Log-APP Detection for FSO Repetition MIMO Links

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***Abstract*—Free-space optical (FSO) data links can provide high bandwidth and secure data transmission. A robust solution on the physical layer to minimize channel impacts, such as atmospheric turbulence, is to use spatial diversity (multi-aperture MIMO) in conjunction with repetition coding. In this contribution, the optimum detector in the sense of minimizing the bit error rate (BER), an a posteriori probability (APP) detector, is adopted to the FSO channel model. We derive a general metric for FSO transmission and present a metric that improves the BER performance if the noise process at the receiver side is correlated. Simulation results verify that the proposed receiver outperforms conventional soft-decision detection if the noise process is colored.**

I. Introduction

A lot of effort has been made to find efficient modulation schemes for FSO links. The major attention has been devoted to non-coherent direct detection schemes in conjunction with intensity modulation, such as simple on-off keying (OOK), Q-ary pulse position modulation (PPM) (e.g. [1], [2]) or Q-ary unipolar amplitude modulation (ASK). By now, it is well known that spatial diversity reception is a robust method for mitigating turbulence-induced fading under the assumption that the spacing between the receiving apertures is greater than the correlation length, which is easily fulfilled by separation distances of some centimeters for reasonable propagation distances [3].

Let us consider M transmit and N receive apertures. In a previous contribution, we have shown for a time-varying scintillation channel that by means of adaptive modulation and channel coding the robustness with respect to maximizing data throughput can be improved [4]. The two extremes are *spatial multiplexing*, where different data is transmitted via all $M \times N$ channels, and *spatial diversity* with repetition coding, where the same data is sent via all $M \times N$ channels. Minor attention has been devoted to an overall coding and optimal detection scheme. Besides simple hard-input decoding schemes, a number of soft-input decoding schemes exist. We will focus on the Log-APP detector, which is known to perform best in terms of minimizing the BER. This detector outputs either probability values or the corresponding log-likelihood ratios (LLRs).

It is well known that in the case of correlated receiver noise, the BER performance degrades when conventional detection is employed. We will show in this contribution that by taking the correlation coefficients at the receiver side into account, a gain in BER performance can be achieved even if the noise samples are highly correlated. In order to illustrate the performance gain in the case of correlated and uncorrelated receiver noise, we will present simulation results that compare the conventional and the modified LLR metric with maximum ratio combining (MRC), which is known to be the optimum method for independent additive white Gaussian noise (AWGN) channels.

The remainder of this paper is organized as followed. The system model for the FSO domain is presented in Section II. In Section III, we introduce the Log-APP detection rule and give analytical results of the optimal algorithm for the FSO channel. We then take into account a 2×2 MIMO channel and derive a general analytical metric that is suitable for correlated and uncorrelated receiver noise. In Section IV the simulation setup is presented and performance results for the considered scheme as a function of different turbulence strengths, multiple transmit and receive apertures and receiver noise correlation conditions are given in Section V. Finally conclusions are drawn in Section VI.

II. System Model

We consider FSO MIMO transmission employing OOK modulation and direct detection with M transmit and N receive apertures. The $M \times N$ channels are assumed to be fully decorrelated and independent which is fulfilled by separating the transmit apertures accordingly.

The vector/matrix notation of the channel model is

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is the received signal vector of size N, $\mathbf{x}$ is the transmitted data symbol vector of size M with $\mathbf{x} \in \{0, 1\}$, and $\mathbf{H}$ is the channel matrix of size $M \times N$. The coefficients $h_{m,n}$ of $\mathbf{H}$ are assumed to be log-normally distributed, are always real valued, and non-negative. $\mathbf{n}$ is the noise vector at the receiver of size N and is assumed to follow the Gaussian distribution with variance σ_{n}^2 and zero-mean μ_n.

The channel matrix $\mathbf{H}$ is assumed to be known at the receiver. The noise contribution $\mathbf{n}$ will be described in more detail in the next paragraph.

Fig. 1 shows the system model under investigation. At first the digital data $\mathbf{u}$ is encoded by a channel encoder. The encoded sequence $\mathbf{u}_c$ is interleaved and distributed to the M

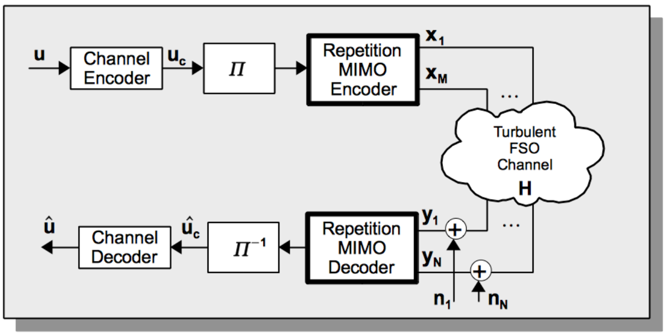

Fig. 1. FSO MIMO System Model

transmit apertures by the repetition MIMO encoder. A robust repetition coding is used (*spatial diversity*).

After passing the multiplicative turbulent channel with channel matrix $\mathbf{H}$, white Gaussian noise is added prior to the repetition MIMO decoder. The repetition detector either performs Log-APP detection or MRC for comparison, which will be presented in Section III. After the repetition detector, the data is de-interleaved before the outer channel code is decoded and the BER performance is evaluated by Monte Carlo simulations, comparing $\hat{\mathbf{u}}$ with $\mathbf{u}$, which will be denoted as *Monte-Carlo simulation based on hard decisions (hard sim)*. In addition a different approach considering the outputs of the Log-APP detector will be used to determine the BER (*Monte-Carlo simulation based on soft decisions (soft sim)*) [5].

Since the Log-APP detector is a soft-input soft-output detector, Turbo processing for the decoding process could be implemented [6]. Several coding scheme results are presented in Section IV.

A. FSO Transmission Channel $\mathbf{H}$

In this contribution, fog or rain as slowly varying channel impacts for FSO transmission are not treated but the scintillation induced amplitude fluctuations observed at the receiver are modelled following the log-normal distribution function. The MIMO approach brings diversity gain considering scintillation effects but will not improve performance under fog or rain conditions.

Many different channel models for the memoryless FSO channel exist, adjusted to the strength of turbulence. The models are mainly based on the work of Kolmogorov. Most popular models are the gamma-gamma or the log-normal fading model which are mentioned e.g. in [7]. Recent measurements taken in the maritime environment have shown that the log-normal model covers a wide range of turbulence strengths [8].

A measure describing the quality of scintillation effects is the *Fried parameter* r_0:

$$r_0 = (0.16 C_n^2 k^2 L)^{-3/5} \tag{2}$$

k denotes the wave number and L the transmission length. C_n^2 is the structure parameter that is another measure for turbulence. For a horizontal transmission path, C_n^2 can be assumed to be constant, whether for vertical transmission it is more complicated to calculate C_n^2. An altitude profile model, e.g. the *Hufnagle-Valley Model*, needs to be implemented in the calculation as well as the influence of ground wind speed [7].

As stated above, scintillation leads to intensity fluctuations at the receiver. The amplitude fluctuation is caused by random index-of-refraction fluctuations.

To extract a measure for atmospheric turbulence out of measurement data, the scintillation index can be calculated which equals the variance of irradiance fluctuations scaled by the square of the mean irradiance [7]. The irradiance fluctuations due to scintillation follow a log-normal distribution for a large range of turbulence conditions.

Considering time-varying amplitude fluctuations well below 1 kHz, the channel can be assumed to be constant for a large number of transmitted bits. Therefore training symbols for estimation of $\mathbf{H}$ only have to be transmitted every thousands of bits which will only cost minor user data bandwidth of the system.

B. Receiver noise $\mathbf{n}$

The receiver lens collects the transmitted optical field and converts it into a corresponding electrical voltage. The optical field source could be the intensity of the transmit beam or background radiation.

We assume photodetectors at the receiver that are sensitive over a large wavelength span without narrow-bandwidth filtering, leading to broadband noise contribution. Furthermore we assume to operate the receiver well above the low photon-counting regime. Thus the noise contribution $\mathbf{n}$ is received signal intensity dependant shot noise. Whenever signal levels are sufficiently large, with respect to circuit or electronic thermal noise contributions, $\mathbf{n}$ can be modelled as additive white Gaussian noise (AWGN). This holds true for all noise sources at the receiver. If signal levels are in the photon-counting regime, poisson statistics would be needed to describe the noise at the receiver (e.g. [1] or [7]). Modelling noise at the receiver as white Gaussian noise is a common approach which can be found e.g. in [3] or [9].

The noise sources from above will be treated as combined additive noise contribution which can be correlated if background light is dominant. The special case of correlated AWGN will be investigated later in this contribution.

On general we make the assumption that the two sources of the received optical field are fully independent (cp. (1)).

III. The LLR metric for FSO channels

The Log-Likelihood Ratio (LLR) or Log-Likelihood metric is a metric to further improve the detection performance in terms of minimizing the bit-error rate (BER). This metric belongs to soft decision decoding since the outputs of the corresponding Log-APP detector are *soft* values. The proposed Log-APP detector takes into account the a priori information of the transmitted code words and the channel information. The latter could be for example extracted at the receiver by evaluating training sequences. The a priori information is not stringently needed. The output of the detector is the a posteriori information, which will be used as decision metric and the extrinsic information which is only available if the a priori information is known.

A. LLR for FSO Transmission

The a posteriori LLR for on-off keying (OOK) modulated signals is given as [1]

$$L(x|y) = \log_b \frac{P_{X|Y}(x=+1|y)}{P_{X|Y}(x=z|y)} \quad (3)$$

In (3) the ratio of probabilities for a transmitted "1", as high level and "z" as low level for a given receive signal is formulated. In FSO systems with OOK and direct detection (DD), the modulation is normally set in a way to completely switch off the transmit power, if a low level signal is transmitted. We set the low level amplitude to z ($1 > z \geq 0$), in order to preserve the degree of freedom if the transmit laser is not fully modulated ($m < 1$), where m is the modulation index.

To switch argumentation in (3), (in this terminology to switch relations between x and y), Bayes equation is used.

Since the receive signal is continuously changing it's value over time due to the channel impact, the probability density function (PDF) $p_{Y|X}$ of the receive signal can be used instead of the probability value $P_{Y|X}$

$$p(X|Y) = \frac{p(Y|X) \cdot P(X)}{p(Y)} \quad (4)$$

Bayes equation is a method to transform the *a-priori probability* $P(X)$ for occasion X into the *a-posteriori probability* $P(X|Y)$. Using (4), under the assumption that $p(Y)$ does not contribute to the LLR, (3) can be written as

$$\begin{aligned} L(x|y) &= \log_e \frac{p_{Y|X}(y|x=+1) \cdot P_X(x=+1)}{p_{Y|X}(y|x=z) \cdot P_X(x=z)} \quad (5) \\ &= \underbrace{\log_e \frac{p_{Y|X}(y|x=+1)}{p_{Y|X}(y|x=z)}}_{L(y|x)} + \underbrace{\log_e \frac{P_X(x=+1)}{P_X(x=z)}}_{L(x)} \\ &:= L(y|x) + L(x) \quad (6) \end{aligned}$$

The expression in (6) states that the a-posteriori LLR equals the extrinsic LLR less the a priori LLR. $L(y|x)$ can be interpreted as the channel information [5].

We can rewrite (5) as

$$L(x|y) = L(x) + \log_e \frac{p_{Y|X}(y|x=+1)}{p_{Y|X}(y|x=z)} \quad (7)$$

The PDF for the Gaussian distribution is defined as

$$p(y) = \frac{1}{\sqrt{2\pi\sigma_{\mathrm{n}}^2}} \cdot \exp\left(-\frac{(y-\mu_n)^2}{2\sigma_{\mathrm{n}}^2}\right) \quad (8)$$

The LLR value for the channel model given in (1) for receiver n is

$$\begin{aligned} L(x|y_n) = \quad & L(x) + \log_e \frac{\frac{1}{\sqrt{2\pi\sigma_{\mathrm{n}}^2}} \cdot \exp\left(-\frac{1}{2\sigma_{\mathrm{n}}^2}(y_n-h_n)^2\right)}{\frac{1}{\sqrt{2\pi\sigma_{\mathrm{n}}^2}} \cdot \exp\left(-\frac{1}{2\sigma_{\mathrm{n}}^2}(y_n-z\cdot h_n)^2\right)} \\ = \quad & L(x) - \tfrac{1}{2\sigma_{\mathrm{n}}^2} \cdot \left((y_n-h_n)^2 - (y_n - z\cdot h_n)^2\right) \\ = \quad & L(x) + \tfrac{1}{2\sigma_{\mathrm{n}}^2}(2\ y_n h_n(1-z) - h_n^2(1-z^2)) \quad (9) \end{aligned}$$

For repetition MIMO, h_n in (9) is the sum of the channel coefficients $h_{m,n}$, corresponding to receiver n. With $E_s/N_0 = 1/2\sigma_{\mathrm{n}}^2$, the extrinsic LLR for the FSO channel with OOK and DD is given by

$$L(y_n|x) = \frac{E_s}{N_0}(2\ y_n h_n(1-z) - h_n^2(1-z^2)) \quad (10)$$

An interesting observation is that (10) holds for any distribution of the channel coefficients $h_{m,n}$. For the optimal Log-APP detector the channel coefficients are estimated by training symbols. The FSO channel distribution changes over time from gamma-gamma in the mid to high turbulent regime with mid to strong irradiance fluctuations to a log-normal distribution in the low turbulent regime with weak irradiance fluctuations [7]. An advantage of the Log-APP detector for FSO transmission systems is that no adjustment to the time-varying channel needs to be done at the receiver side to find the optimal decision rule. This is different, e.g. in the case of implementing quantization levels for soft decision detection that need to be adjusted to the dynamics of the channel to always find optimum levels. The effort though needed at the receiver is to extract the channel coefficients.

For the special case of full OOK modulation $m = 1, z = 0$, (10) reduces to

$$L(y_n|x) = \frac{E_s}{N_0}(2\ y_n h_n - h_n^2) \quad (11)$$

B. LLR for FSO MIMO Systems (uncorrelated receiver noise)

After calculating the extrinsic LLR for the case of $M = N = 1$ with M number of transmit and N number of receive apertures, the LLR metric for the FSO MIMO channel will be presented. The most obvious question is how the single N LLR values are handled in a spatial diversity scheme.

For the case of $M = N = 4$, the PDF will change to

$$p_{Y1,Y2,Y3,Y4|X}(y1, y2, y3, y4|x) \quad (12)$$

Using the chain rule of probability for an incident with 4 random variable (A,B,C,D) and assuming that the 4 channels are statistically independent, such that they are fully decorrelated and considering the conditional independence between $A_1, A_2, \cdots A_n$, Bayes equation can be written as

$$\begin{aligned} & P(X|Y_1, Y_2, \cdots, Y_n) = \\ & P(Y_1|X) \cdot P(Y_2|X) \cdots P(Y_n|X) \cdot P(X) \quad (13) \end{aligned}$$

Now it is straight forward to write down the LLR values for

[1] Without loss of generality the basis b of $\log_b$ is assumed to be e (natural logarithm).

the n receiver MIMO case.

$$L(x|y_1, y_2, \cdots, y_n) = \\ L(x) + L(y_1|x) + L(y_2|x) + \cdots + L(y_n|x) \quad (14)$$

where the single n LLR values are calculated using (10). For MIMO transmission the LLR values of the single receiving apertures sum up which means that the decision will become more reliable with increasing number of receivers. This is true if the receiver noise sources are uncorrelated.

C. LLR for FSO MIMO Systems (correlated receiver noise)

As shown above, the LLR values sum up for the independent channels in a MIMO transmission scheme. A different result is expected if the receiver noise is correlated. It has to be noted that the correlation behavior under investigation takes into account receiver noise $\mathbf{n}$ and not the channel coefficients $h_{m,n}$. Furthermore the assumptions from II-B need to be met.

So far we have calculated the LLR values by using the Gaussian distribution PDF. If we consider a correlation between the channels (in this example a 2×2 MIMO channel), the two-dimensional Gaussian distribution function needs to be regarded.

$$p(y_1, y_2|x) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho}} \cdot \\ \exp\left(-\frac{\sigma_2^2(y_1-x)^2 - 2\rho\sigma_1\sigma_2(y_1-x)(y_2-x) + \sigma_1^2(y_2-x)^2}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right) \quad (15)$$

The correlation coefficient ρ denotes zero correlation if $\rho = 0$ and thus uncorrelated noise with variances σ_1^2 and σ_2^2 at the receiver. The correlation between the two channels increases with increasing value of ρ until both channels equal at $\rho = 1$.

The PDF from (15) considers white Gaussian noise at the receiver which has its origin mainly in ambient light, as described in II-B. The goal is to find the optimal LLR rule if this receiver noise is correlated. This noise is added to the channel coefficients $h_{m,n}$, that are assumed to be known at the receiver. Similar to (7), we can write

$$L(x|y_1, y_2) = L(x) + \\ \log_e \frac{\exp\left(-\frac{\sigma_2^2(y_1-h_1)^2 - 2\rho\sigma_1\sigma_2(y_1-h_1)(y_2-h_1) + \sigma_1^2(y_2-h_1)^2}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right)}{\exp\left(-\frac{\sigma_2^2(y_1-zh_2)^2 - 2\rho\sigma_1\sigma_2(y_1-zh_2)(y_2-zh_2) + \sigma_1^2(y_2-zh_2)^2}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right)} \quad (16)$$

After some conversions and under the assumption that $z = 0$, (16) reduces to

$$L(x|y_1, y_2) = L(x) + \\ \frac{1}{1-\rho^2}\left[\underbrace{\frac{2y_1h_1 - h_1^2}{2\sigma_1^2}}_{L(y_1|x)} + G_1 + G_2\right] \quad (17)$$

where G_1 and G_2 are given as

$$G_1 = \frac{2y_2h_1 - h_1^2}{2\sigma_2^2} \\ G_2 = \rho\frac{h_1}{\sigma_1\sigma_2}(h_1 - y_1 - y_2) \quad (18)$$

G_1 and G_2 are mixed expressions that depend on the channel information h_1, that is known at the receiver and the two receiver noise sources with variances σ_1, σ_2. The noise sources are weighted by the correlation coefficient ρ.

Besides G_1 and G_2, the LLR value $L(y_1|x)$ is weighted by the term $1/(1-\rho^2)$. The correlation coefficient matrix is symmetric. For the 2×2 MIMO case it is given by

$$\begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix} \quad (19)$$

or for the 4×4 MIMO

$$\begin{pmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 \end{pmatrix} \quad (20)$$

Due to the symmetry, only 6 coefficients have to be calculated. In the case of 4×4 MIMO, (17) will be evaluated for all 6 channel constellations. Again, the sign of the sum of the single results is evaluated for decision. Simulation results will show the increase in performance when using (17) in Section V.

IV. Simulation Setup

Different sets of simulations are performed to show the performance gain for the derived LLR metric under different channel conditions. The main simulation structure is based on a simple source pattern generator that generates uniformly distributed binary data. This data is distributed to the M transmit apertures and distorted by uncorrelated log-normal FSO characteristic channel coefficients. At the receiver, additive white noise is added to each N receive aperture, which could be correlated. $\mathbf{H}$ is assumed to be known at the receiver side. The detector type is realized as a Log-APP detector for soft decision, that takes into account the correlation, a MRC detector and a simple majority detector for reference purposes.

As mentioned in Section II, the overall coding scheme includes an outer convolutional encoder with adjustable rates by puncturing and a random interleaver before the data is fed into the MIMO repetition encoder.

According to the work in [5], we perform Monte Carlo simulations to determine the BER. Two different simulation approaches exist that both are based on the soft output of the Log-APP detector.

A. Simulation Method A - Log-APP (hard sim)

The first set takes into account the hard-decision of the LLR values. If more than one channel exists, the sum or the weighted sum of LLR values is regarded, depending on the correlation of the receiver noise sources. In this method the sign of the LLR values is evaluated and compared to the sign of the transmitted info bit. For a MIMO setup with $M = N > 1$ the sign of the sum of the single LLR values will be evaluated. This first set is denoted as *Log-APP (hard sim)*. As in most BER simulations the transmitted bits have to be known at the receiver to calculate the BER. This is usually not the case in real transmission systems. To still get a measure of transmission quality (besides receive power) a second method has been developed in [5].

B. Simulation Method B - Log-APP (soft sim)

The second set takes into account soft-decision, based on the probability of the LLR values to determine the BER. Each LLR value will provide a probability value. The mean of the sum of several probability values will give a BER value for that particular duration. This method allows to measure the BER at the receiver without knowing the transmitted sequence which has advantages in terms of simulation complexity.

In a real transmission environment the BER calculation without knowing the transmit sequence could be used for extracting channel state information with respect to BER performance and could thus be used as a threshold to change code rates at the transmitter for a more robust transmission. For a MIMO setup with $M = N > 1$ the sum of the LLR values will be used for calculating the probability values. Considering (14), spatial diversity gain is included in the results if $M = N > 1$. This set of simulation is denoted with *Log-APP (soft sim)*.

C. Simulation Method C - Majority decision

The *Hard dec (majority)* case is based on a simple majority decision of the received data. The sum of the received data is evaluated as

$$D(k) = \frac{1}{N} \sum_{N} y_N(k) \tag{21}$$

where N is the number of receivers. The hard decision rule is

$$E_D[k] = \begin{cases} 0 & \text{if} \quad D[k] \leq P_b[k]/2 \\ 1 & \text{else} \end{cases} \tag{22}$$

Where $E_D[k]$ is the hard decision at time index k, and $P_b[k]$ is the amplitude of the transmitted signal in the non-disturbed case. The simple majority decision case is implemented for comparison to show the gain of the Log-APP detector.

V. Performance Results

In this chapter we present performance results for the FSO MIMO channel and the optimal detection rule for the Log-APP detector, adjusted to the FSO characteristics. Especially the LLR metric, as derived in Section III, will be compared to different channel conditions and turbulence strengths. The additional gain in detection performance if the receiver noise is correlated will be presented for different numbers of transmit and receive apertures M and N. For all simulation results the total transmit power is kept constant for a fair comparison. The analytical results found in (17) and (18) are used in the detector and the simulation results over different correlation coefficients are shown in Fig. 2. The correlation coefficients are set in a way that the noise between receiver 1 and 2 and between receiver 2 and 4 are most correlated. Correlation of the remaining 4 constellations is negligible. In Fig. 2, the correlation coefficient denotes the strongest correlation between two channels. The 4×4

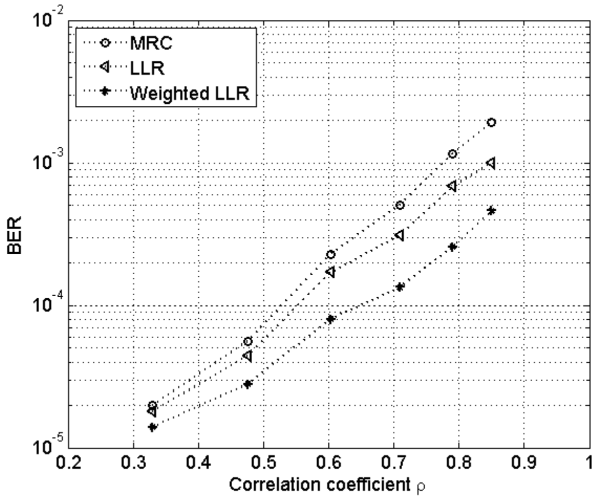

Fig. 2. Receiver Correlation impact on uncoded 4×4 MIMO scheme with (weighted) LLR and MRC detection at target BER of 10^{-6} at a SNR of 11 dB

MIMO simulation uses white Gaussian noise sources at the receiver that are correlated by the correlation coefficient ρ. The y-axes shows the BER. To achieve a BER of 10^{-6} an SNR of 11 dB was used. With increasing correlation, the BER performance decreases and the gap between LLR, MRC and weighted LLR (metric that takes into account the correlation coefficients) gets larger. For uncorrelated receiver noise the three detection schemes show the same performance. The LLR metric outperforms the MRC metric when considering receiver noise correlation. The most important result is that the new LLR metric that takes into account the single correlation coefficients shows an improvement compared to the LLR and MRC metric.

After presenting the behaviour of the LLR values if the receiver noise is correlated, we further compare the different BER simulation types, using the LLR decision metric as derived in (10) for the uncorrelated case.

Fig. 3 shows the comparison of the Monte-Carlo simulation results for the Log-APP detector for an uncoded transmission with uncorrelated receiver noise. The BER curves of the Log-APP detector with *hard* and *soft* simulation align, which is a similar result as published in [5], stating that the BER probability prediction is as accurate as the comparison between transmit and receive bits.

The scintillation index has been set to 0.18, which equals a moderate turbulent FSO channel. In Fig. 3(a) a simple 1×1 transmission is assumed. The majority decision case shows a BER floor at BER of $> 10^{-2}$. Log-APP detection gain comes from the fact that the channel coefficients are available and used in the detection process. The MRC detector performs as good as the Log-APP detector, since the channel coefficients are not correlated.

In Fig. 3(b), the 2×2 MIMO transmission shows the additional spatial diversity gain (about 12 dB at a BER of 10^{-6}). The fading mitigation effect of MIMO transmission can also be seen at the majority decision where the BER floor now reaches a BER of 10^{-4}. Further diversity gain is achieved with a 4×4 MIMO setup as shown in Fig. 3(c) which brings an additional diversity gain of about 3 dB at a BER of 10^{-6}, compared to the 2×2 MIMO case.

The diversity gain is not significantly increased by increasing the aperture number from 2 to 4. This comes from the fact that turbulence induced fading at a scintillation index of about $\mathrm{SI} = 0.18$ is nearly sufficiently compensated by a 2×2 setup. With increasing SI, the diversity gain between 2×2 and 4×4 will also increase. Spatial diversity gain in MIMO systems has been for example also studied in [1], [4].

For the uncoded case, the *soft Log-APP* calculation matches the common Monte-Carlo simulation case where the transmitted bits need to be known at the receiver.

To further increase the performance of the transmission system a convolutional code is implemented, followed by an interleaver, as described in Section IV. In this last set of simulations we show how the FSO MIMO transmission performance can be further boosted by outer coding schemes. The convolutional code has a rate of 1/3. Decoding is performed by a Viterbi decoder that uses the soft outputs (LLR values) of the Log-APP detector. Fig. 4 compares the coded and uncoded version of the Log-APP detector for different $M \times N$ constellations at a fixed scintillation index. The code gain for the 1×1 scheme is about 20 dB at a goal BER of 10^{-6}. Increasing the number of transmit and receive apertures induces additional spatial diversity gain. The difference between coded and uncoded gain becomes smaller and levels at about 12 dB.

VI. Conclusions

In this contribution we have investigated the powerful Log-APP detector for a repetition coded FSO MIMO scenario. To achieve a certain reliability, it is inevitable to make use of scintillation mitigating techniques such as spatial diversity.

We concentrated on the comparison between the well known MRC detector and the Log-APP detector that has been adopted to take into account receiver noise correlation. Our results show that over a wide realistic range of scintillation impacts following the log-normal distribution, the use of four transmit and four receive apertures is well suited to achieve a saturation in spatial diversity gain. This means that by further increasing the number of transmit and receive apertures no significant performance gain can be achieved. Basically we have shown that the Log-APP detector and the MRC detector have a similar performance if the receiver noise is uncorrelated.

The new metric for the Log-APP detector that takes into account receiver noise correlation outperforms simple Log-APP or MRC detection. In a scenario where the correlation due to ambient light is dominant at the receiver, the new metric can increase the receiver performance by about 2 dB at a worst case correlation coefficient of 0.9 between two channels for the uncoded 4×4 MIMO case.

In conjunction with FEC coding and interleaving in combination with Log-APP detection, the repetition MIMO performance is further improved and short term signal losses can be covered by a suitable interleaver length. Operating points well below 0 dB can be achieved.

Future work will address higher order modulation schemes, such as QAM in combination with repetition coding and Log-APP detection for the FSO channel to further increase the data rate.

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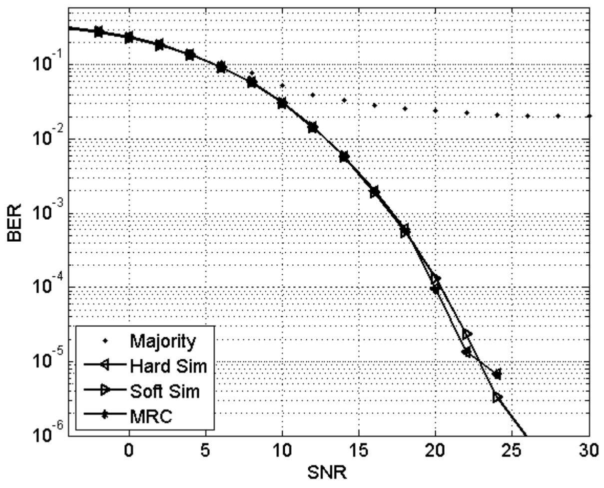
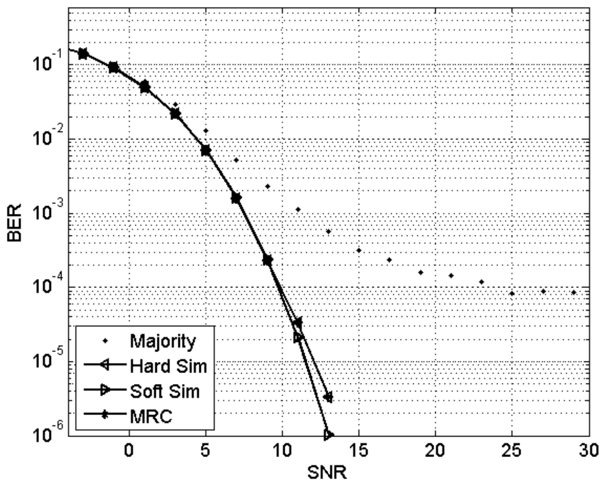
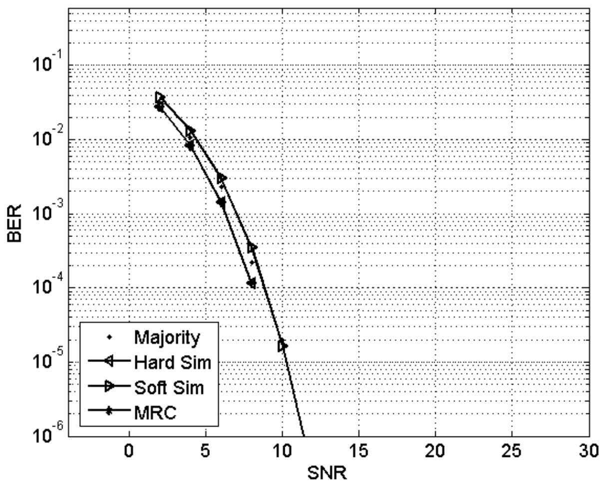


(a) 1×1 SISO

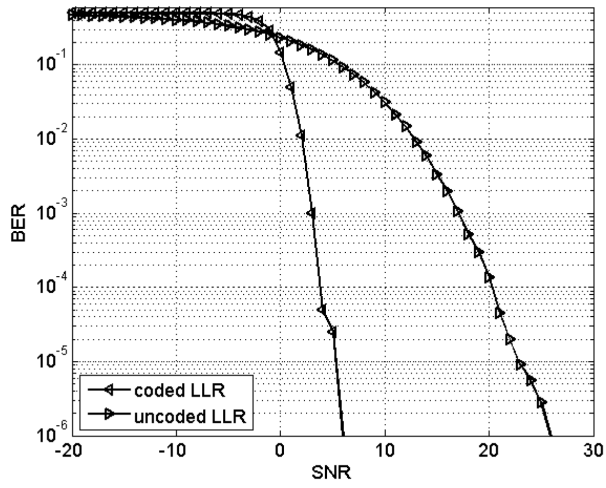


(b) 2×2 MIMO

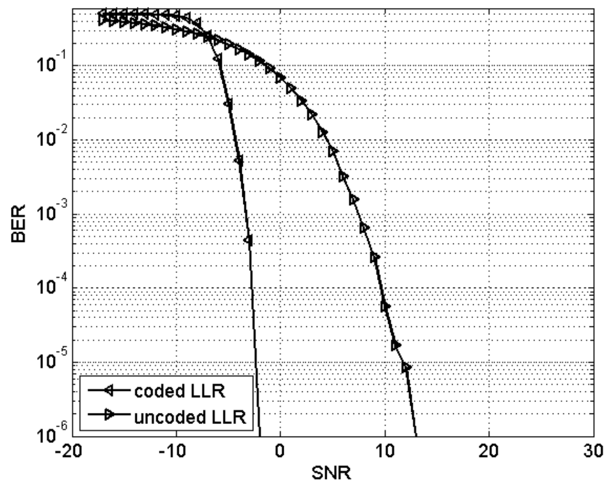


(c) 4×4 MIMO

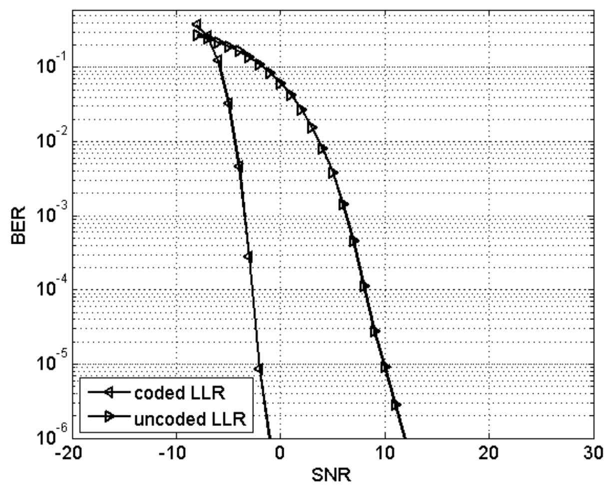
Fig. 3. Uncoded $M \times N$ repetition MIMO with Log-APP detection (soft/hard sim) for moderate turbulence channels (SI=0.18). Comparison of two methods to determine the BER. Majority decision and MRC detection for comparison.



(a) BER simulation using Log-APP detection (1×1 MIMO)



(b) BER simulation using Log-APP detection (2×2 MIMO)



(c) BER simulation using Log-APP detection (4×4 MIMO)

Fig. 4. Code gain for $M \times N$ repetition MIMO with Log-APP detection. Data is coded by convolutional encoder with rate 1/3 and interleaved by random interleaver before distributed to M transmitters.