three-dimensional wind fields (*u*, *v*, *w*), the monostatic Doppler velocity (V_1) measured by the transmitting radar and the bistatic Doppler velocity (V_2) represented in the above are synthesized as follows:

$$\begin{cases} V_1 = \sin(a_1)\cos(e_1)\,u + \cos(a_1)\cos(e_1)\,v + \sin(e_1)\,w_p \\ V_2 = \dfrac{\sin(a_1)\cos(e_1)+\sin(a_2)\cos(e_2)}{2\cos(\beta/2)}\,u + \dfrac{\cos(a_1)\cos(e_1)+\cos(a_2)\cos(e_2)}{2\cos(\beta/2)}\,v + \dfrac{\sin(e_1)+\sin(e_2)}{2\cos(\beta/2)}\,w_p \end{cases}$$

where a_1 and e_1 are the azimuth and elevation angles of the main radar antenna, a_2 and e_2 are the azimuth and elevation angles of the direction from the bistatic antenna to the target echo, and w_p is the sum of vertical air motion w and terminal falling velocity w_t, that is $w_p=w+w_t$. From these equations, the horizontal wind components (*u* and *v*) are calculated by

$$\left.\begin{aligned} u &= \frac{1}{\sin(a_1-a_2)}\left[\left(\frac{\cos(a_1)}{\cos(e_2)}+\frac{\cos(a_2)}{\cos(e_1)}\right)V_1-\left(\frac{2\cos(\beta/2)\cos(a_1)}{\cos(e_2)}\right)V_2\right] \\ &\quad+\frac{1}{\sin(a_1-a_2)}\left[\left(\frac{\cos(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\cos(a_1)\sin(e_2)}{\cos(e_2)}\right)-\left(\frac{\cos(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\cos(a_2)\sin(e_1)}{\cos(e_1)}\right)\right]w_p \\ v &= \frac{1}{\sin(a_1-a_2)}\left[\left(\frac{2\cos(\beta/2)\sin(a_1)}{\cos(e_2)}\right)V_2-\left(\frac{\sin(a_1)}{\cos(e_2)}+\frac{\sin(a_2)}{\cos(e_1)}\right)V_1\right] \\ &\quad+\frac{1}{\sin(a_1-a_2)}\left[\left(\frac{\sin(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\sin(a_2)\sin(e_1)}{\cos(e_1)}\right)-\left(\frac{\sin(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\sin(a_1)\sin(e_2)}{\cos(e_2)}\right)\right]w_p \end{aligned}\right\}$$

$$\left.\begin{aligned} u &= \frac{1}{\sin(a_1-a_2)}\left[\left(\frac{\cos(a_1)}{\cos(e_2)}+\frac{\cos(a_2)}{\cos(e_1)}\right)V_1-\left(\frac{2\cos(\beta/2)\cos(a_1)}{\cos(e_2)}\right)\left(\frac{V_2}{\cos(\beta/2)}\right)\right] \\ &\quad+\frac{1}{\sin(a_1-a_2)}\left[\left(\frac{\cos(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\cos(a_1)\sin(e_2)}{\cos(e_2)}\right)-\left(\frac{\cos(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\cos(a_2)\sin(e_1)}{\cos(e_1)}\right)\right]w_p \\ v &= \frac{1}{\sin(a_1-a_2)}\left[\left(\frac{2\cos(\beta/2)\sin(a_1)}{\cos(e_2)}\right)\left(\frac{V_2}{\cos(\beta/2)}\right)-\left(\frac{\sin(a_1)}{\cos(e_2)}+\frac{\sin(a_2)}{\cos(e_1)}\right)V_1\right] \\ &\quad+\frac{1}{\sin(a_1-a_2)}\left[\left(\frac{\sin(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\sin(a_2)\sin(e_1)}{\cos(e_1)}\right)-\left(\frac{\sin(a_1)\sin(e_1)}{\cos(e_2)}+\frac{\sin(a_1)\sin(e_2)}{\cos(e_2)}\right)\right]w_p \end{aligned}\right\}$$

Variance of synthesized horizontal winds

The variance of the estimates of (u,v) are expressed by

$$
\begin{cases}
\sigma_u^{2} = \dfrac{1}{\sin^2(a_1-a_2)}\left[\left\{\cos^2(a_1)+\cos^2(a_2)+2\cos(a_1)\cos(a_2)\right\}\sigma_1^{2}+\left\{4\cos^2(\beta/2)\cos^2(a_1)\right\}\sigma_{a2}^{2}\right] \\
\sigma_v^{2} = \dfrac{1}{\sin^2(a_1-a_2)}\left[\left\{\sin^2(a_1)+\sin^2(a_2)+2\sin(a_1)\sin(a_2)\right\}\sigma_1^{2}+\left\{4\cos^2(\beta/2)\sin^2(a_1)\right\}\sigma_{a2}^{2}\right]
\end{cases}
$$

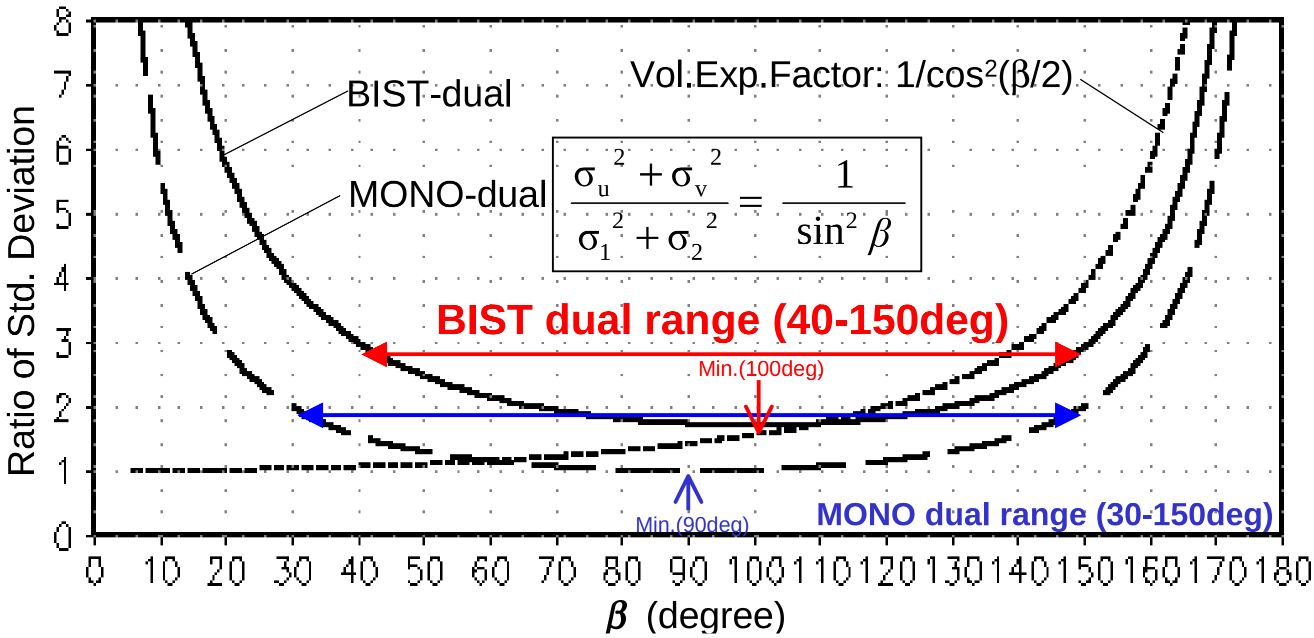
Since the bistatic velocity variance should be multiplied by the velocity expansion factor $(\cos(\beta/2))^{-1}$, the sum of the variance are expressed by

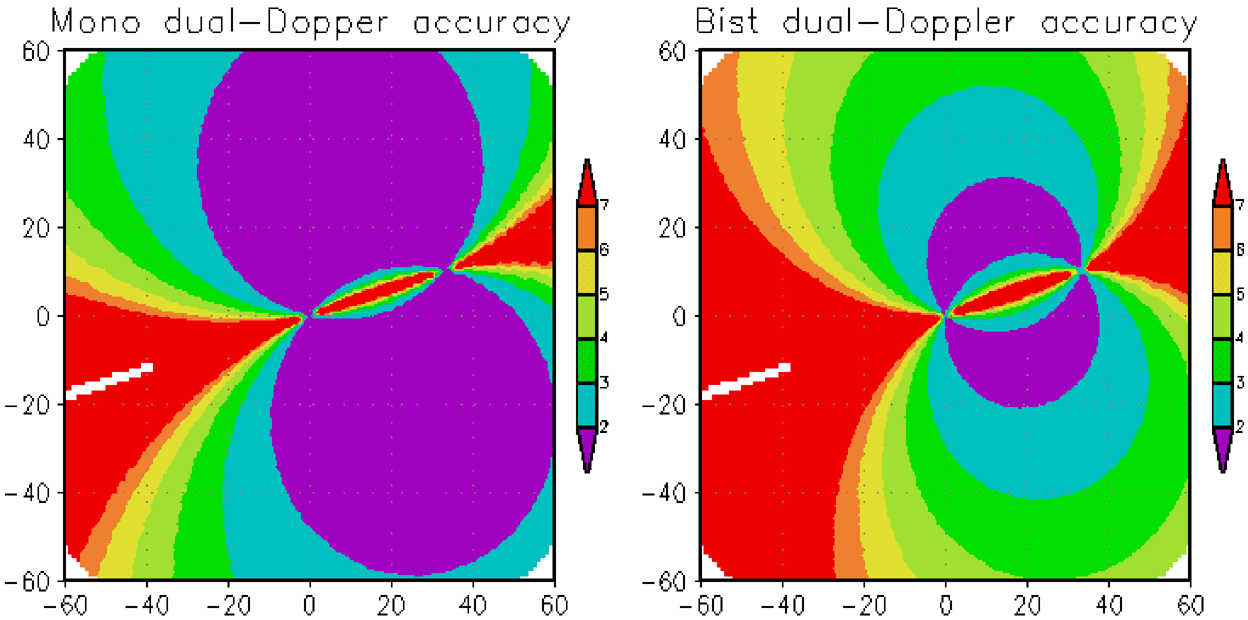
$$
\sigma_u^{2}+\sigma_v^{2} = \frac{\sigma_1^{2}+\sigma_{a2}^{2}}{\sin^2(\beta/2)}, \qquad \sigma_{a2} = \sigma_2 / \cos(\beta/2)
$$

$$
\sigma_u^{2}+\sigma_v^{2} = \frac{4\cos^2(\beta/2)\sigma_1^{2}+4\sigma_2^{2}}{\sin^2\beta} \quad \Longrightarrow \quad \text{Assuming } \sigma_1=\sigma_2,
$$

BISTATIC-dual

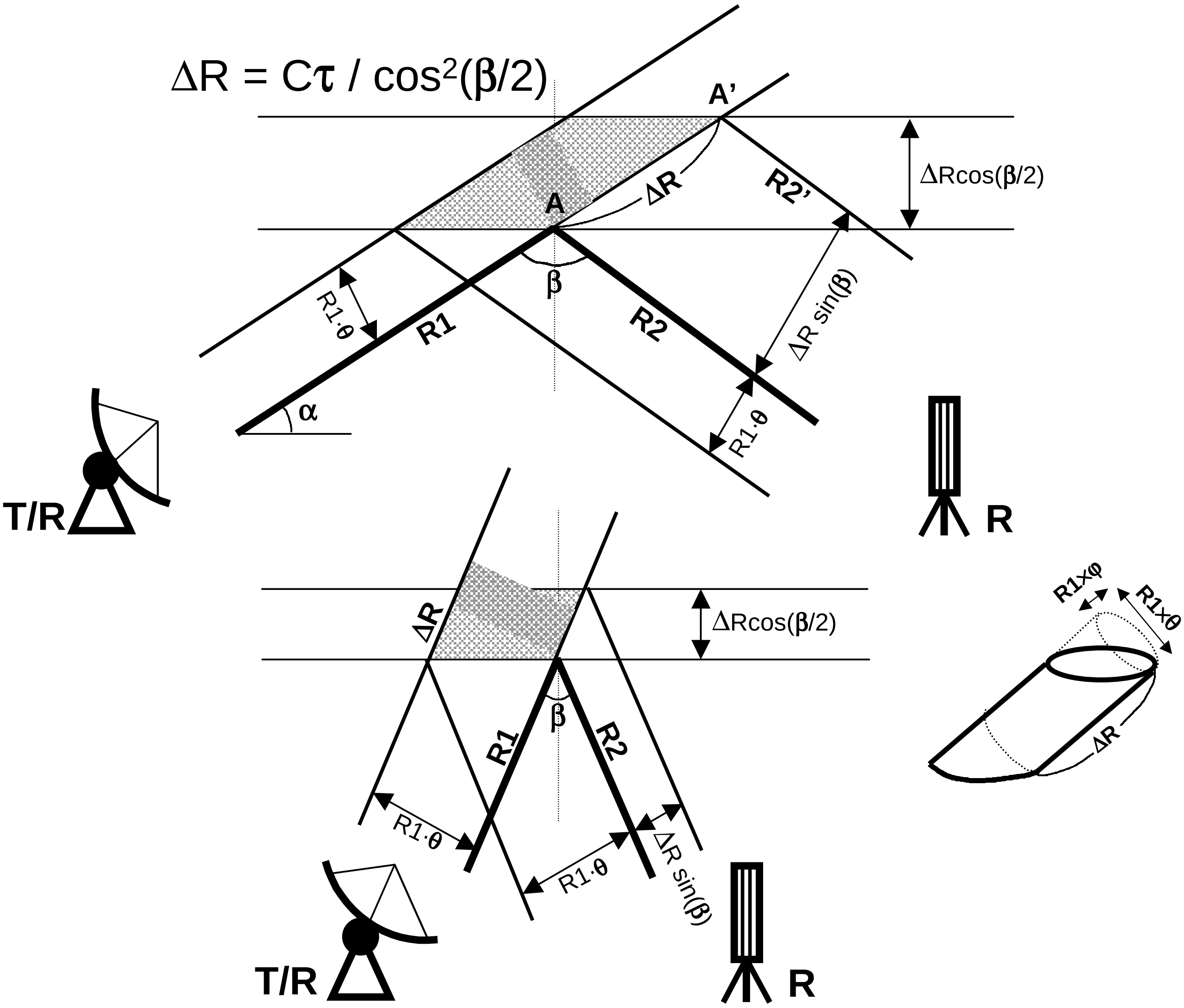
$$
\boxed{\frac{\sigma_u^{2}+\sigma_v^{2}}{\sigma_1^{2}+\sigma_2^{2}} = \frac{2\left\{\cos^2(\beta/2)+1\right\}}{\sin^2\beta}}
$$





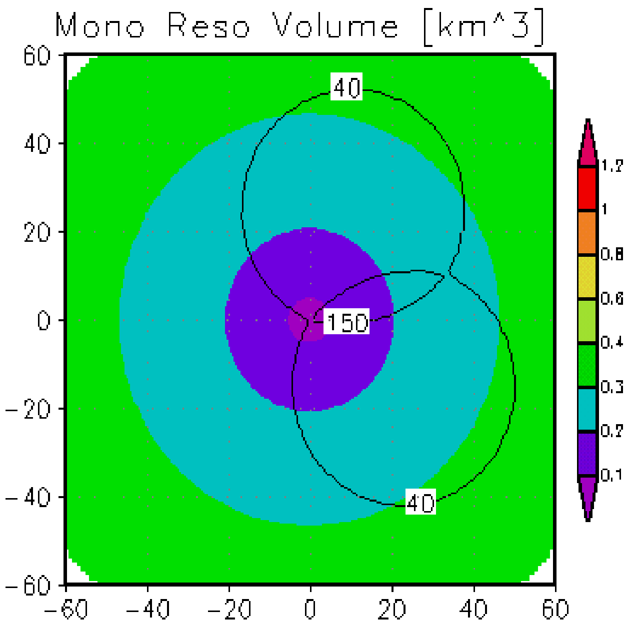
Bistatic resolution volume

$$\Delta R = C\tau / \cos^2(\beta/2)$$



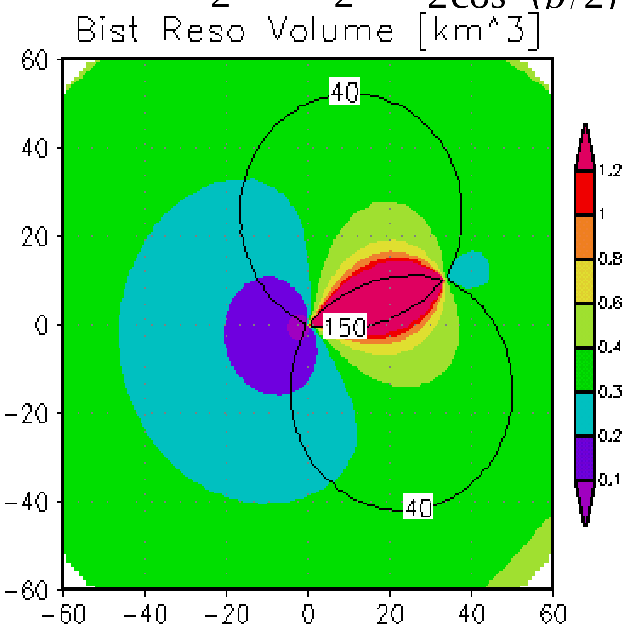
MONOSTATIC

$$V_c = \pi \times \frac{R1\cdot\theta}{2} \times \frac{R1\cdot\phi}{2} \times \frac{C\cdot\tau}{2}$$



BISTATIC

$$V_c = \pi \times \frac{R1\cdot\theta}{2} \times \frac{R1\cdot\phi}{2} \times \frac{C\cdot\tau}{2\cos^2(\beta/2)}$$



Bistatic radar equation

The bistatic resolution volume is calculated using Gaussian shape for the transmitting beam pattern as follows,

$$V = \frac{\pi \, R1^2 \, \theta \, \phi \, C \, \tau}{16 \ln(2) \cos^2 (\beta/2)}$$

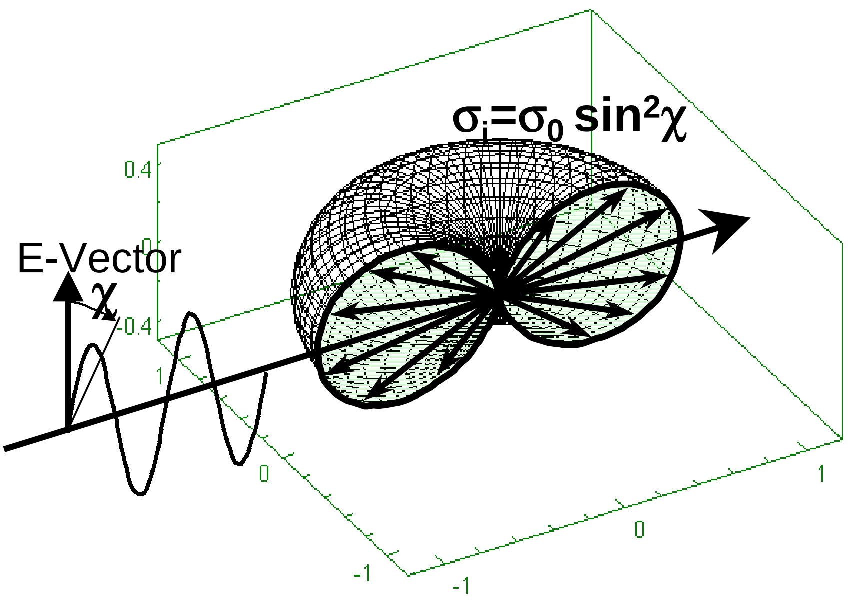
A bistatic radar equation is expressed by

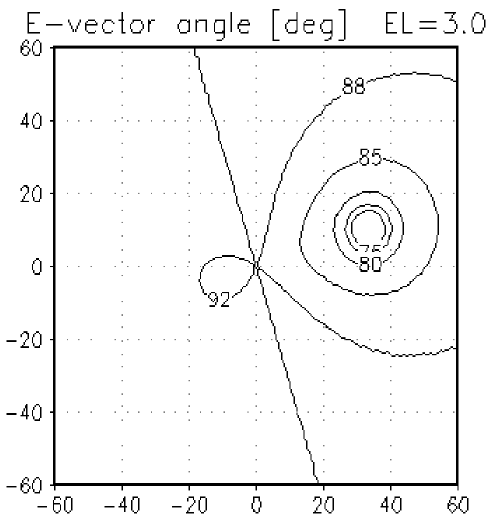
$$P_r = \frac{P_t \, G_1}{4 \pi \, R1^2} \cdot \frac{A_e}{4 \pi \, R2^2} \cdot V \cdot \sum_{vol} \sigma_i , \qquad A_e = \frac{\lambda^2 \, G_2(\theta_B)}{4 \pi}$$

where G_1 and G_2 are antenna gain of transmitter and receiver, respectively. A_e is the effective area of the receiver. $\Sigma\sigma_i$ is the total of the individual oblique-scattering cross-section areas in a unit volume. Inserting the sample volume V,

$$P_r = \frac{P_t \, G_1 \, G_2(\theta_B) \, \lambda^2 \, \theta \, \phi \, C \, \tau}{1024 \ln(2) \pi^2 \, R2^2 \cos^2 (\beta/2)} \sum_{vol} \sigma_i , \qquad \sigma_i = \sigma_0 \sin^2 \chi$$

Therefore the so-called range-correction term should be expressed by 20log[R2cos(β/2)], instead of 20logR1 in a well-known monostatic radar equation.



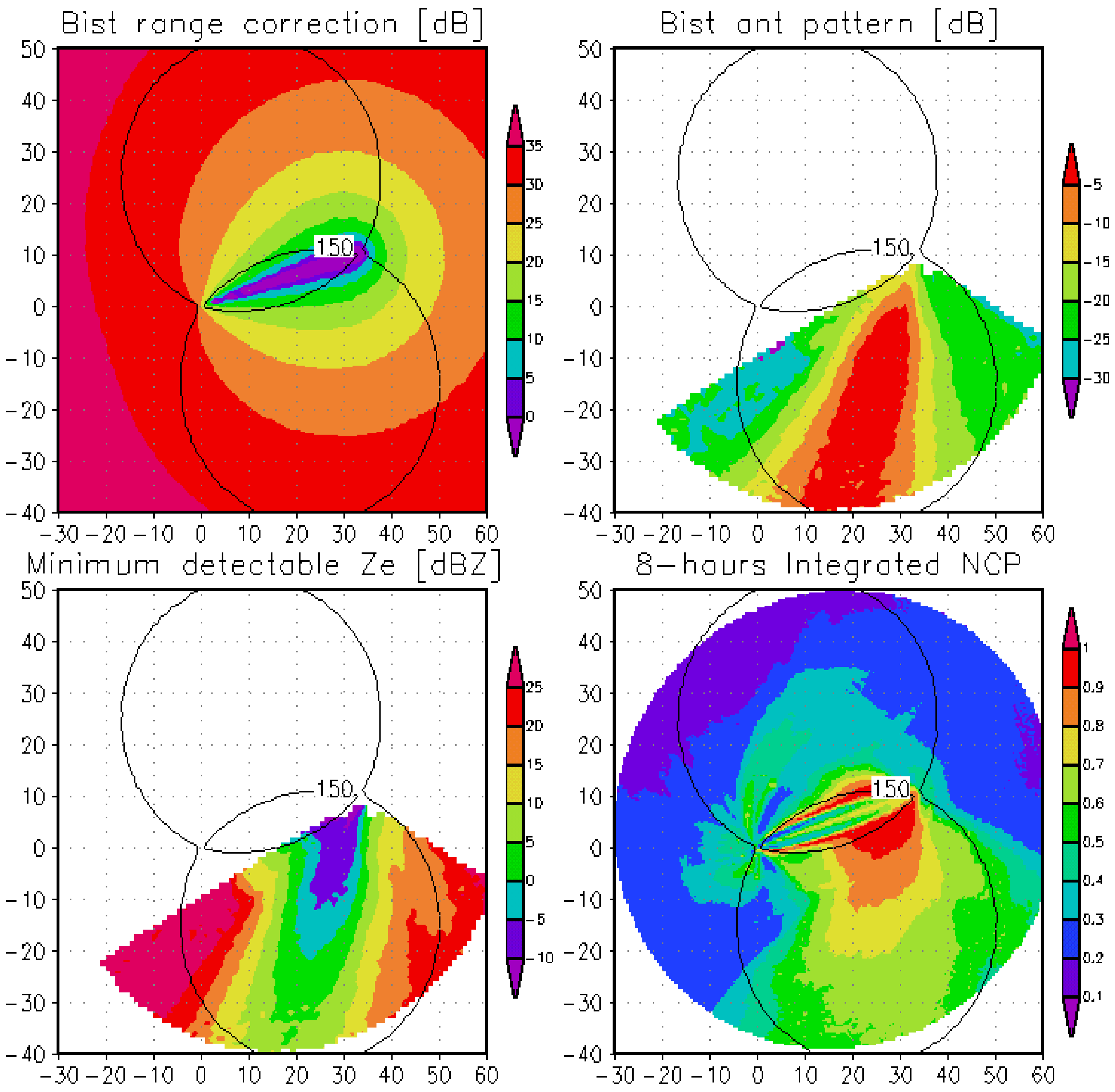


$\sin^2(85\text{deg})=0.99$

$\sin^2(80\text{deg})=0.97$

$\sin^2(75\text{deg})=0.93$

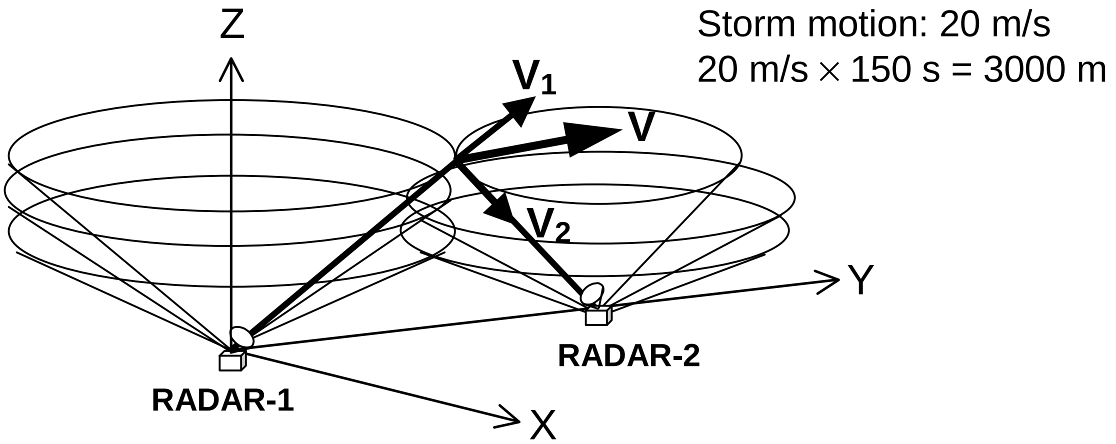
Minimum detectable reflectivity



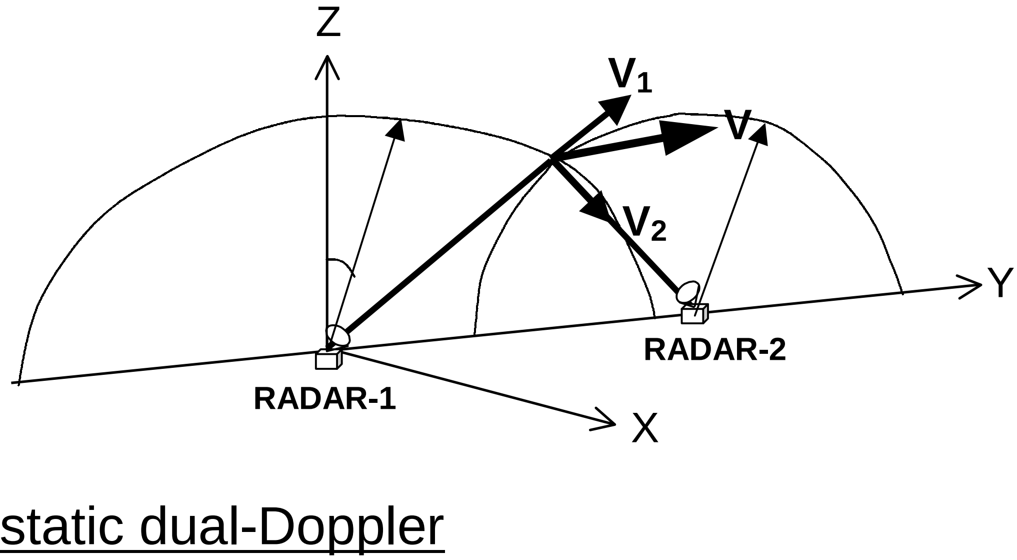
Difference of observation time

Monostatic dual-Doppler

(1) Volume scan: time difference = 0 ~ 300 sec

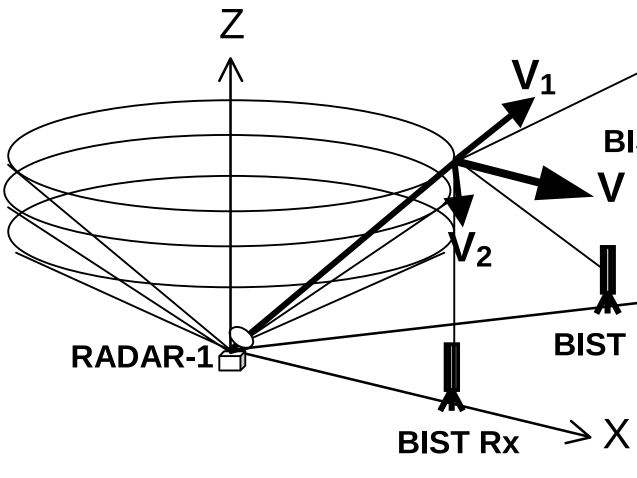


(2) COPLAN: time difference = 0 ~ 20 sec

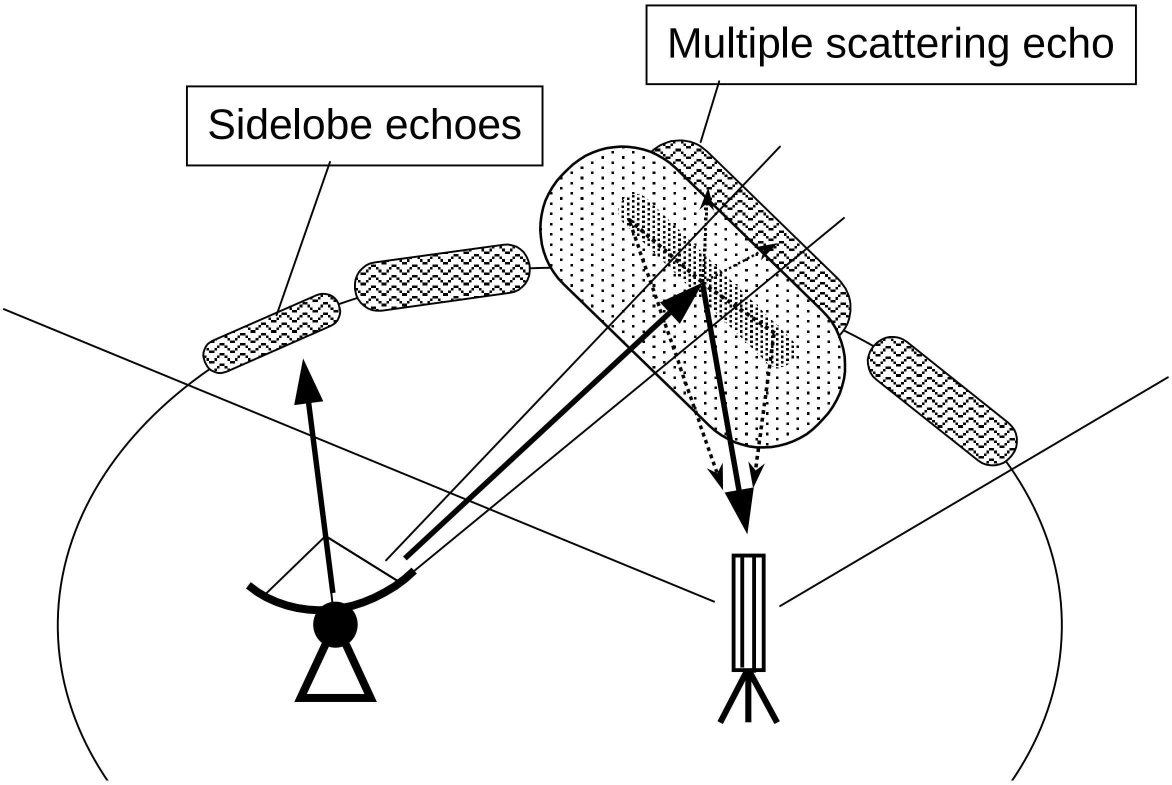


Bistatic dual-Doppler

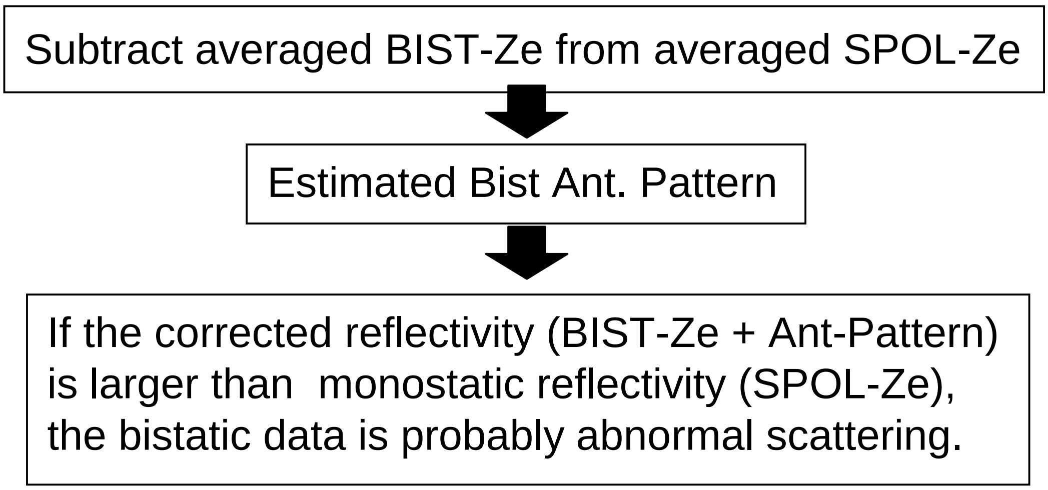
(3) Bistatic: time difference = 0



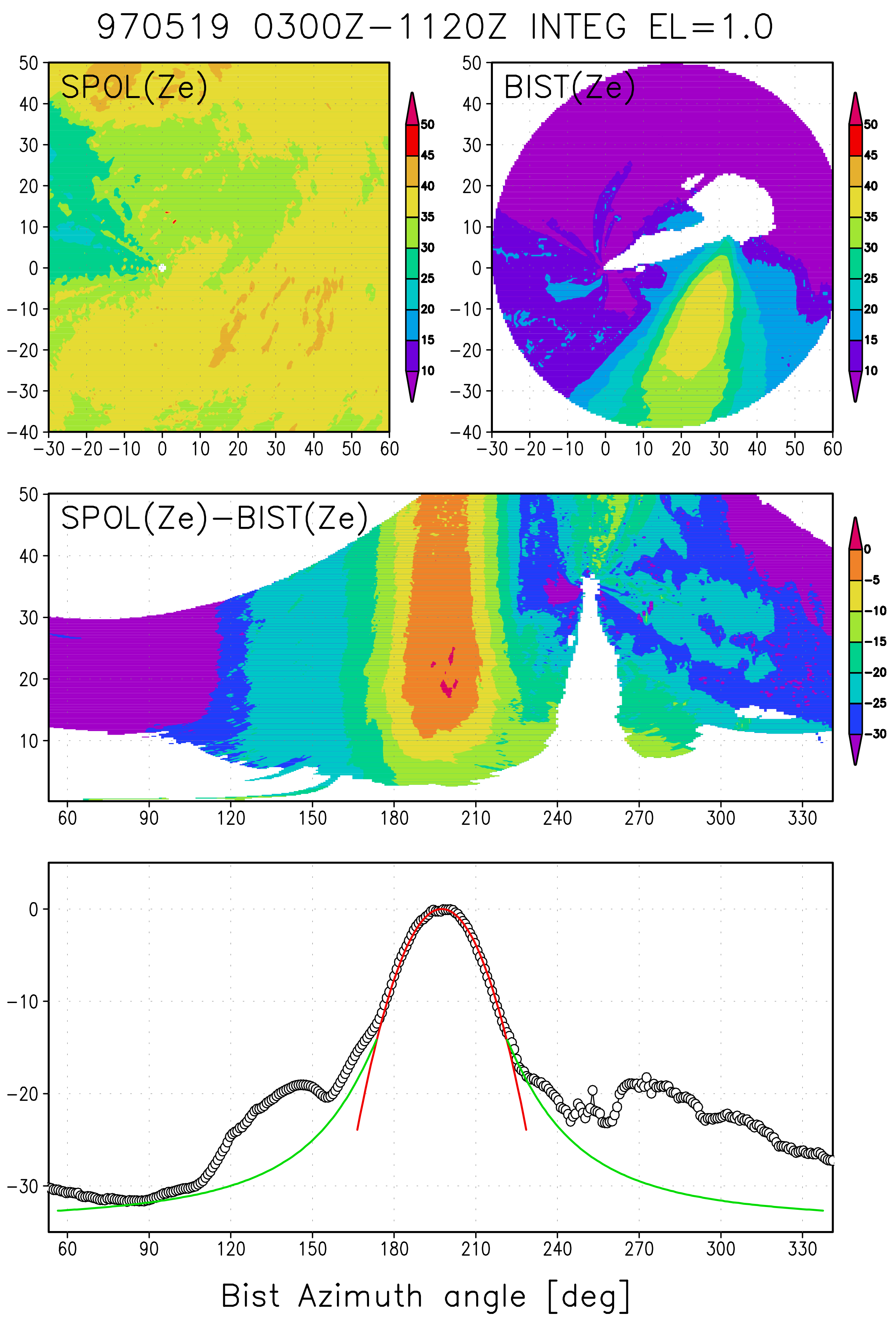
Sidelobe and multiple scattering contamination



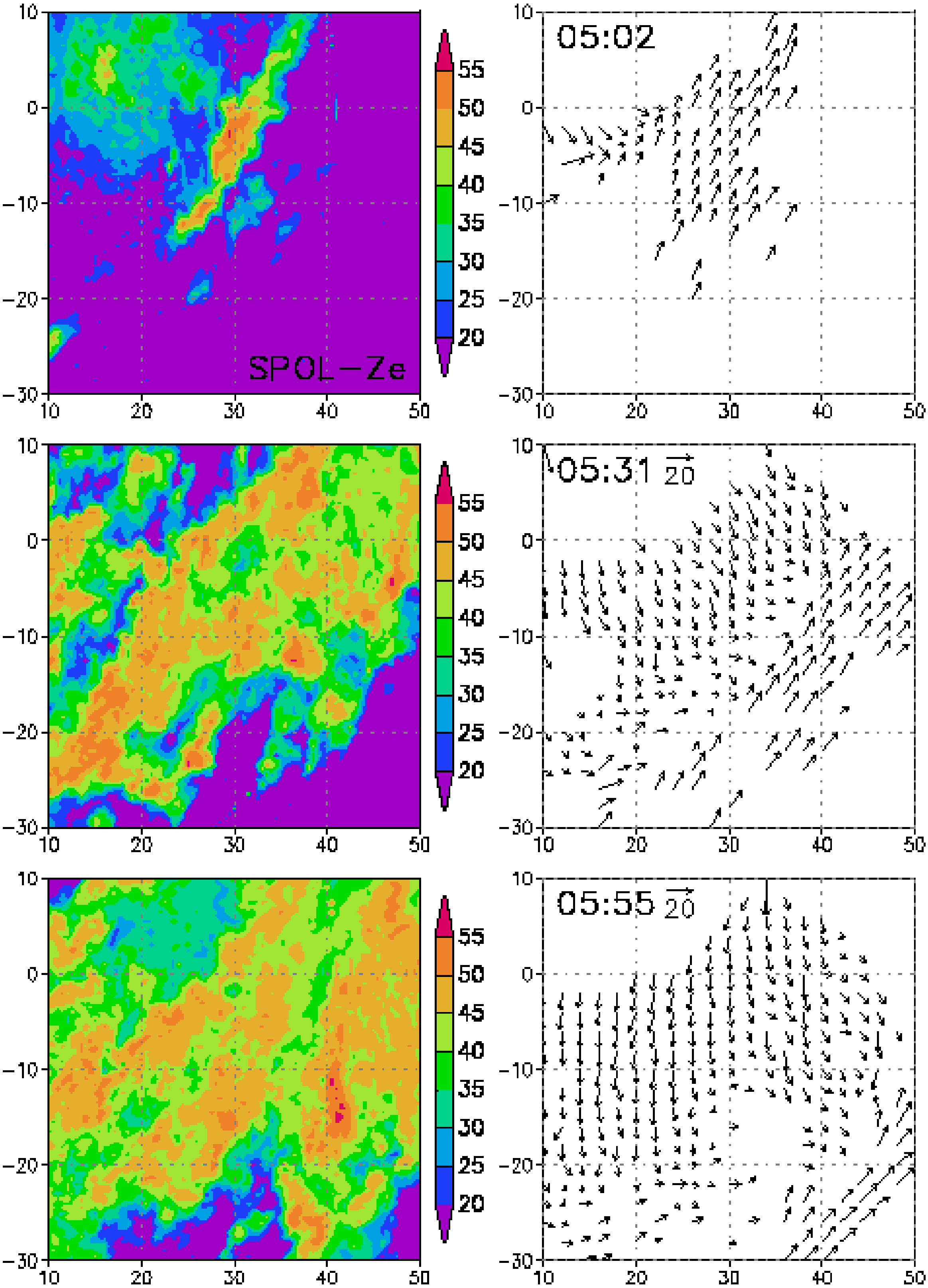
Strong echo may cause multiple and sidelobe scattering. These abnormal scattering will disappear due to time-integration of bistatic Ze. The distribution of enough time-average Ze indicates the bistatic antenna pattern.



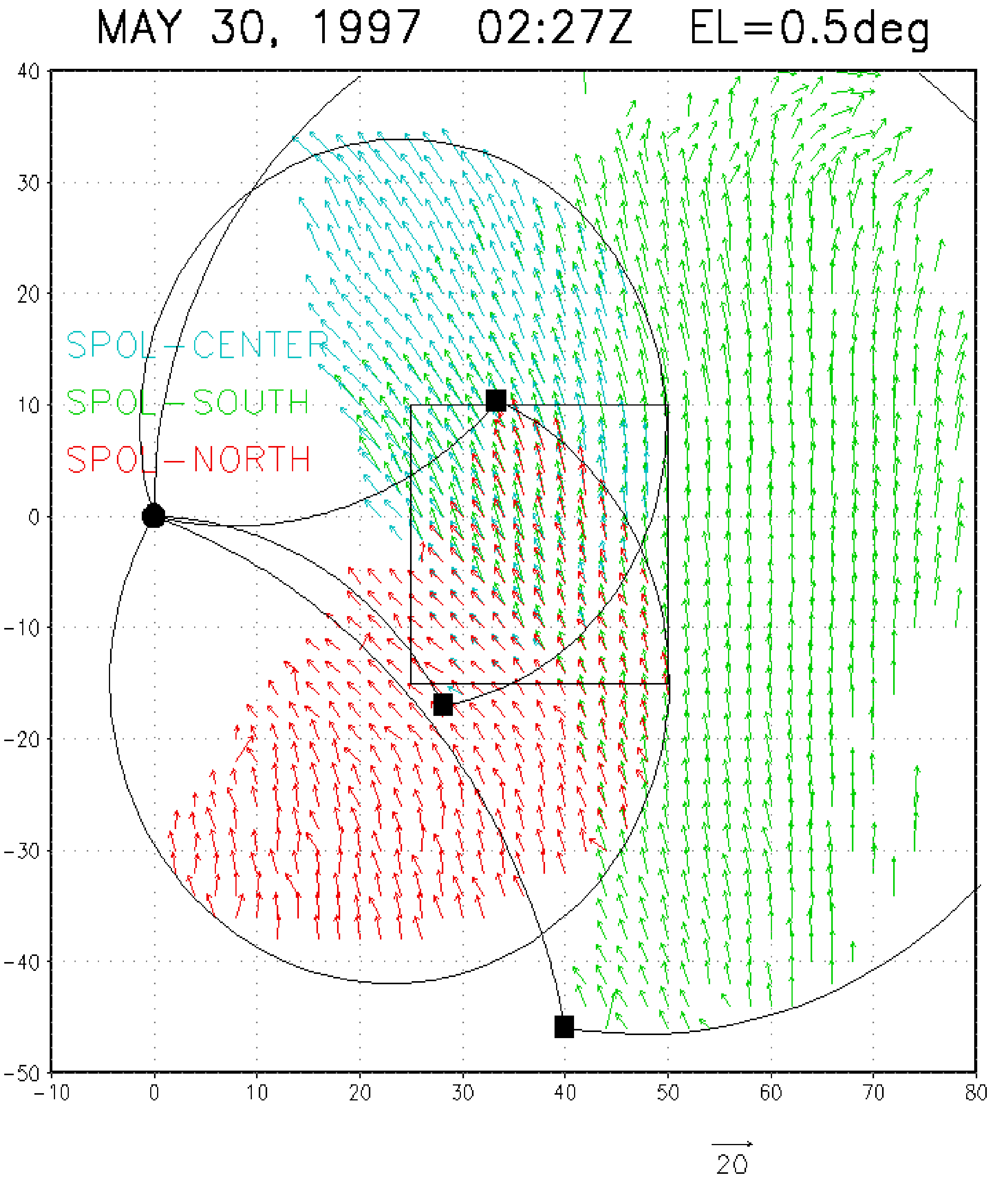
Bistatic antenna pattern



CONVECTIVE CASE (97/05/19) EL=1.0



Three pairs of dual-Doppler analysis



Variational Method in over-determined cases

Continuity equation as a strong constraint
Observed winds from dual-Doppler pairs as weak constraints

$$J \equiv \int \Big[\alpha_{u1}(u-u_1)^2 + \cdots + \alpha_{un}(u-u_n)^2 + \alpha_{v1}(v-v_1)^2 + \cdots + \alpha_{vn}(v-v_n)^2$$
$$+ \alpha_w (w-w_0)^2 + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{w}{\rho}\frac{\partial \rho}{\partial z} \right) \Big] dv$$

α_{un} : Gauss precision moduli, $\alpha_{un} = \left(2\,\sigma_u^{\,2}\right)^{-1}$

λ : Lagrange undetermined multiplier

In the case of low elevation angles ignoring vertical components,

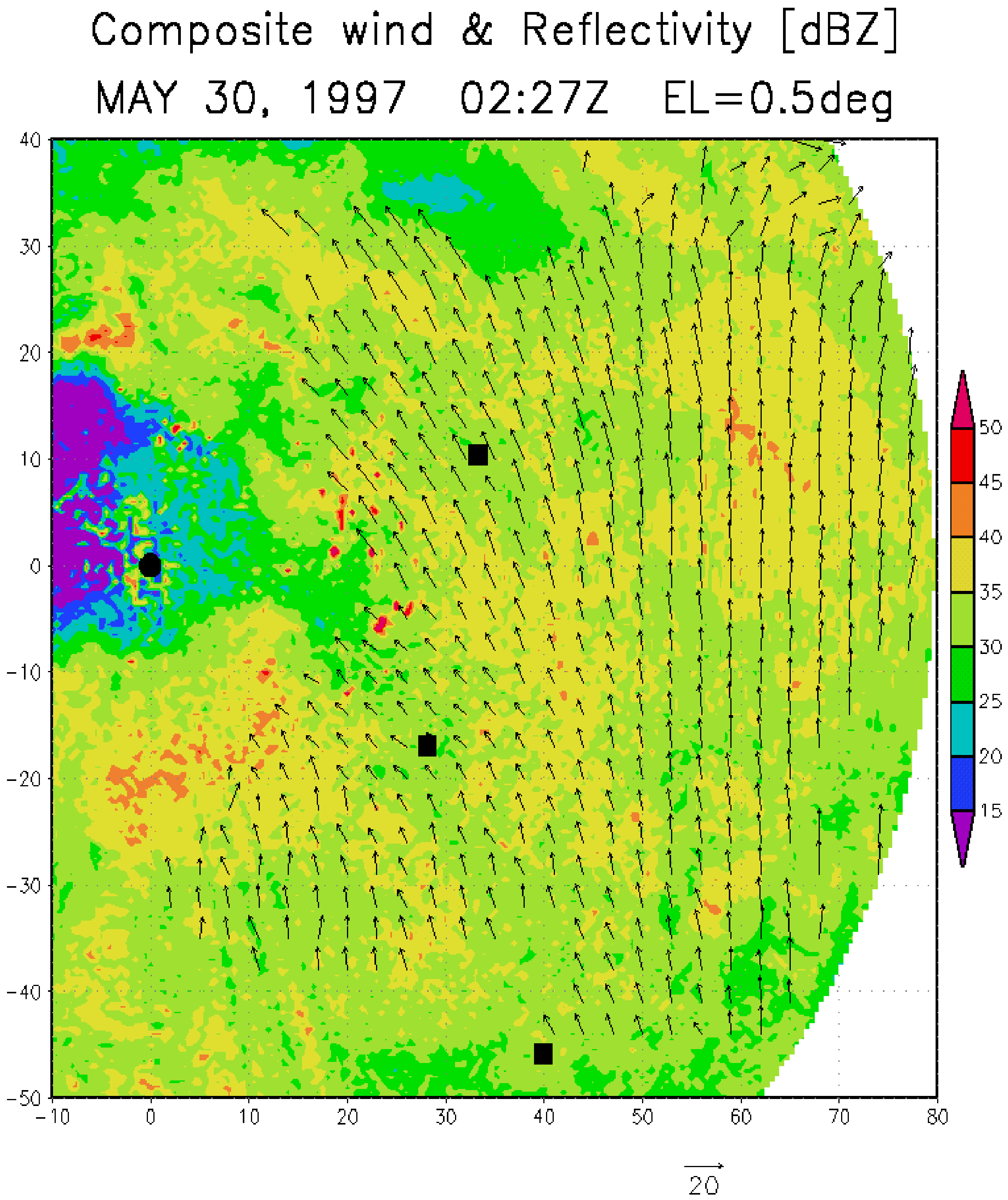
$$J_H \equiv \alpha_{u1}(u-u_1)^2 + \cdots + \alpha_{un}(u-u_n)^2 + \alpha_{v1}(v-v_1)^2 + \cdots + \alpha_{vn}(v-v_n)^2$$

$$u_0 = \frac{\alpha_{u1}\,u_1 + \alpha_{u2}\,u_2 + \cdots + \alpha_{un}\,u_n}{\alpha_{u1} + \alpha_{u2} + \cdots + \alpha_{un}}, \quad v_0 = \frac{\alpha_{v1}\,v_1 + \alpha_{v2}\,v_2 + \cdots + \alpha_{vn}\,v_n}{\alpha_{v1} + \alpha_{v2} + \cdots + \alpha_{vn}}$$

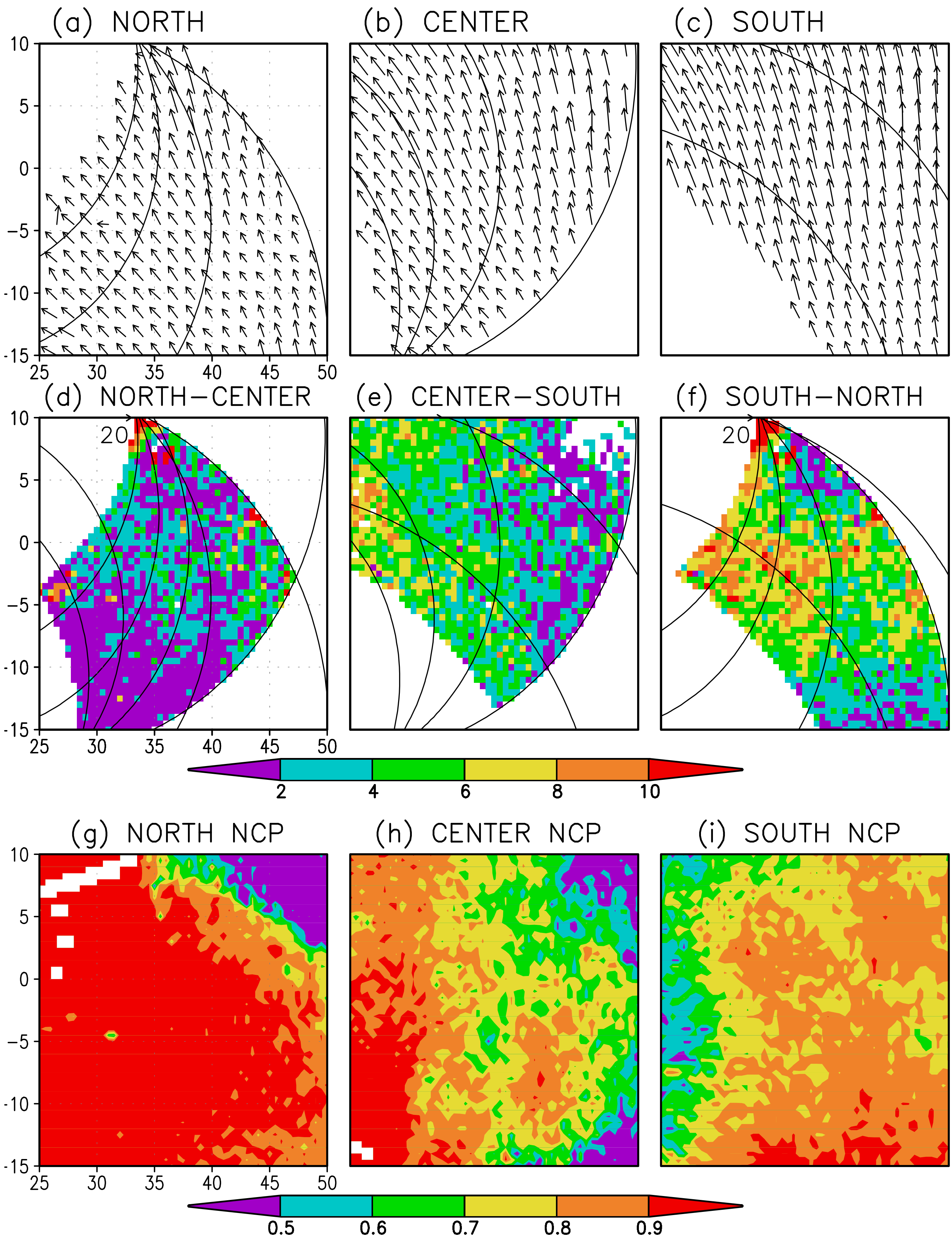
$$\alpha_{un} = \alpha_{vn} = \frac{\sin^2(\beta/2)}{\cos^2(\beta/2)+1}, \ \text{becasue} \ \frac{\sigma_u^{\,2} + \sigma_v^{\,2}}{\sigma_1^{\,2} + \sigma_2^{\,2}} = \frac{2\left\{\cos^2(\beta/2)+1\right\}}{\sin^2(\beta/2)}$$

where $\sigma_1^2 = \sigma_2^2 = 1$ is assumed.

Composite wind vectors using variational method



Comparison among three pairs of dual-Doppler analysis



CONCLUSIONS

(1) The standard deviation of the synthesized horizontal wind vectors is less than three times of observed Doppler velocities within a range of $40 < \beta < 150$ degree. The minimum error appears at $\beta \approx 100$ degree.

(2) The accurate bistatic radar equation including the bistatic resolution volume, the polarization scattering angle, and the estimated antenna-pattern leads to the right bistatic reflectivity and the minimum detectable reflectivity.

(3) The sidelobe contamination can be eliminated using the difference between the transmitting radar reflectivity and the corrected bistatic reflectivity.

(4) In strong (convective) echoes, the sidelobe contamination is the prime cause of the wind error. In week (stratiform) echoes, the minimum detectable power dominates the wind accuracy.