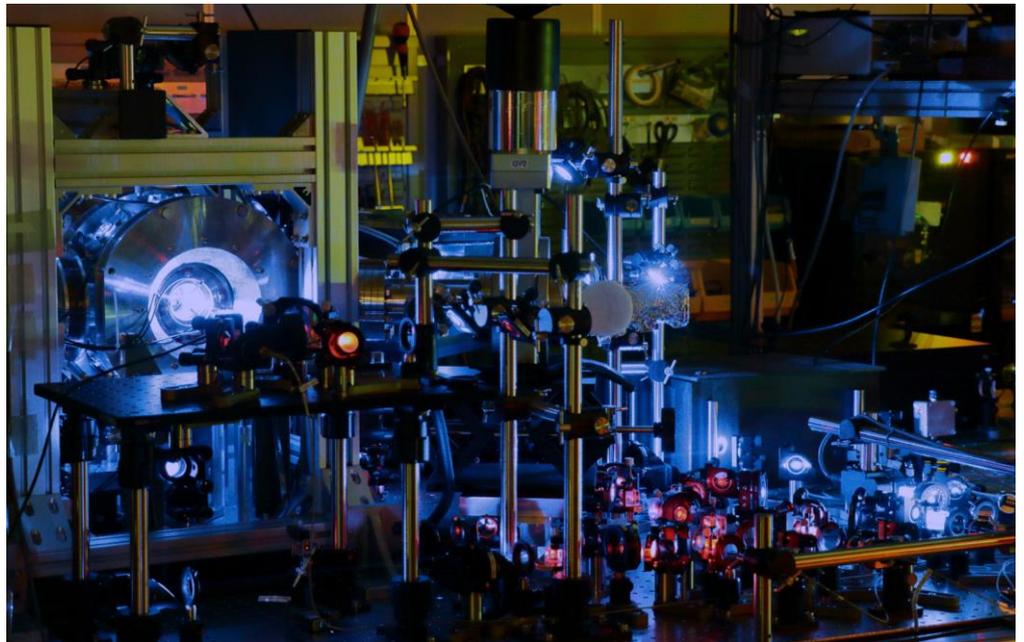


Evaluation of the TAI scale interval using an optical clock

National Institute of Information
and Communications Technology
(NICT)
Space-Time Standards Laboratory

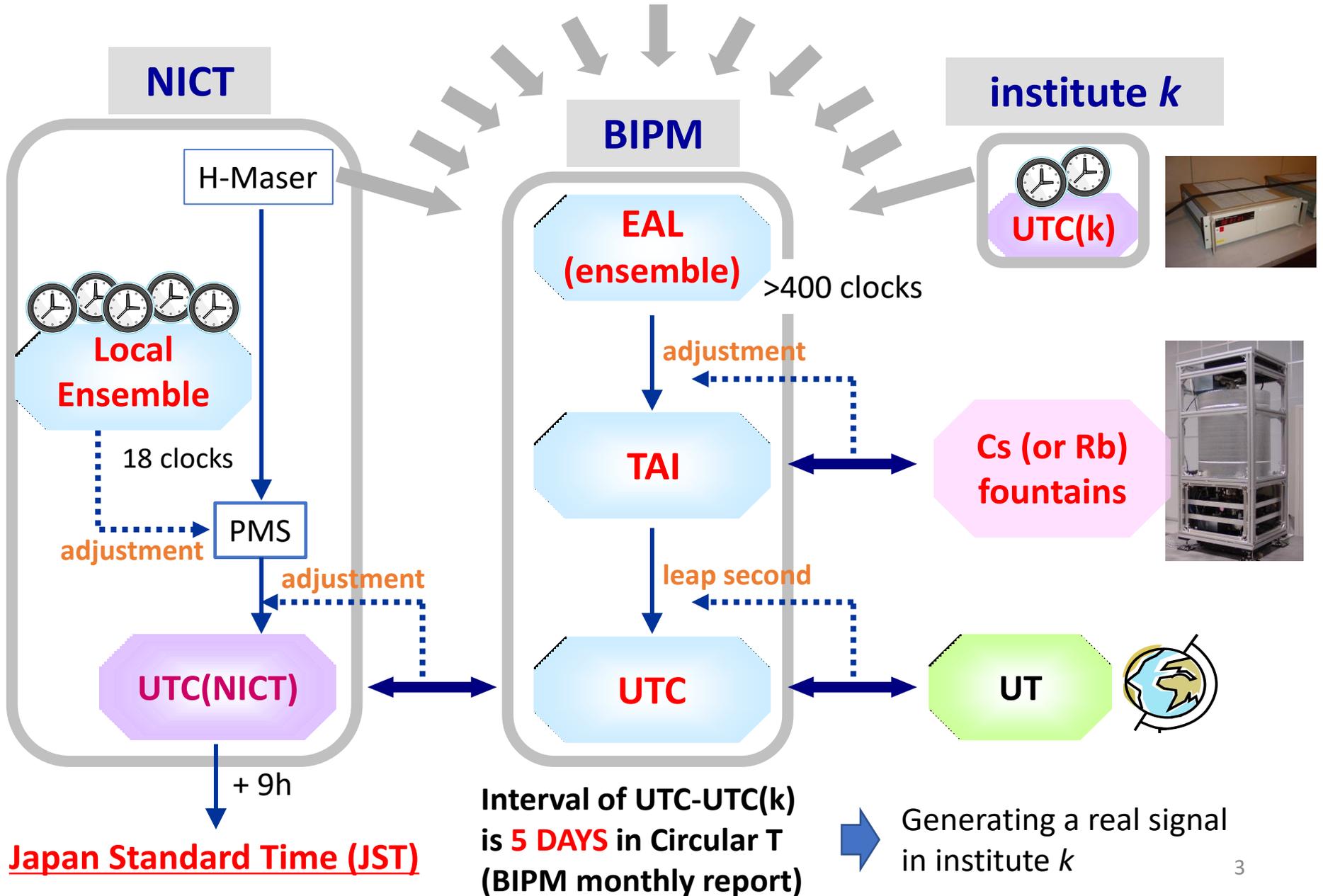
T. Ido

H. Hachisu, N. Nemitz,
F. Nakagawa, T. Gotoh,
Y. Hanado

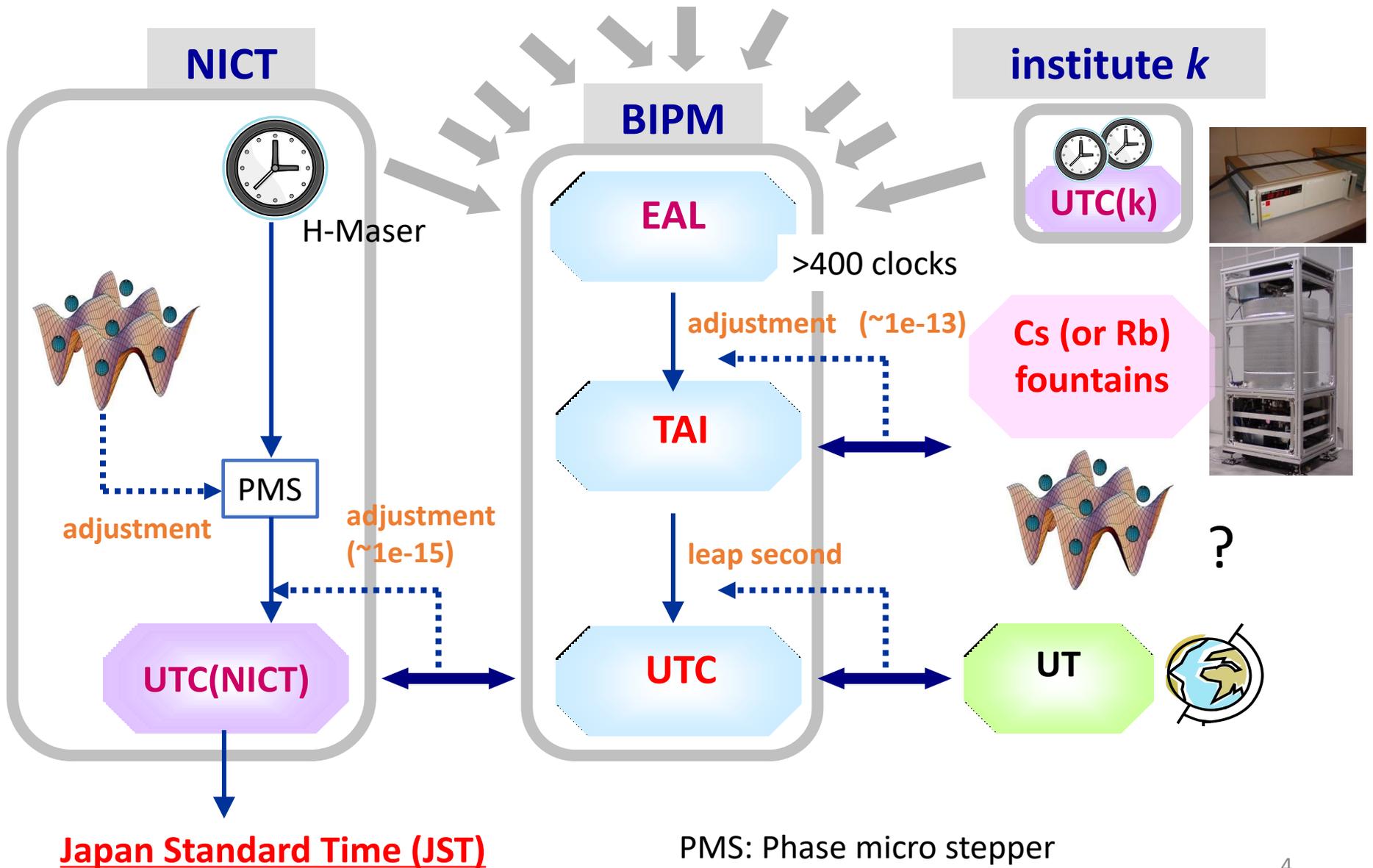


Optical – microwave hybrid time scale using Sr lattice clock

Time scale



Optical clocks utilized for time scale



How much clocks deviate from linear drift?

Allan variance (= two sample variance) is insensitive to constant frequency difference.



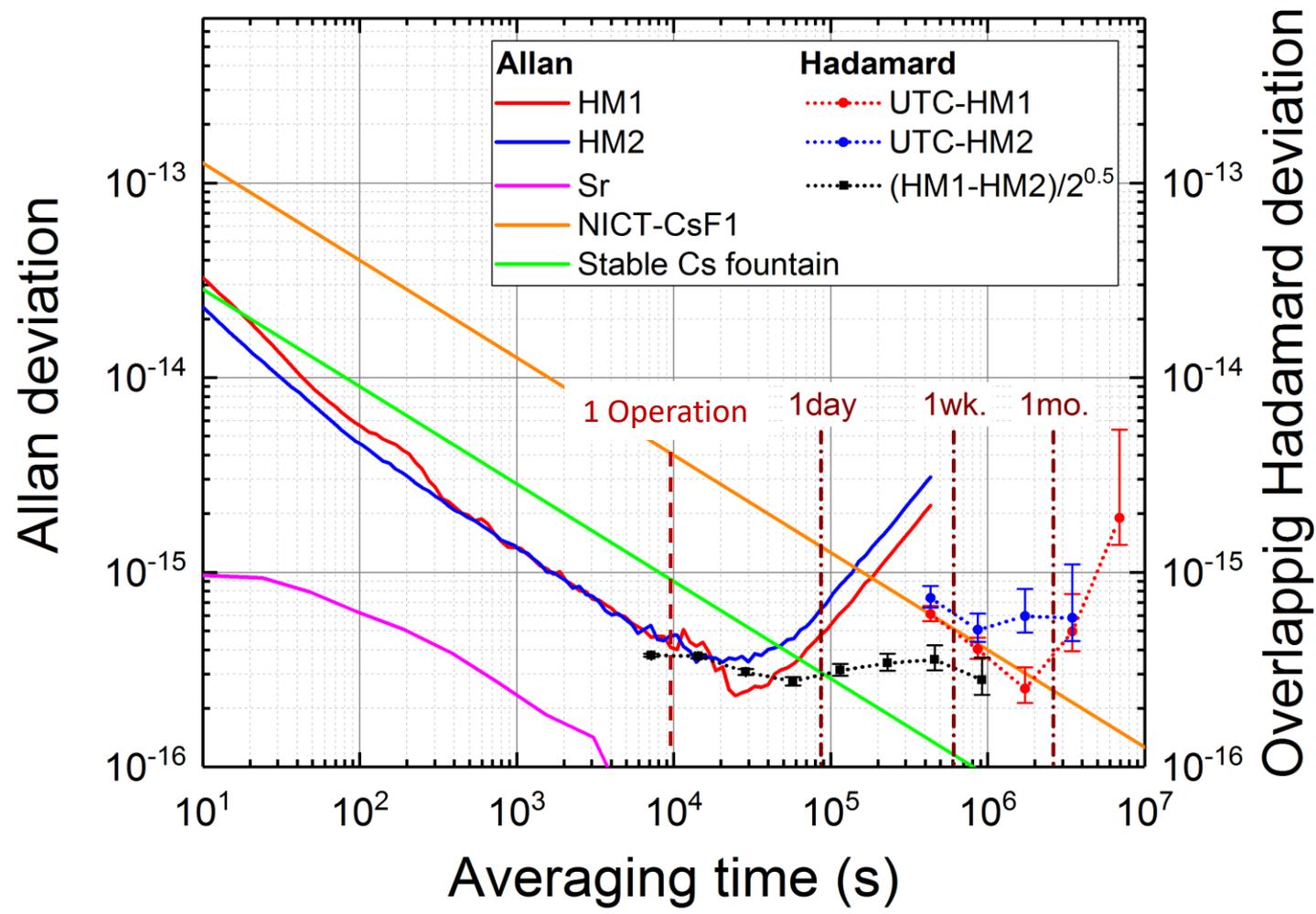
Raise the # of variance.

Three sample variance (Hadamard variance) is insensitive to constant difference of frequency drift rate.

3-sample variance (Hadamard variance)

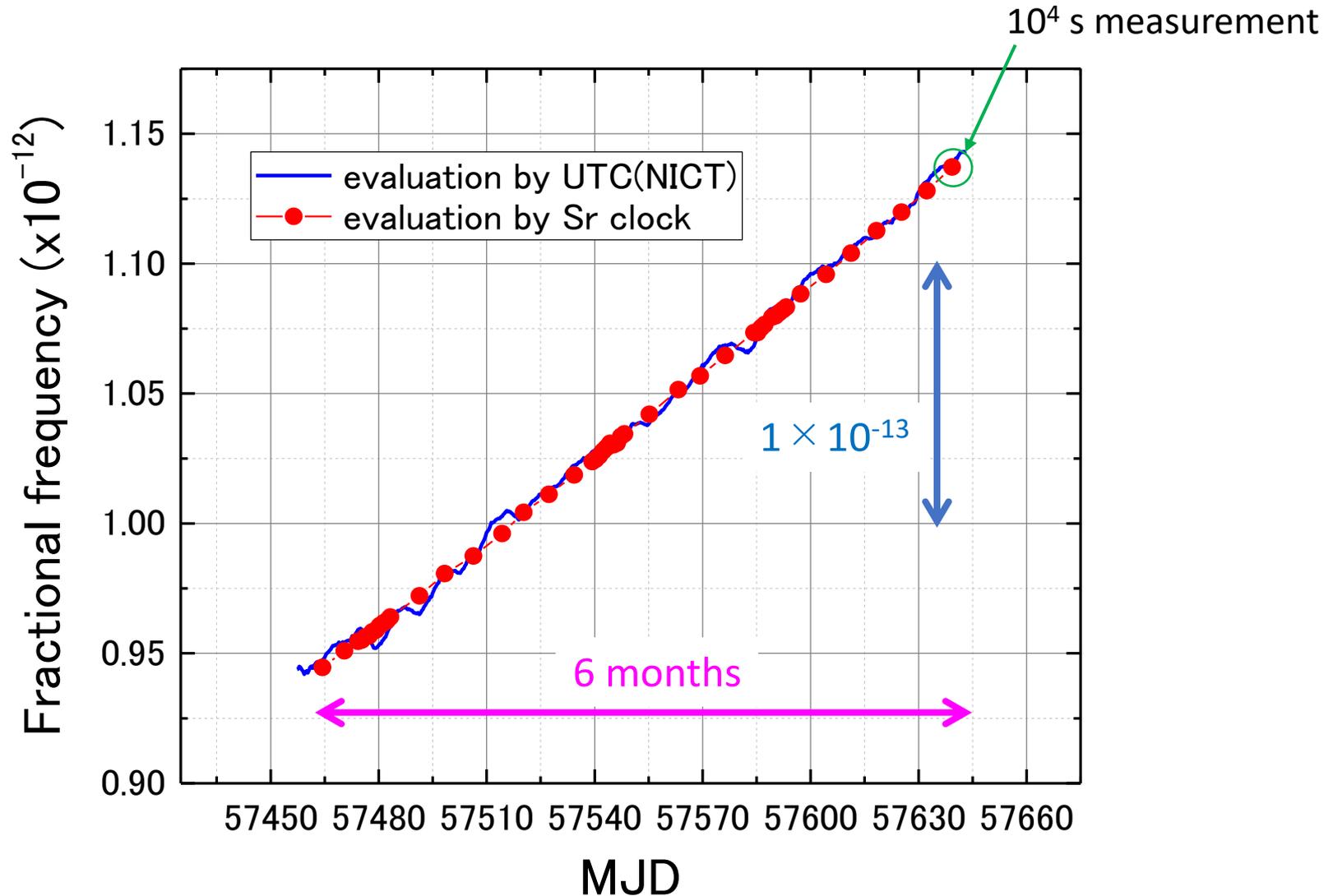
$$\frac{1}{6(M-2)} \sum_{i=1}^{M-2} [y(i+2) - 2y(i+1) + y(i)]^2 = \frac{1}{6(N-3)\tau^2} \sum_{N=1}^{N-3} [x(i+3) - 3x(i+2) + 3x(i+1) - x(i)]$$

Advantage of “optical” steering



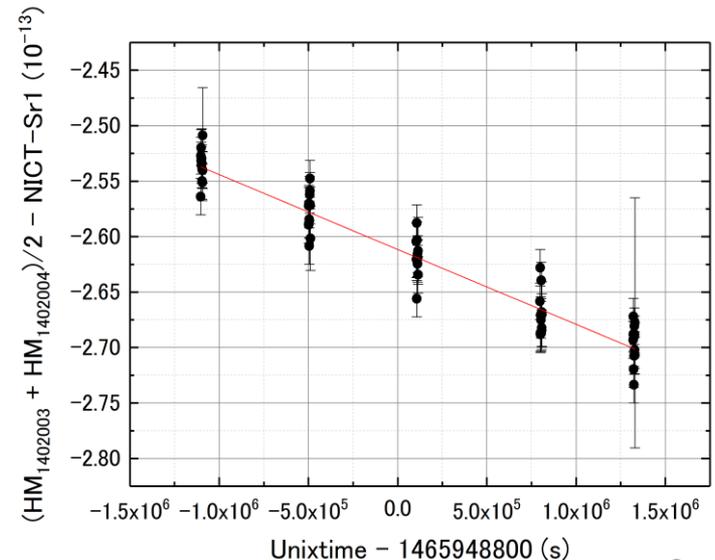
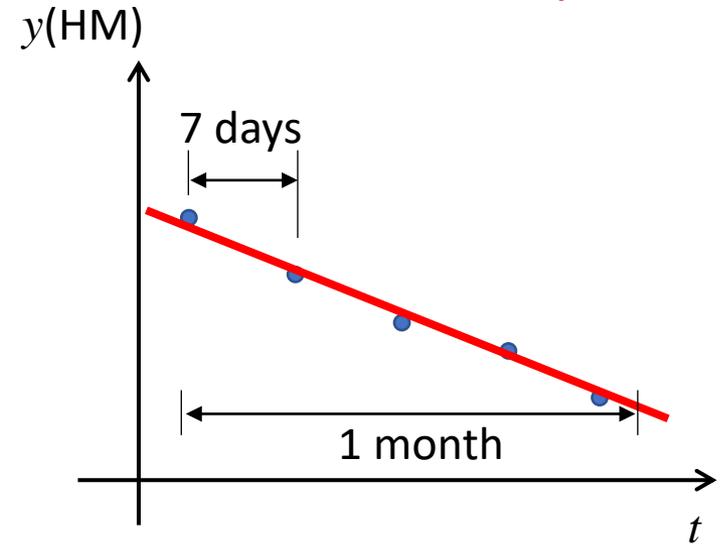
- 10^4 s of operation is sufficient to evaluate the scale unit of a HM at mid- 10^{-16} level
- Short term fluctuation of HM may be compensated by an optical clock.
- Not necessary to operate all the time.
once in a week for 3 hours

INTERMITTENT Evaluation of a HM for 6 months by Sr and UTC(NICT)

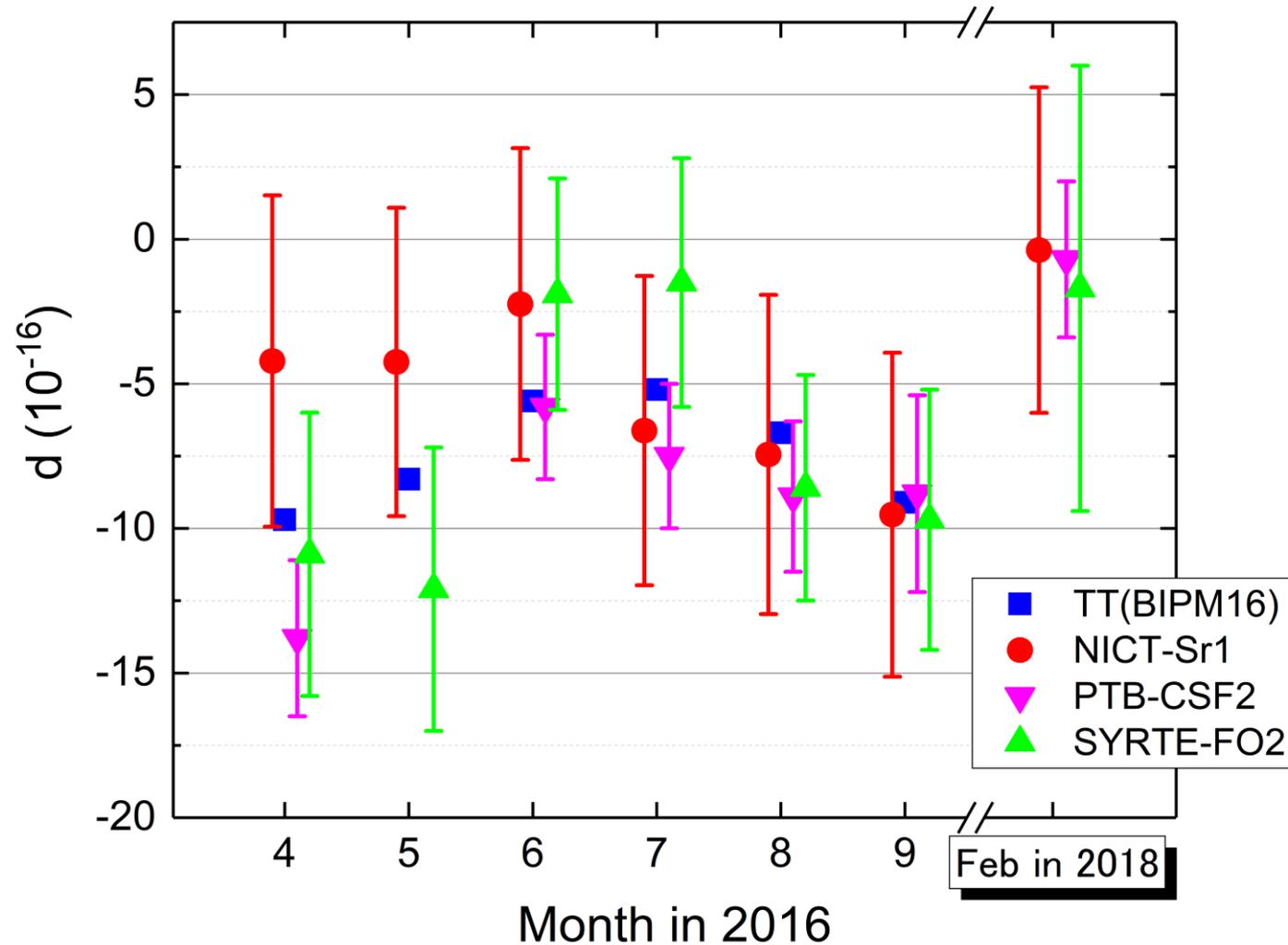


Evaluation of one-month mean TAI scale interval by Sr

Effects	Uncertainty (10^{-17})
Sr systematic	6
Gravity	2.2
Hydrogen maser	
deterministic	25
stochastic (dead time)	18
Phase measurement	5
UTC-UTC(NICT) link	26 (30 days average)
Sr frequency (uSrep)	40 (CIPM 2017)
Total	57 (40 w/o uSrep)

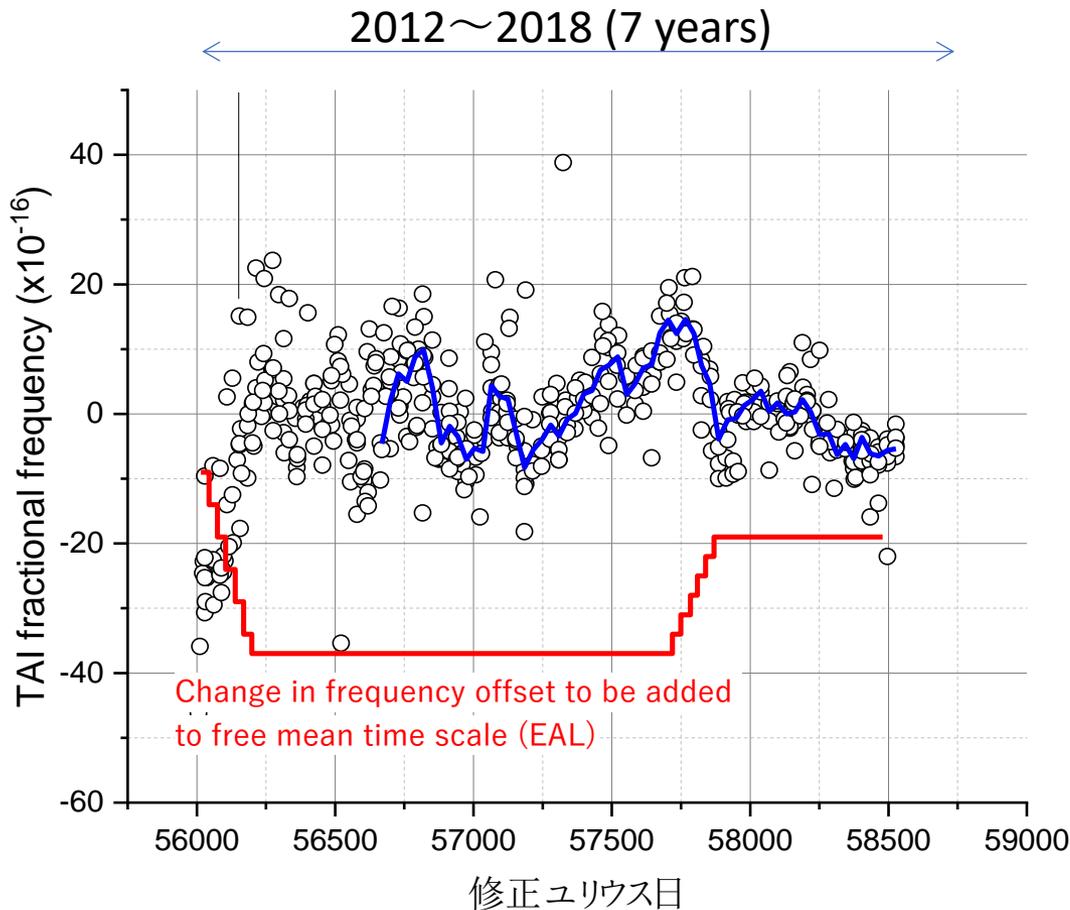


Result: Evaluation of TAI scale interval by Sr



Sr standard frequency: 429 228 004 229 873.0 Hz
(2017 CIPM#)

History of TAI scale interval (evaluated by Cs or Rb fountains)



- Error less than 1×10^{-15} in most of time
- BIPM manually tunes TAI referring primary (Cs) and secondary (Rb) microwave standards validated by CCTF-WG
- Only those in PTB and SYRTE routinely evaluate these days (Difficulties in regular operations?)

Evaluation of TAI scale interval

Table 1: Estimate of d by individual PSFS measurements and corresponding uncertainties.

All values are expressed in 10^{-15} and are valid only for the stated period of estimation.

Standard	Period of Estimation	d	u_A	u_B	u_{lab}	u_{Tai}	u	u_{Srep} Ref(u_S)
PTB-CS1	58664 58694	-4.80	8.00	8.00	0.00	0.07	11.31	PFS/NA
PTB-CS2	58664 58694	-7.38	5.00	12.00	0.00	0.07	13.00	PFS/NA
NICT-Sr1	58644 58679	0.68	0.01	0.07	0.21	0.17	0.28	0.4 [1]
NIM5	58679 58694	-0.25	0.30	0.90	0.20	0.37	1.04	PFS/NA
SYRTE-FO1	58664 58694	0.62	0.30	0.32	0.10	0.20	0.49	PFS/NA
SYRTE-FO2	58664 58694	0.95	0.20	0.24	0.16	0.20	0.40	PFS/NA
SYRTE-FOM	58664 58694	0.56	0.25	0.61	0.06	0.20	0.69	PFS/NA
SYRTE-FORb	58664 58694	0.87	0.15	0.25	0.07	0.20	0.36	0.6 [1]
SU-CsFO2	58664 58694	0.86	0.21	0.22	0.12	0.46	0.56	PFS/NA

Table 2: Estimate of d by the BIPM based on all PSFS measurements identified to be used for TAI steering over the period MJD58304-58694, and corresponding uncertainties.

Period of estimation	d	u
58664-58694	0.67x10**-15	0.21x10**-15 (2019 JUN 30 - 2019 JUL 30)

PTB-CS1, Cs2: not fountain, but Cs beam

Circular T (Aug, 2019)

NICT-Sr1 provides the least $u = 2.8 \times 10^{-16}$

Normally, fountains in PTB report with the smallest $u = 0.2 \sim 0.3 (\times 10^{-16})$

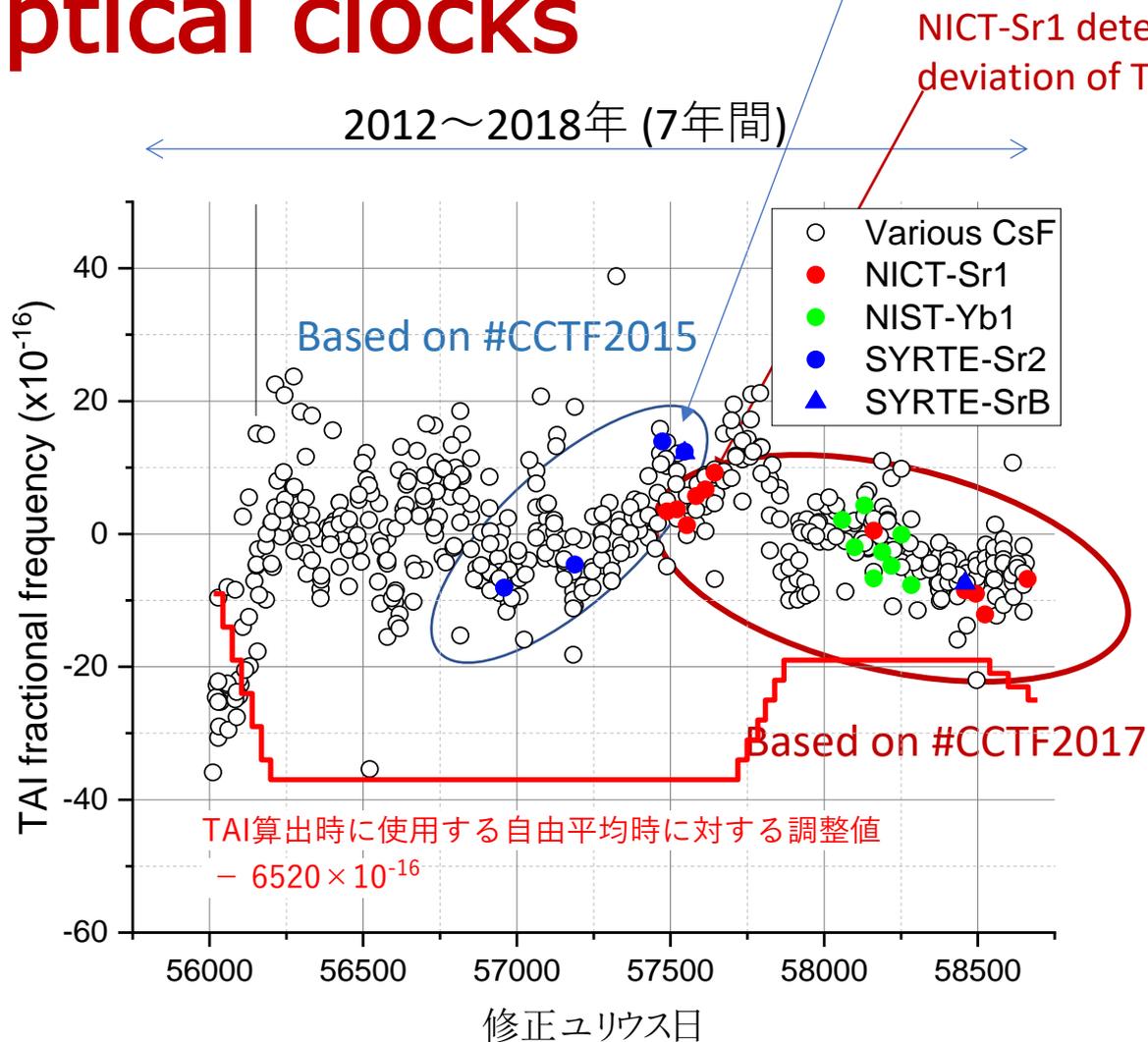
TAI calibration using optical lattice clocks

- Working group in CCTF validates frequency standards which can be usable to calibrate the one second of TAI.

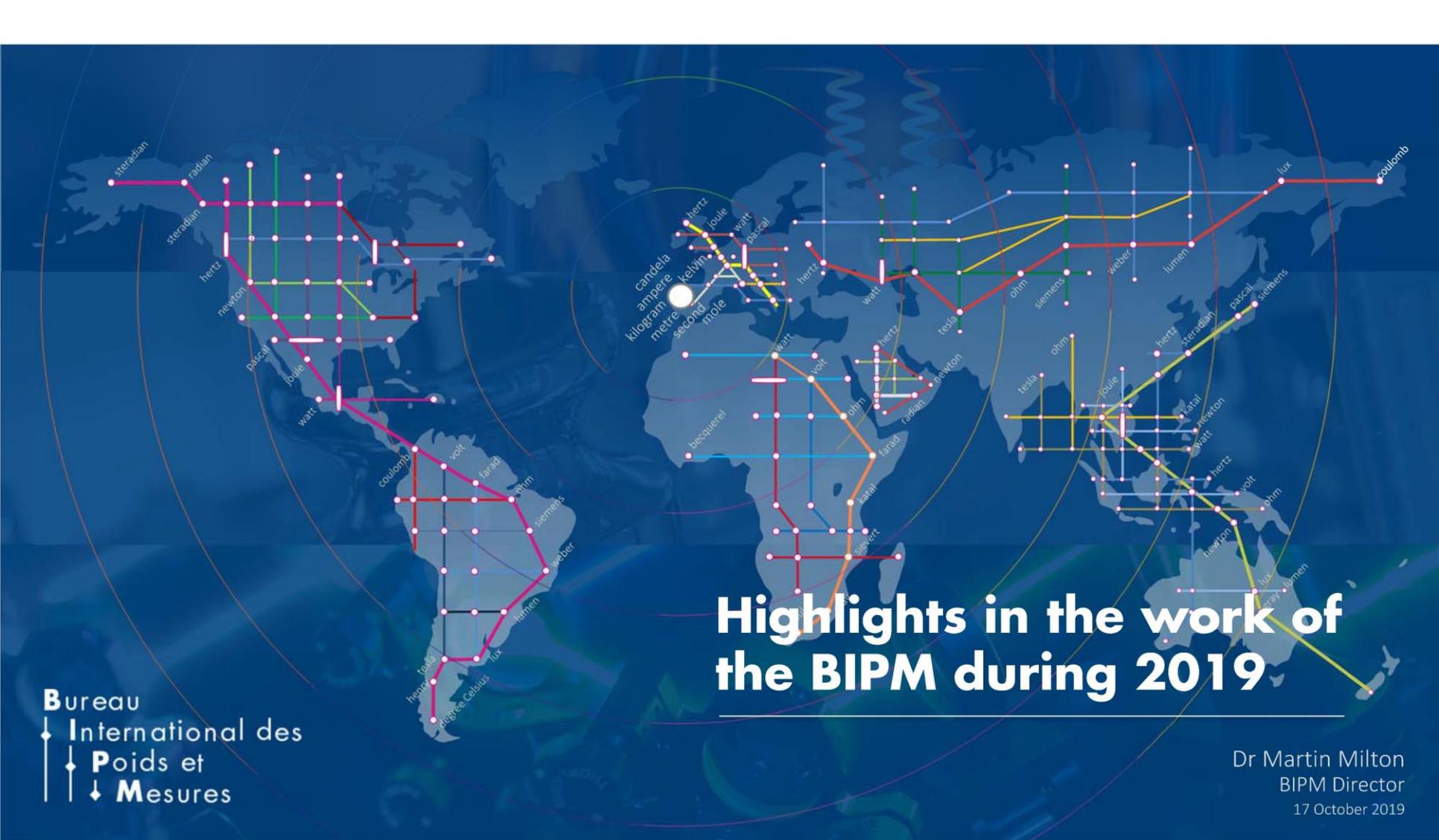
Optical lattice clocks accepted by CCTF working group

	SYRTE-Sr2 & SrB	NICT-Sr1	NIST-Yb1	INRIM-Yb1
Test calibration (check using past data)	2014/10 (10d) 2015/06(20d) 2016/03(10d) 2016/05(15d)	2016/04~09 (30dX6) 2018/02 (30d)	2017/11~ 2018/06 (30dX8)	
WG validation	2017/02	2018/11	2019/02	2019/11
“on-time” calibration	2018/12	2018/12 2019/01 2019/02 2019/07	N/A	N/A

TAI calibrations provided by optical clocks



- Latest two points of NICT-Sr1 reflect the manual adjustment of TAI by BIPM, which Cs fountains don't show clearly
- These data may demonstrate the advantage of optical clocks over fountains. → Another driving force to the redefinition of the second



Highlights in the work of the BIPM during 2019

Bureau
International des
Poids et
Mesures

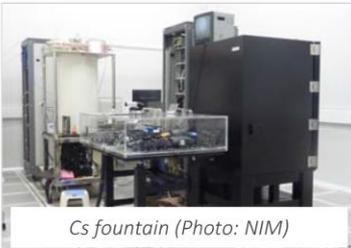
Dr Martin Milton
BIPM Director
17 October 2019

Search: **Highlights BIPM 2019**

New frequency standards contribute to UTC

In August 2019 the relative accuracy of UTC with respect to the SI second was:

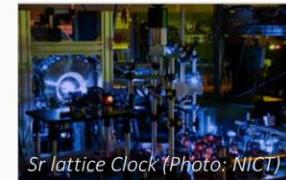
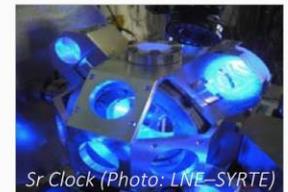
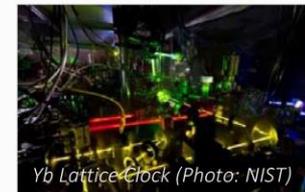
$$0.32 \pm 0.13 \times 10^{-15}$$



- The accuracy of UTC is based on the steering versus the primary realizations of the SI second.
Primary realizations from NMIs are 10 Cesium fountains (accuracy 10^{-16}) and 2 traditional Cesium beam (accuracy 10^{-14}). 6 additional Cs fountains under development.

- **Secondary representation of the second are also contributing:**

- July 2015, first contribution from the SYRTE Rb fountain
- March 2017, first results from the SYRTE Strontium lattice standards
- Dec 2018, the NICT and SYRTE Strontium standards and
- Feb 2019, the NIST Ytterbium lattice standard entered in the UTC computation.



Summary

- Opt. –MW hybrid timescale generated in real time for half a year, demonstrated stably and accurately
- Optical clocks started contributions to maintain UTC, which is social time employed in society. **First application of optical clocks to our daily life**

Team

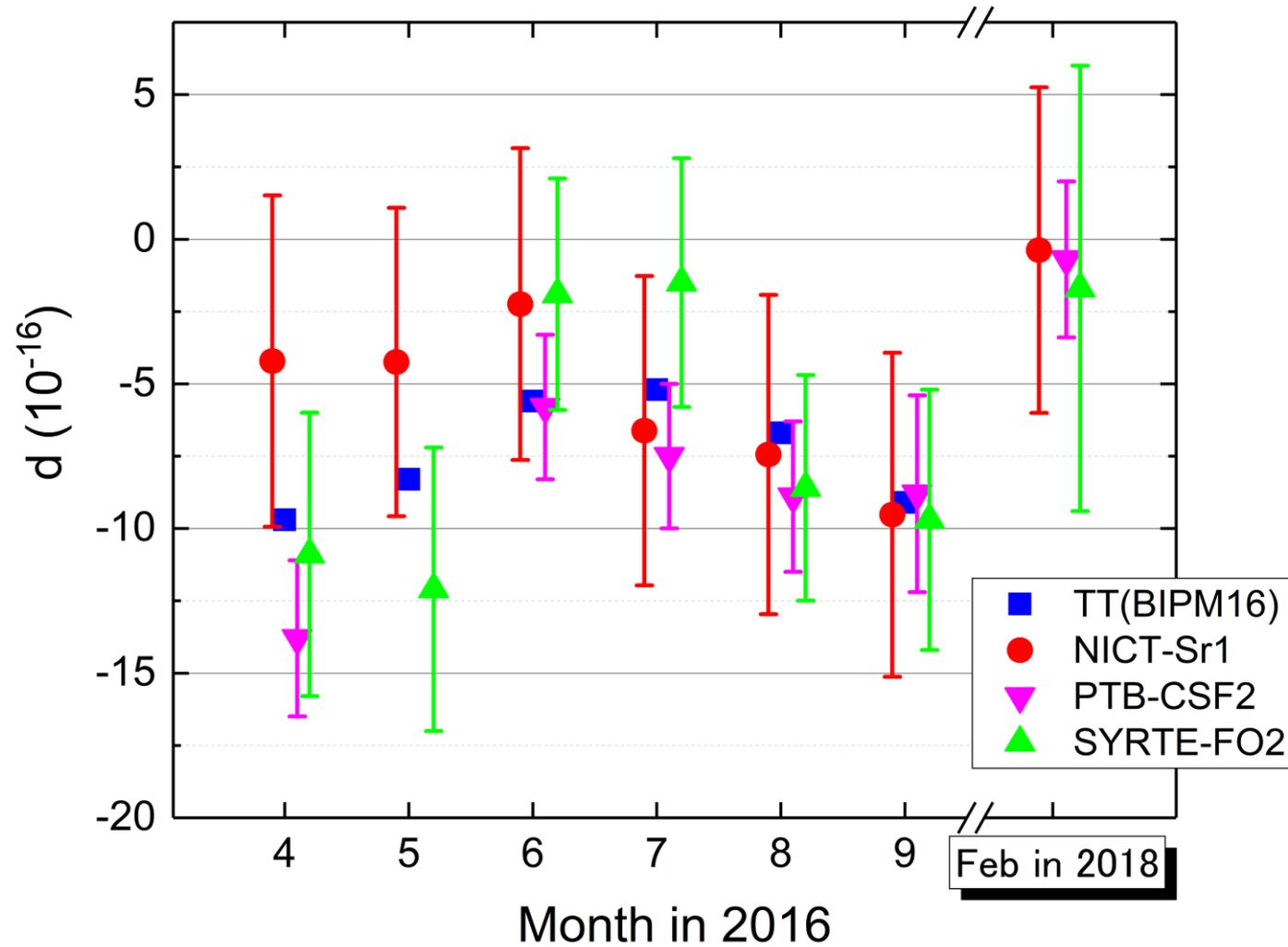
Particularly, strong contribution to the work presented here

Sr: H. Hachisu, N. Nemitz

Time scale: F. Nakagawa Y. Hanado

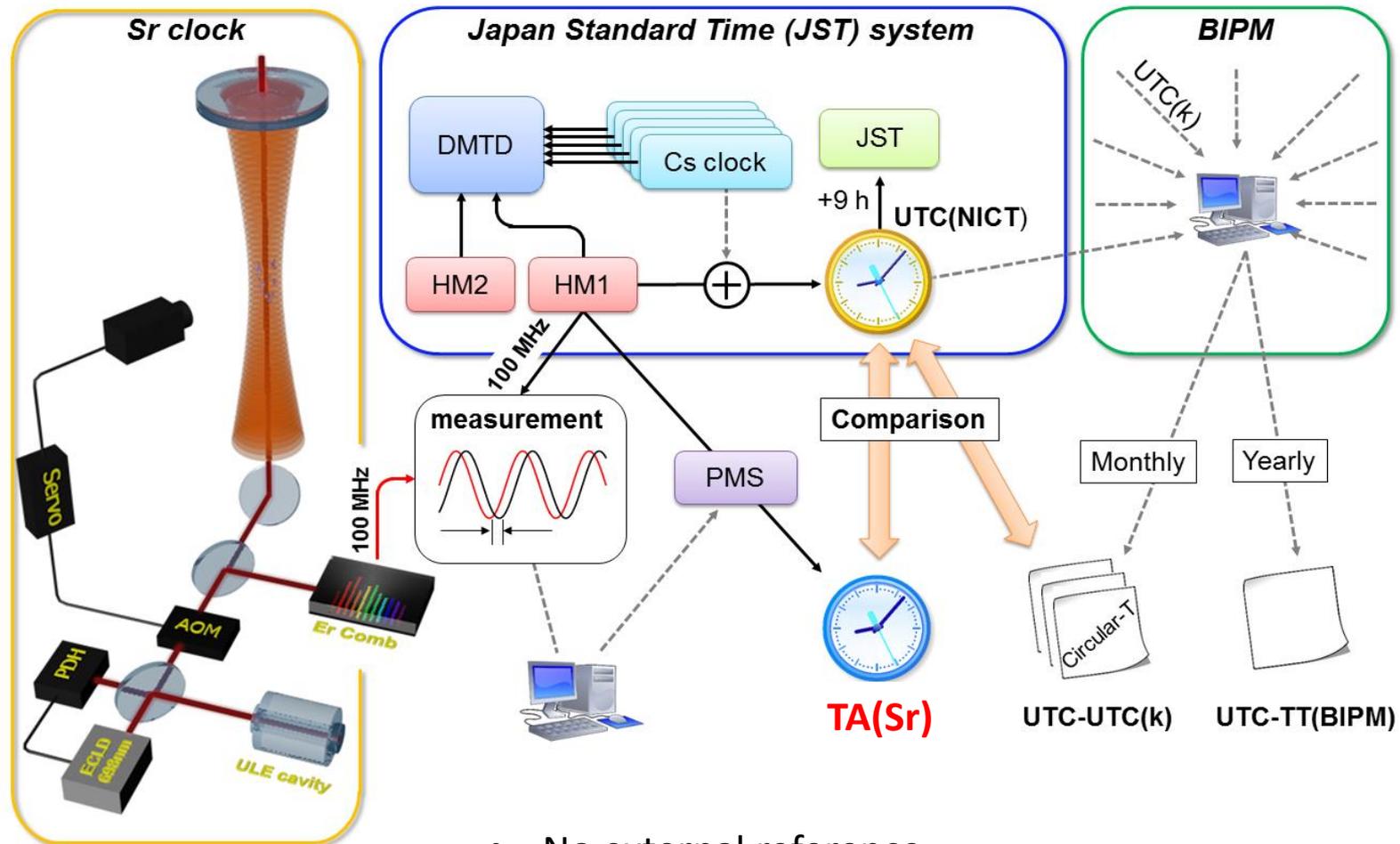


Result: Evaluation of TAI scale interval by Sr



Sr standard frequency: 429 228 004 229 873.0 Hz
(2017 CIPM#)

Optically steered time scale



- No external reference
- Reference to compare with: UTC, TT(BIPM16)

What do we need to reach the redefinition of the SI second?

Prediction of the timeline for redefinition of the second

- 3 clocks
 $\Delta\nu_i/\nu_i \sim 10^{-18}$
- 3 comparisons
 $\Delta\nu_i/\nu_i < 5 \times 10^{-18}$
- 3 clocks
 $\Delta\nu_i/\nu_i < 3 \times 10^{-16}$
- Regular contribution to TAI
- 2 ratio comparisons agree among 5 transitions
 $\Delta(\nu_i/\nu_j)/(\nu_i/\nu_j) < 5 \times 10^{-18}$

Uncertainties ~ **two order of magnitude** better than Cs

agreements of the same optical clocks in **different institutes**

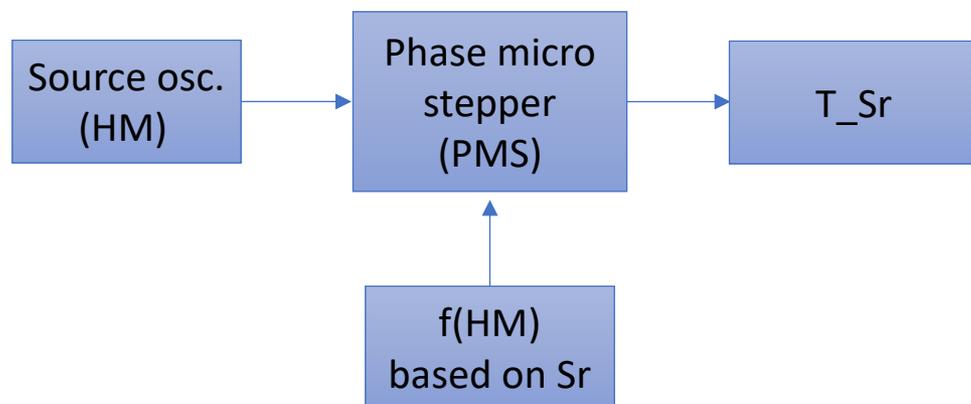
Continuity with present definition: independent measurements w.r.t. three independent Cs primary clocks

Measurement reports covering at least ten days

Locally available frequency ratios in >2 laboratories (Yb/Sr, Yb+/Sr, Hg/Sr, Al+/Sr, In+/Sr)



Steering by intermittent operation of Sr lattice clock: “Opt. – μ wave hybrid time scale”



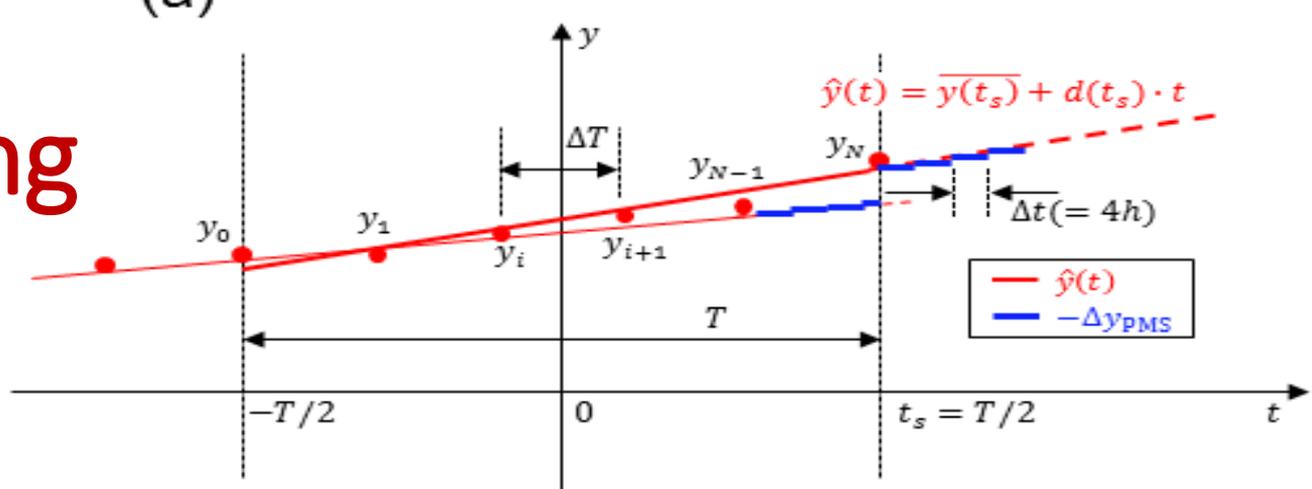
- HM frequency and drift rate calibrated by Sr
- **Adjustment** of PMS offset frequency **every 4 hours**
- No servo to reduce the time offset UTC-T_{Sr}
- **Based on our frequency*** obtained in 2015



Intermittent operation more than **once a week for 10^4 s** continued

*429 228 004 229 872.97 Hz (Hachisu et al., Appl. Phys. B **34**, 123 (2017))
... 873.2 Hz (CIPM2015) → ... 873.0 Hz (CCTF WGFS, Jun 2017)

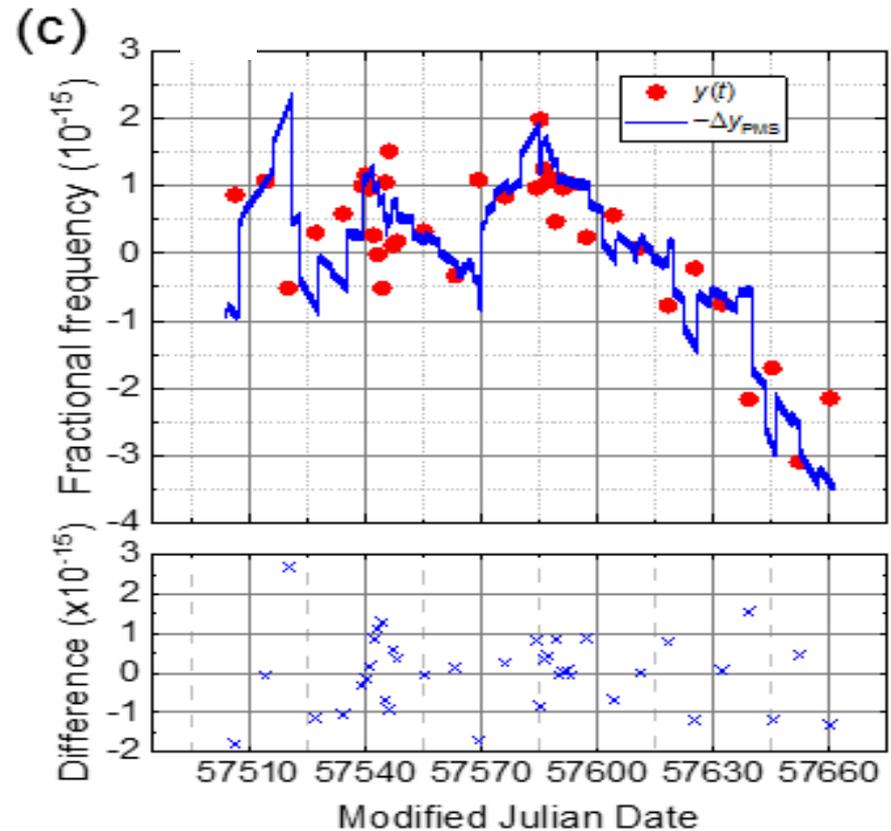
Steering



Linear drift estimation interval: $T=25$ days

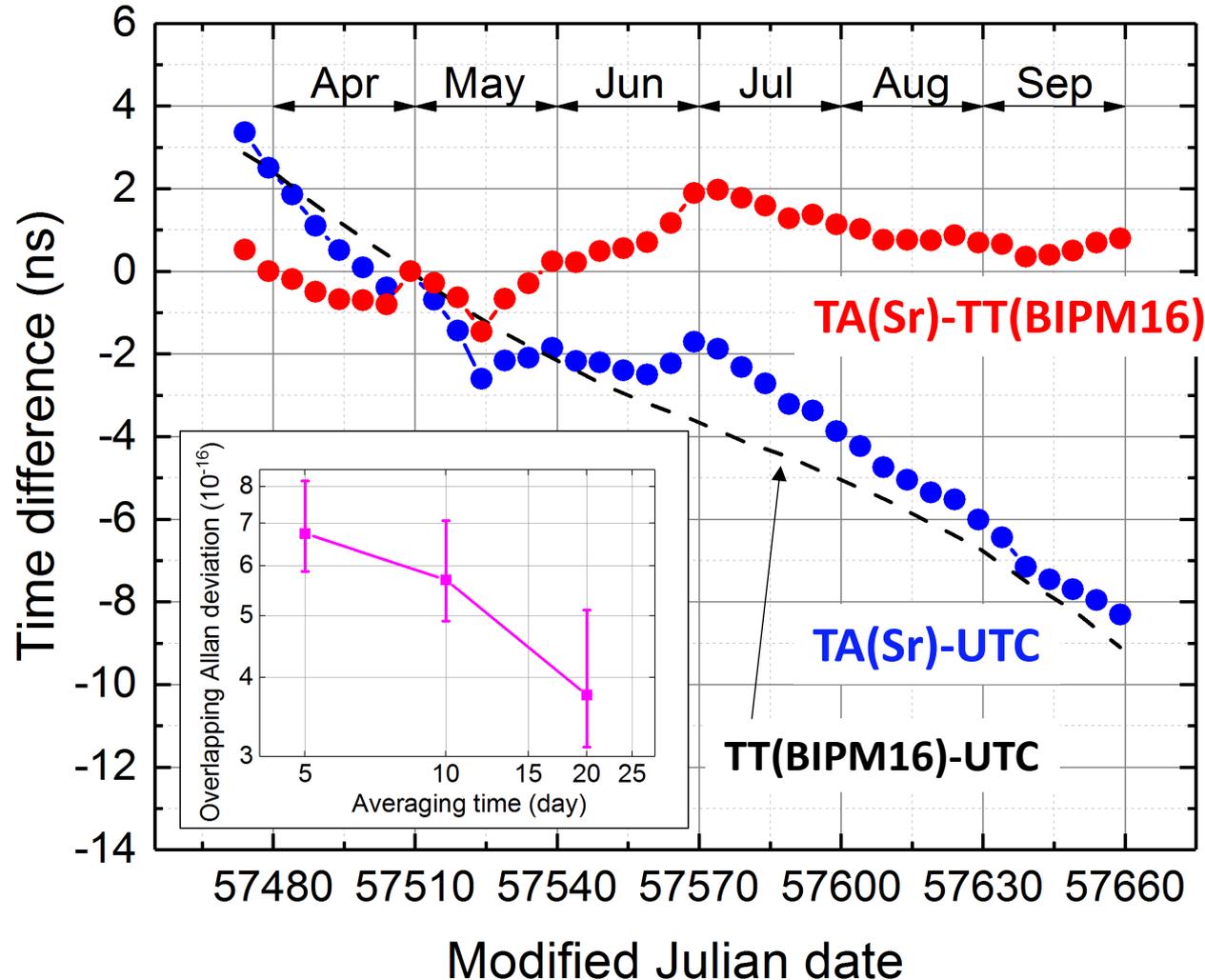
Number of OFS operation in T : $N+1 > 4$
(once per week or more frequently)

One HM free evolution time: $\Delta T = T/N$



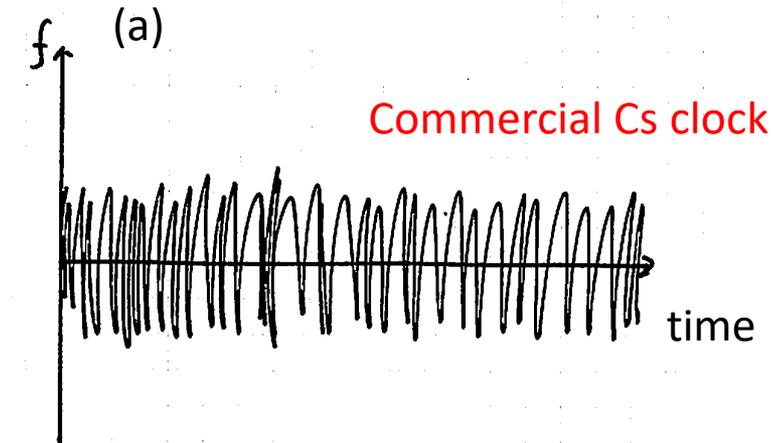
Comparison against UTC & TT(BIPM16)

- Clearly detect the frequency offset of UTC
- Phase difference against TT(BIPM16)
 - < 1ns after 5months
- Stability $4e-16$ @ 20 days

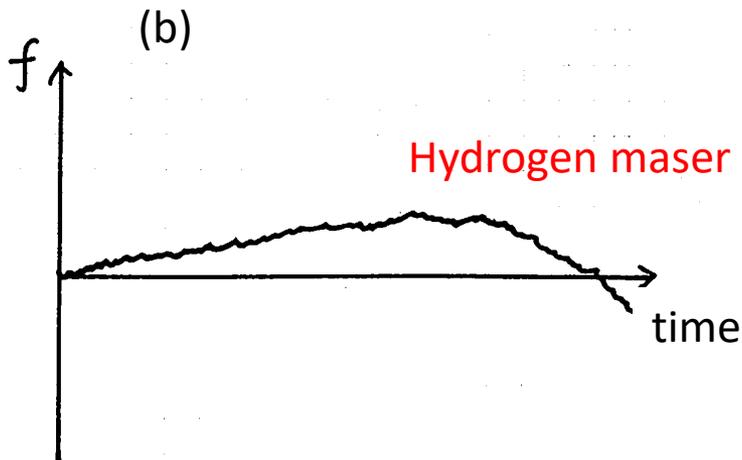


Japan Standard Time (JST)

Commercial Cs clock & H maser



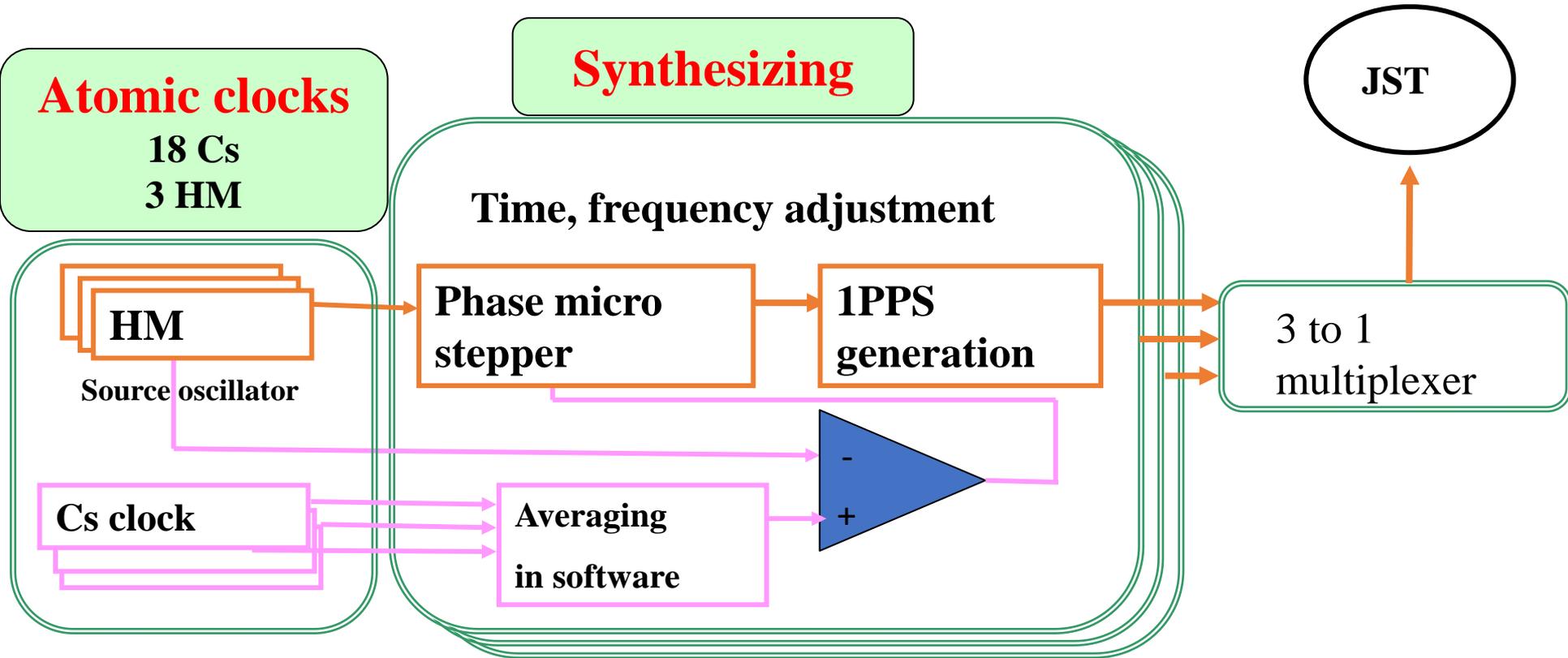
9.2GHz
Noisy, but no drift



1.42GHz
Less (short-time) noise, but drifty

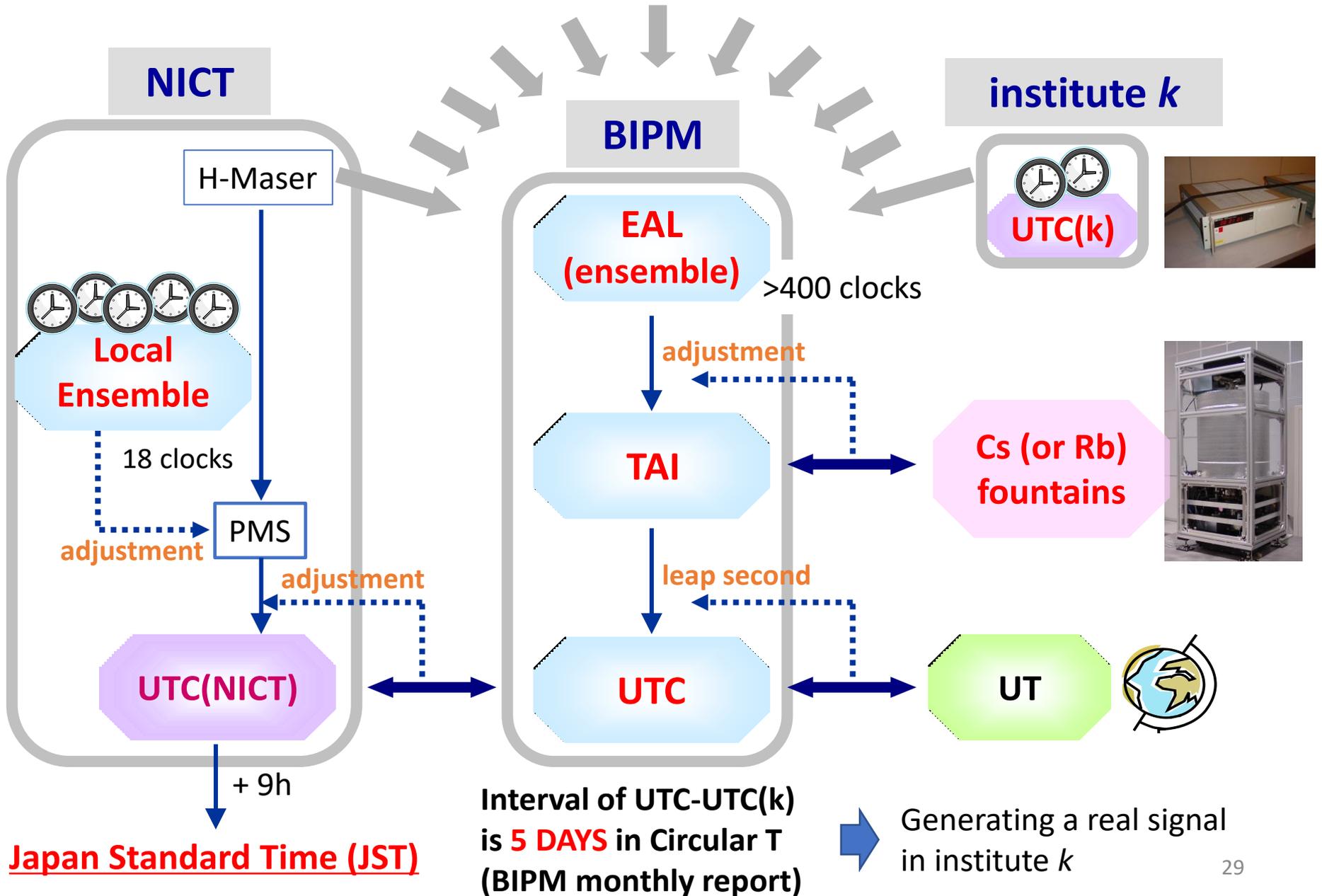
JST employed only Cs clocks before 2006. But since 2006, Cs and H-maser have been combined to get better stability in short term too.

JST generation



HM frequency was steered to ensemble of 18 Cs clock

Time scale



JST : Time scale generation

Timescale

■ Clocks for generating UTC(NICT) :

Cs 5071A : **18** (ensemble timescale)

Anritsu H-Masers : **1**(source) + **2**(backup)

■ The behavior of UTC(NICT) :

| UTC – UTC(NICT) | < **20 ns.**

Stability ~ **2E-15 @ 10-30d.**



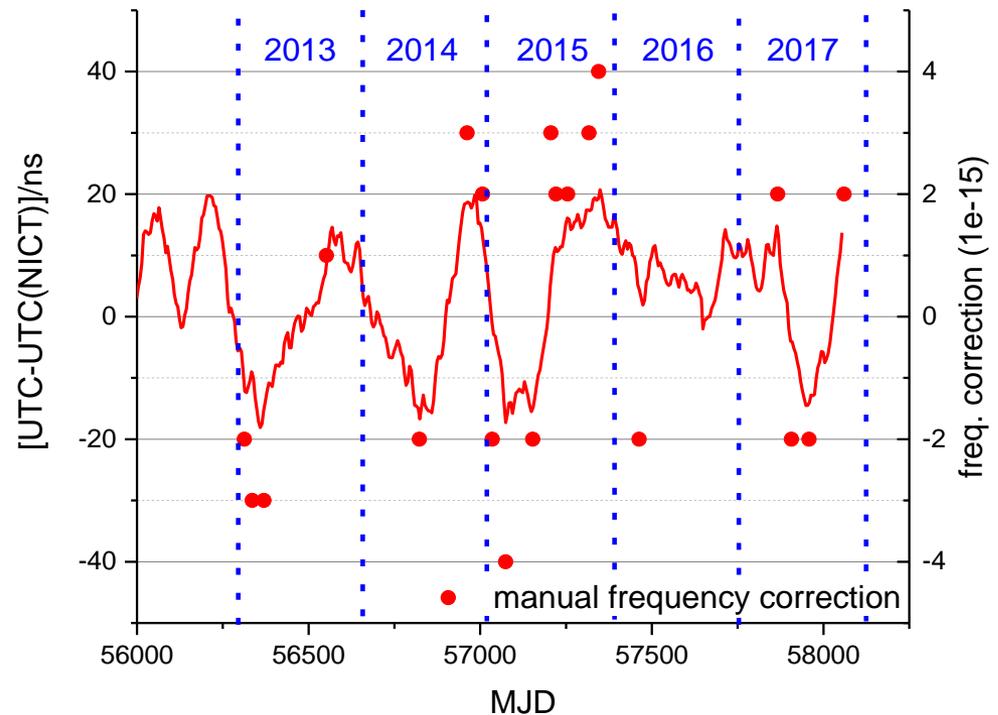
Accuracy

conservative: $5e-14$

(employed for calibration service)

Standard deviation: $<4e-14$

24hours, 7days a week

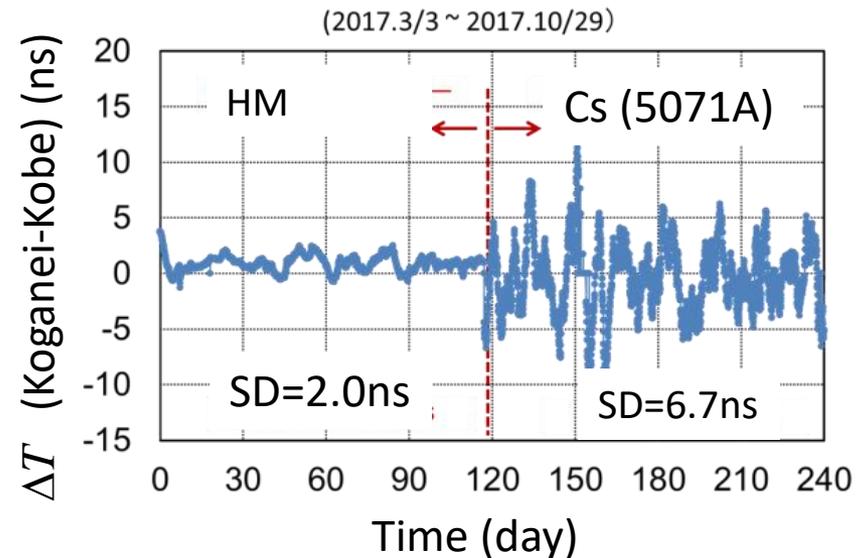


NICT JST Kobe sub-station in becoming ready

Distributed generation of JST

- Kobe sub-station is scheduled to begin time-keeping in June 2018
- 2 H-maser & 5 Cs clocks
- Primary purpose is a backup of Koganei HQ against disasters

(JST has never stopped in more than 40 years)



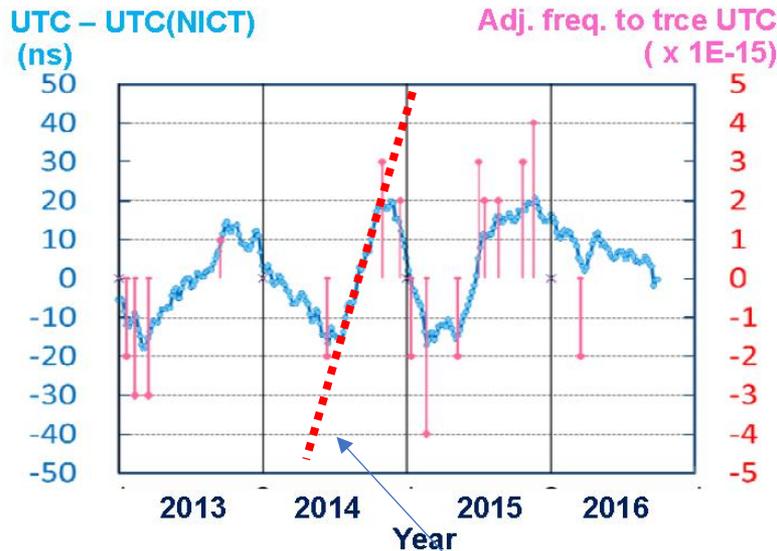
Operation mode:

1. Copy of Koganei HQ
2. Independent operation
3. JST as an ensemble of all clocks operated in 4 stations

JST : Time scale generation

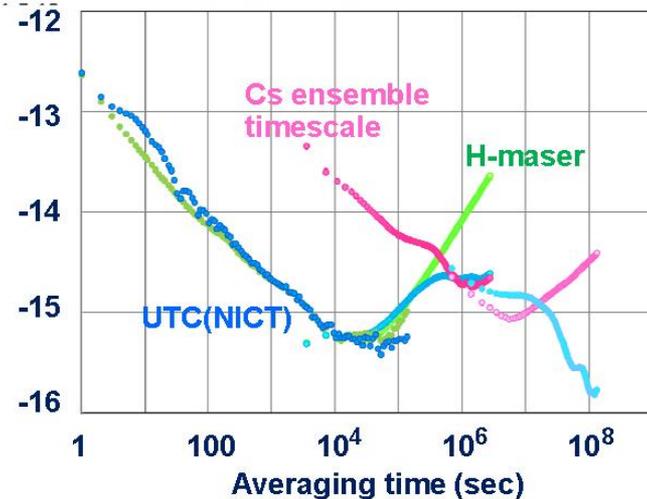
Timescale

- Clocks for generating UTC(NICT) :
 - Cs 5071A : **18** (ensemble timescale)
 - Anritsu H-Masers : **1**(source) + **2**(backup)
- The behavior of UTC(NICT) :
 - | UTC – UTC(NICT) | < **20 ns.**
 - stability $\sim 5 \times 10^{-16}$ @1d, 2×10^{-15} @10~30d



$$80 \text{ ns} / 0.6 \text{ year} = 4 \times 10^{-15}$$

Allan deviation (log scale)



clocks
n of a time

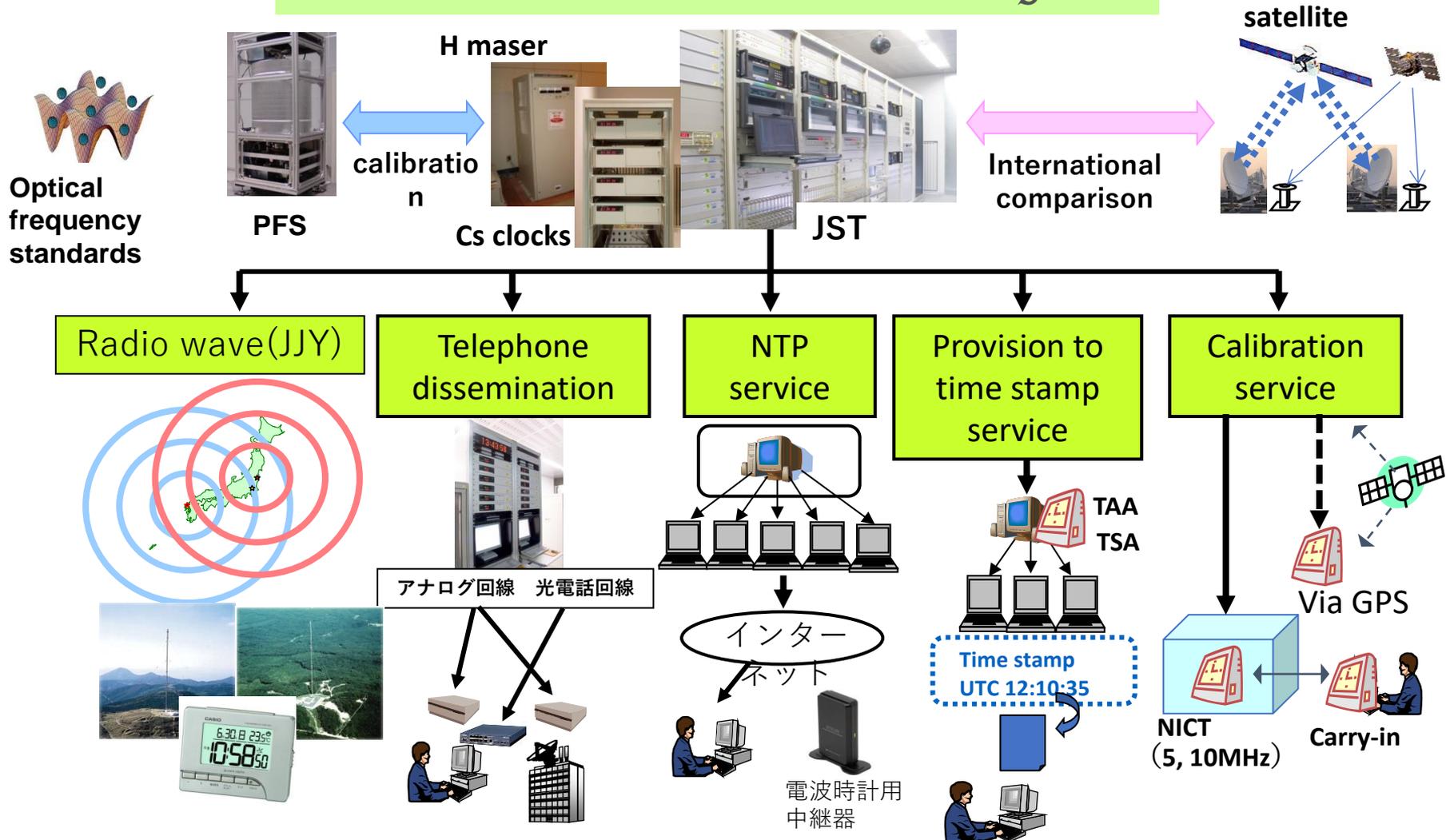
Tetsuya Ido



Space-Time Standards Laboratory
National Institute of Information and Communications Technology
Japan

NICT Dissemination service of Japan Standard Time

JST Generation and measurement system



Secondary representation of the second (SRS)

Frequency combs proposed in late 90s easily provide coherent link of optical frequency to (microwave) SI second.

→ Optical clocks became SI traceable.

→ Transitions with uncertainty in same level as the best Cs recognized as SRS

Ion trap

	u_{SRS} (E-16)	u_{sys} (E-17)	# of labs
Al+	19	0.9	1
Hg+	19	1.9	1
Yb+(E3)	6	0.32	2
Yb+(E2)	6	3.2	2
Sr+	15	2.2	2

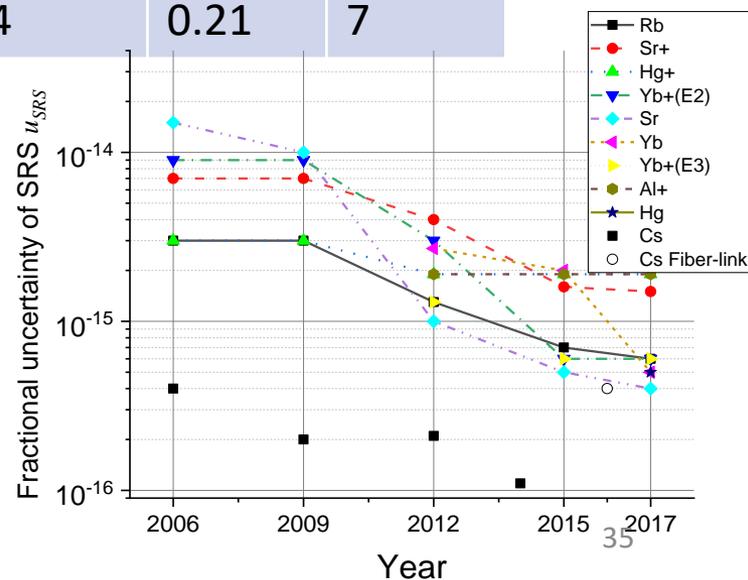
Lattice clock

	u_{SRS} (E-16)	u_{sys} (E-17)	# of labs
Hg	5	7.2	2
Yb	5	0.14	4
Sr	4	0.21	7

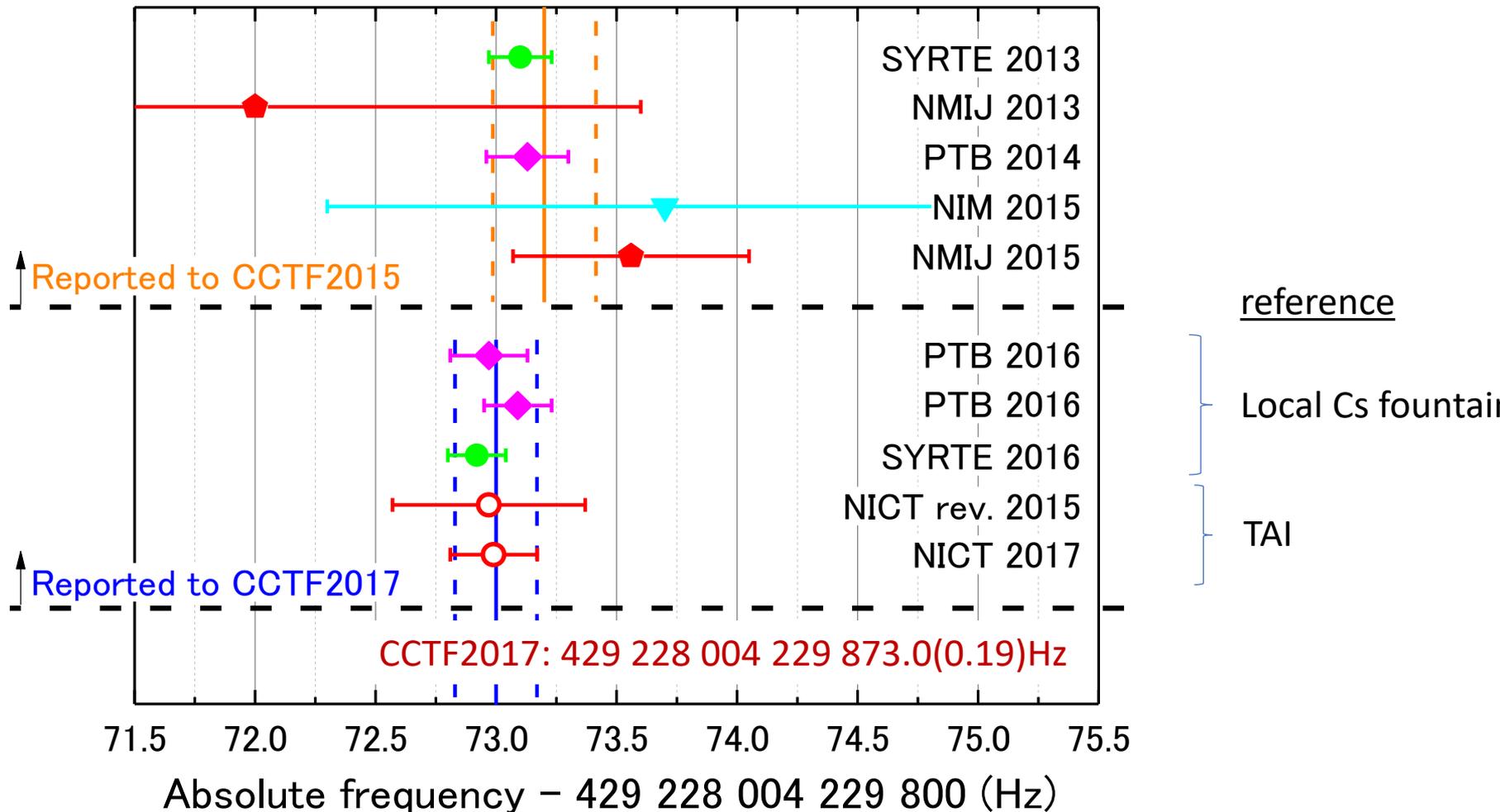
u_{SRS} : uncertainty as a substitute of Cs.
Limited by Cs

u_{sys} : uncertainty as an optical frequency standard

of labs: # which reported frequency to CCTF.



Continuity of the S second



Last 5 data: std. deviation $6e-17 < \text{Cs type-B uncertainty}$

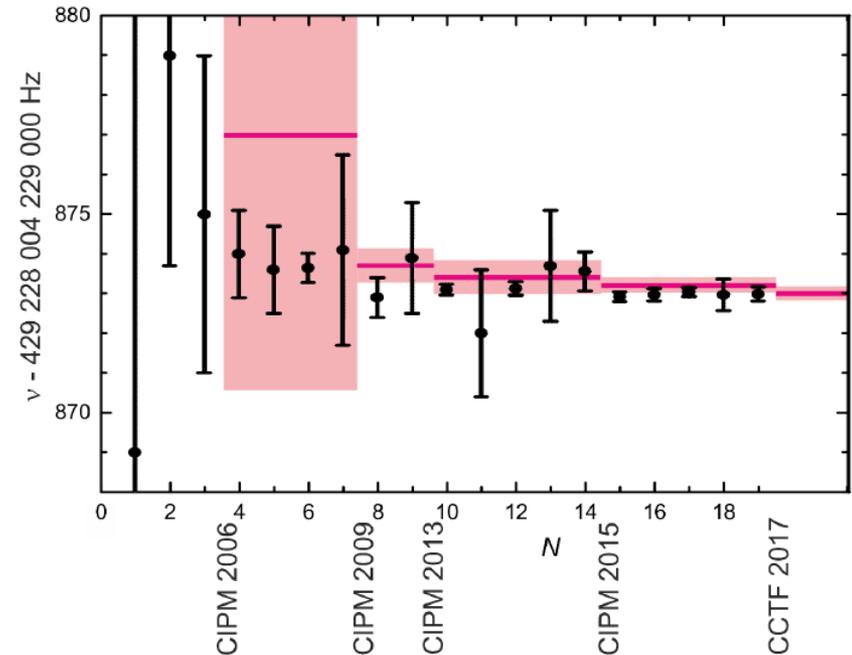
Will be no jump on the redefinition (Possible jump inside the current Cs ability of realization)

But contribution of more Cs clocks is expected.

Present status as of 2017

- Absolute frequency meas

Continuity with present definition: independent measurements w.r.t. three independent Cs primary clocks



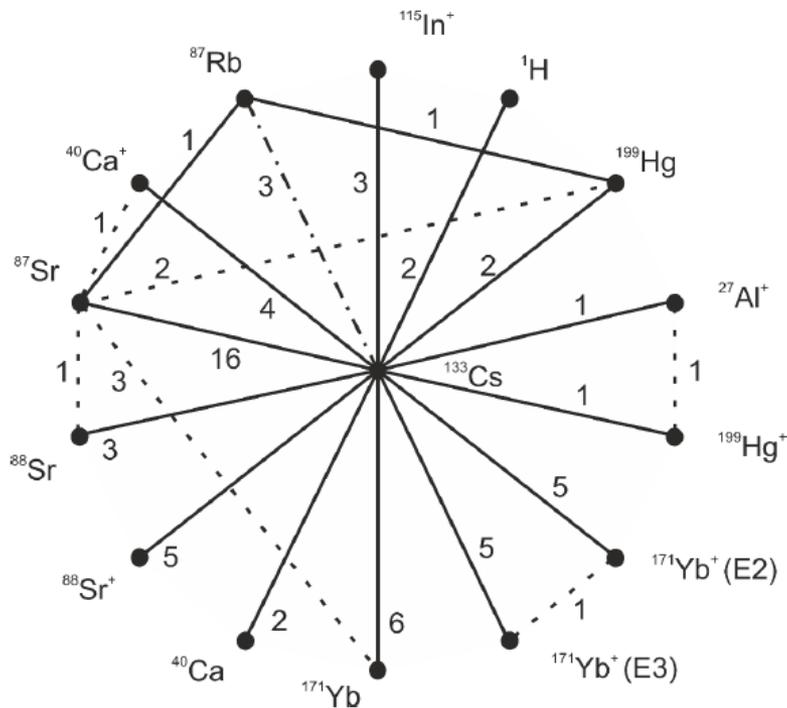
Convergence of the absolute frequency value of the Sr clock transition after 10 years of measurements!

Only measurements w.r.t. SI second contribute to New definition frequency?

No. Optical – optical ratio between two transitions also do.

Absolute frequency measurement
= a ratio measurement against Cs

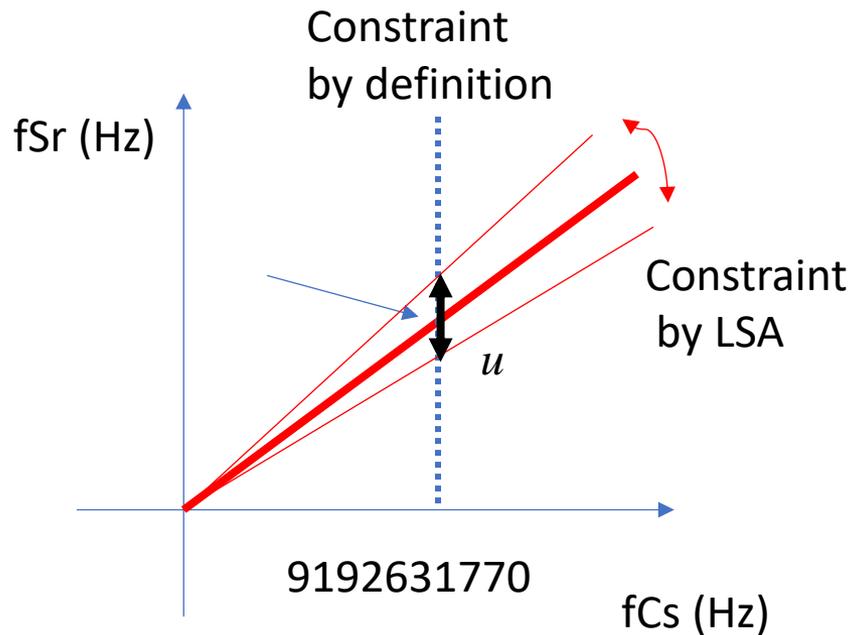
With opt-opt. ratio measurements which provide
overdetermined set of data,
Most probable ratios is determined by numerical
technique such as least square method



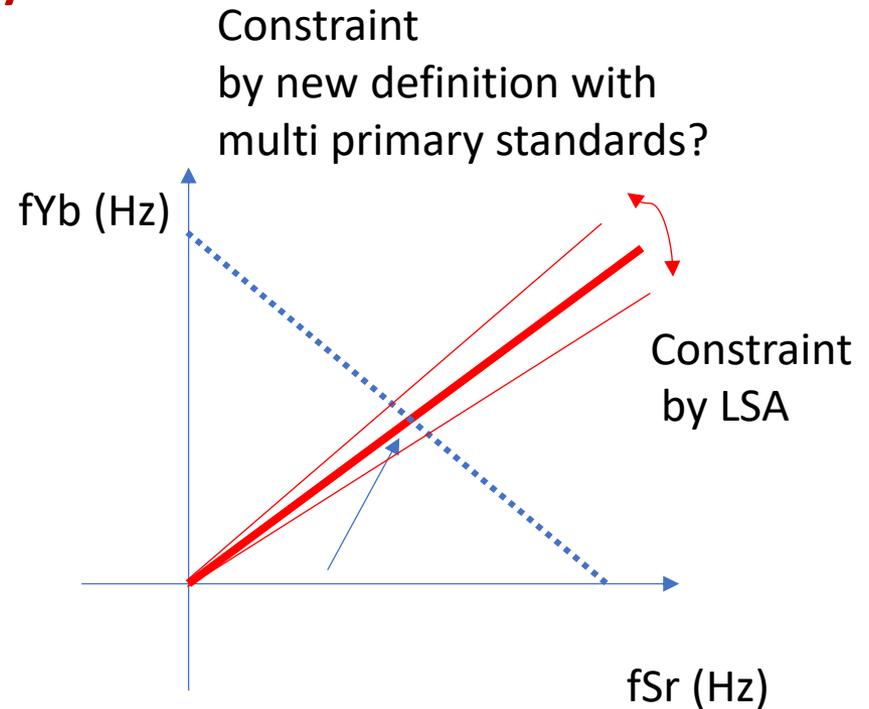
Status in 2017

Then, Cs is currently set to be
9,192,631,770 Hz

Choosing a few transitions as primary transitions?



$$\frac{f_{Cs}}{f_{Cs0}} = 1, f_{Cs0} = 9192631770$$



$$\frac{f_{Sr}}{f_{Sr0}} + \frac{f_{Yb}}{f_{Yb0}} = 2$$

Definition by a linear combination of two frequencies?

Pros
No hierarchy among a few transitions

Cons
Fluctuation of frequencies of primary transitions

Evaluation of TAI scale interval

Strontium is not the definition of the second.

Does it calibrate TAI scale interval equivalently or even more accurately?

Table 1: Estimate of d by individual PSFS measurements and corresponding uncertainties.
All values are expressed in 10^{-15} and are valid only for the stated period of estimation.

Standard	Period of Estimation	d	u_A	u_B	$u_{1/lab}$	$u_{1/Tai}$	u	u_{Srep}	Ref(u_S)	Ref(u_B)
PTB-CS1	58299 58329	-1.03	8.00	8.00	0.00	0.13	11.31	PFS/NA		T148
PTB-CS2	58299 58329	-5.28	5.00	12.00	0.00	0.13	13.00	PFS/NA		T148
SYRTE-FO1	58299 58329	0.56	0.25	0.34	0.06	0.26	0.50	PFS/NA		T301
SYRTE-FO2	58314 58329	0.30	0.30	0.23	0.07	0.49	0.62	PFS/NA		T301
PTB-CSF2	58294 58314	1.15	0.10	0.21	0.09	0.19	0.31	PFS/NA		T287
SU-CsFO2	58299 58329	0.24	0.28	0.24	0.13	0.85	0.93	PFS/NA		T315

Notes:

Circular T (July 2018)

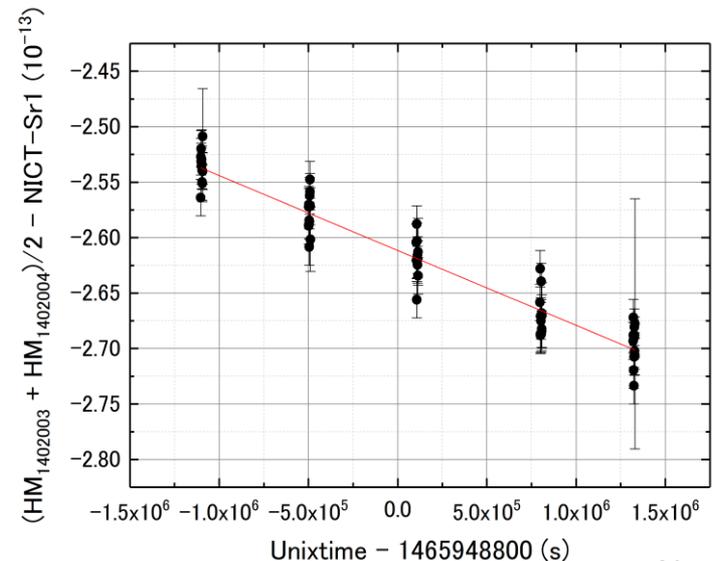
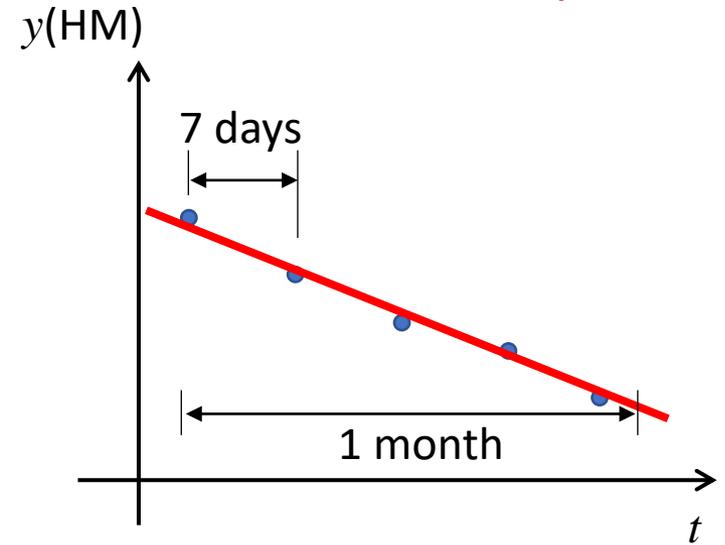
SYRTE reported the calibration by Sr in 4 of (<20day) estimation period.

Longer estimation period is better owing to reduced $u_{1/Tai}$

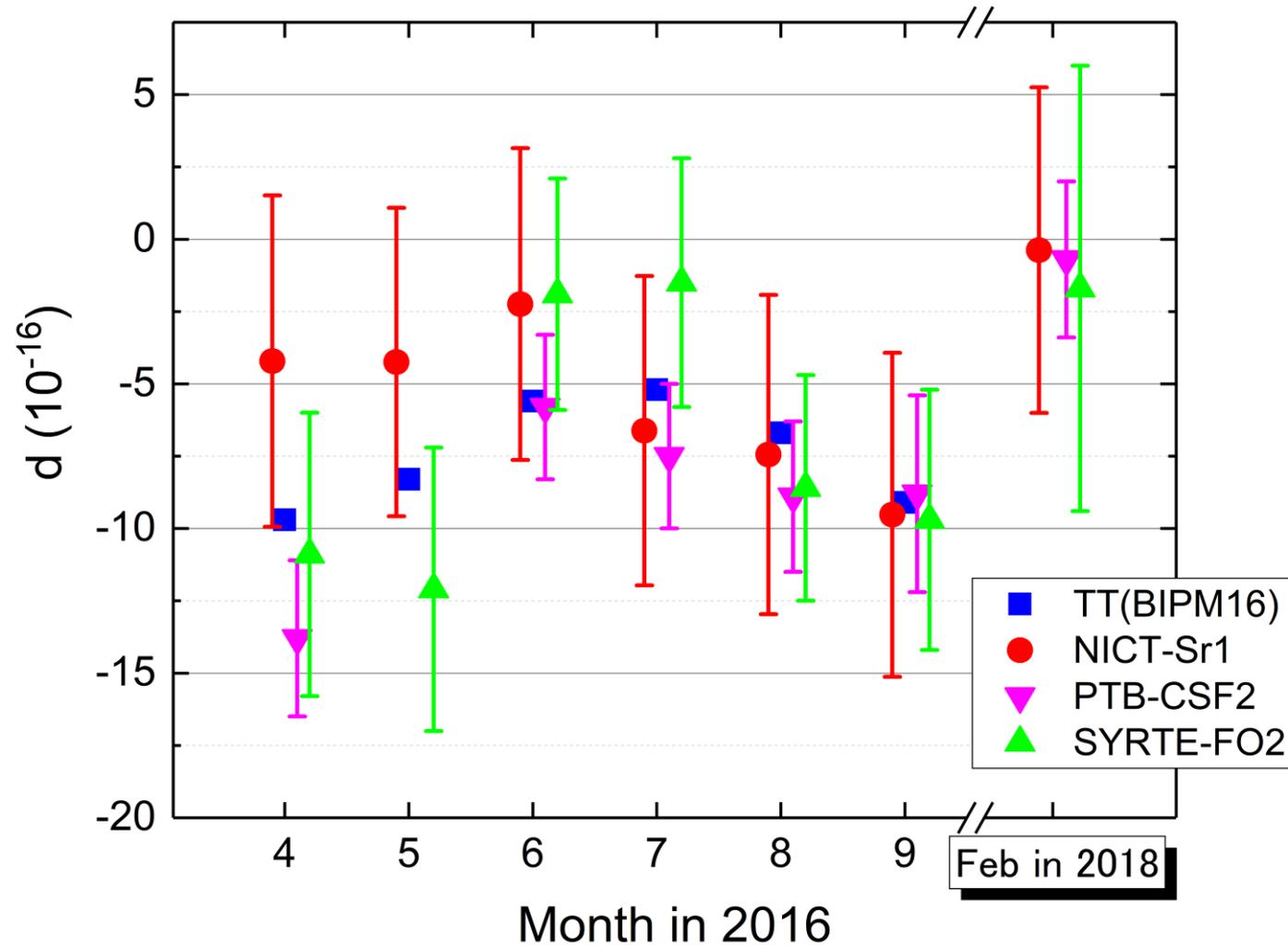
Secondary representation of the second suffers u_{Srep} of 0.40 (CIPM2017#), which is 0.17Hz.

Evaluation of one-month mean TAI scale interval by Sr

Effects	Uncertainty (10^{-17})
Sr systematic	6
Gravity	2.2
Hydrogen maser	
deterministic	25
stochastic (dead time)	18
Phase measurement	5
UTC-UTC(NICT) link	26 (30 days average)
Sr frequency (uSrep)	40 (CIPM 2017)
Total	57 (40 w/o uSrep)



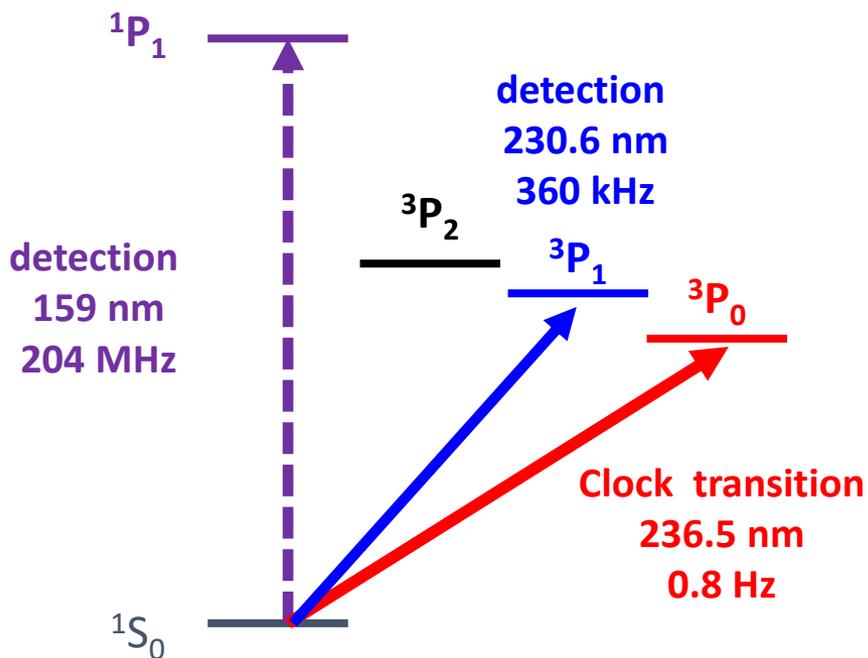
Result: Evaluation of TAI scale interval by Sr



Sr standard frequency: 429 228 004 229 873.0 Hz
(2017 CIPM#)

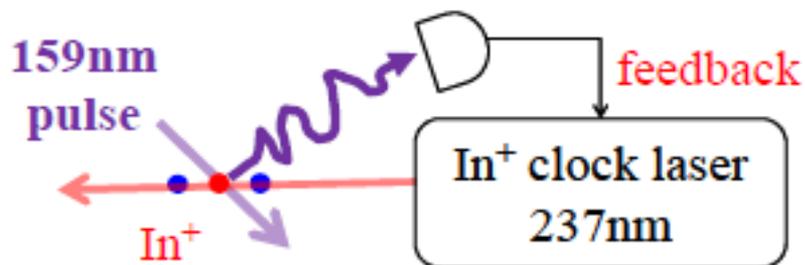
In+ single-ion clock

Motivation: Optical clock based on In^+



VUV excitation

In^+ State readout by VUV pulses



Advantages of In^+

- no electric quadrupole shift
- small Zeeman shifts
- small BBR shift
(17mHz@300K, PRL107, 143006, 2011)

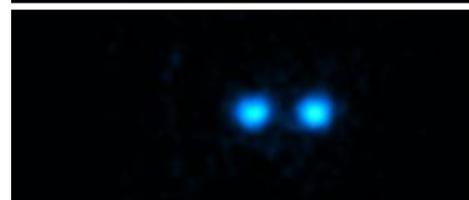
Disadvantage

detection/cooling is slow

Sympathetic cooling of $^{115}\text{In}^+$ with laser-cooled $^{40}\text{Ca}^+$



$(\text{Ca}^+, \text{In}^+, \text{Ca}^+)$
 $\nu_0 = 100.5 \text{ kHz}$



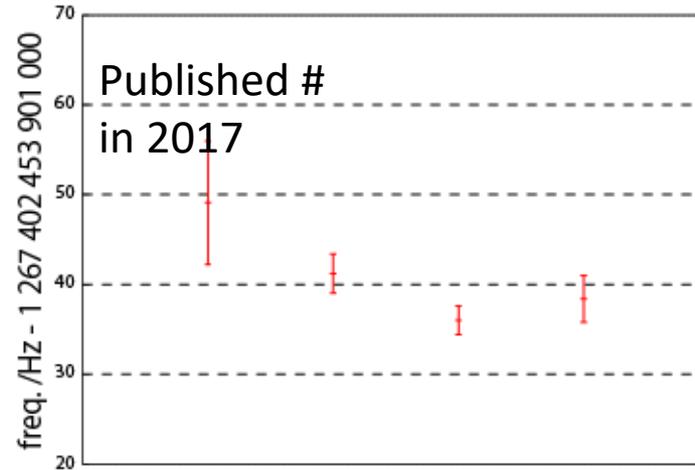
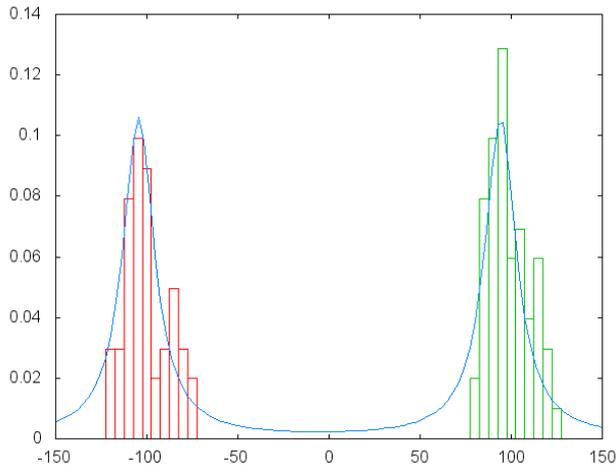
$(\text{In}^+, \text{Ca}^+, \text{Ca}^+)$
 $\nu_0 = 98.5 \text{ kHz}$



$(\text{Ca}^+, \text{Ca}^+, \text{In}^+)$
 $\nu_0 = 98.5 \text{ kHz}$

Clock operations

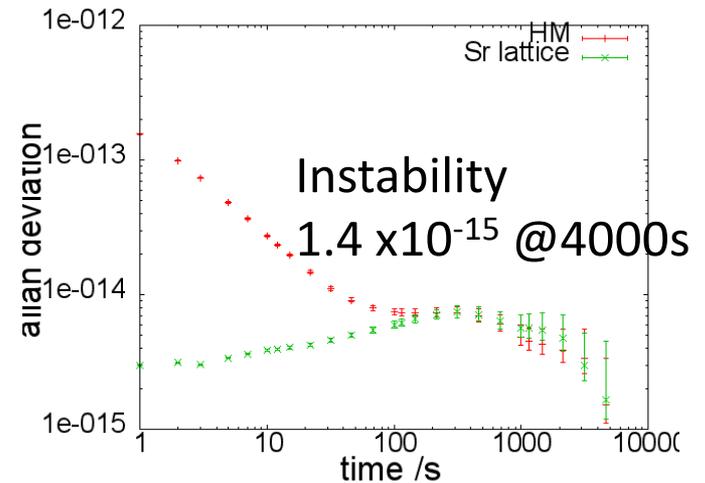
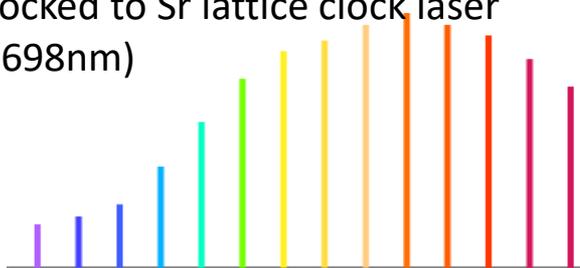
Clock laser frequency are locked to the center of the two Zeeman components(#2,#3).



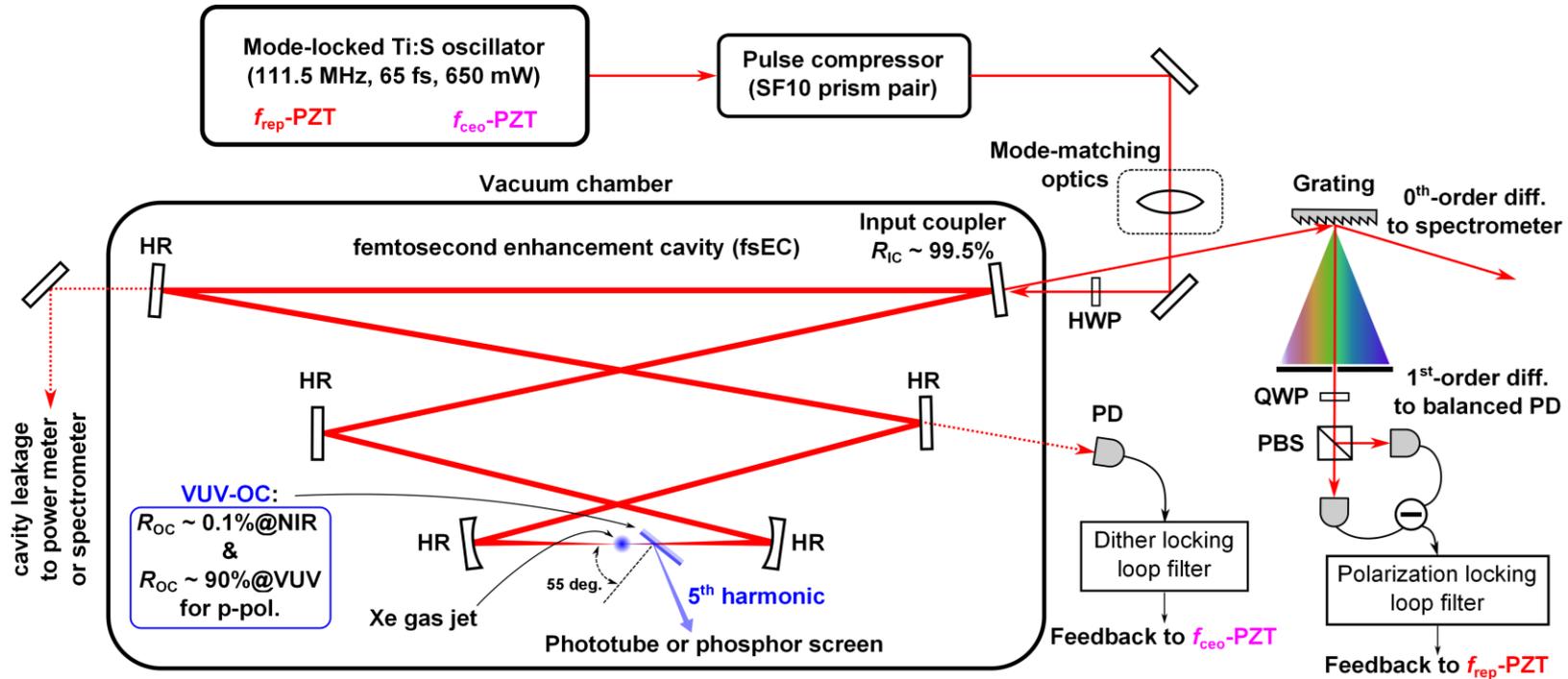
In⁺/Sr ratio measurements

Optical frequency comb is locked to Sr lattice clock laser (698nm)

Count beat note to In⁺ clock fundamental laser (946nm)

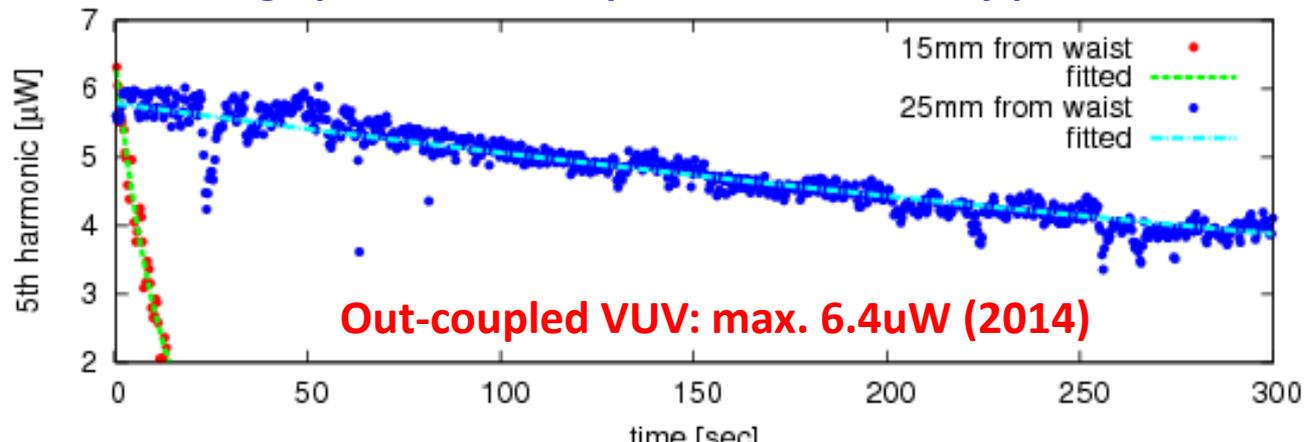


Experimental setup - intracavity high-harmonic generation



Femtosecond enhancement cavity (fsEC)

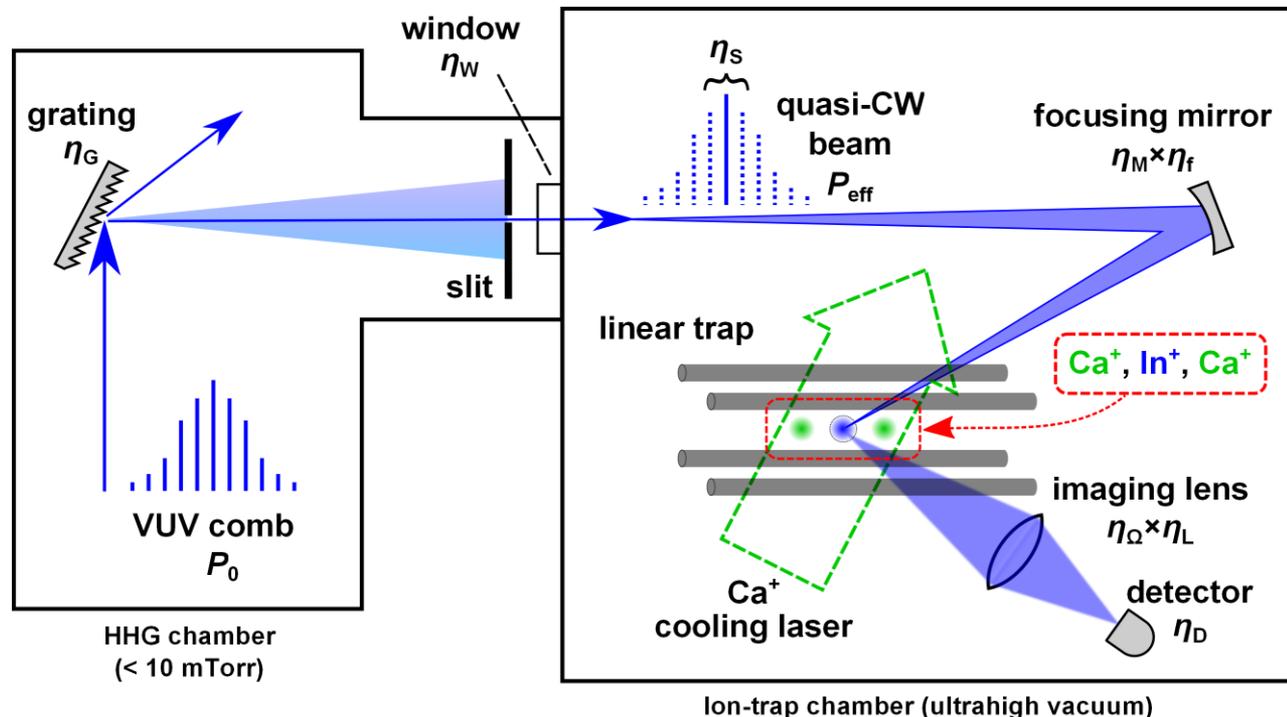
Average power of VUV pulses measured by phototube



Feasibility of VUV detection of a single In^+

Proposed setup
for VUV detection
→
in progress @NICT

Estimated detection rate
per input VUV comb
 $n_0/P_0 \sim 87 \text{ cps}/\mu\text{W}$
↓
 $\sim 1000 \text{ cps}$ photons
from $11\mu\text{W}$ VUV excitation



Description	Symbol	Estimation
Spontaneous emission rate	Γ	$2\pi \times 204 \text{ MHz}$
Saturation intensity	I_s	6.67 W/cm^2
Spot radius	r_0	$0.5 \mu\text{m}$
Grating efficiency	η_G	0.34
Window transmittance	η_W	0.85
Spectrum occupancy	η_s	5.3×10^{-6}

Description	Symbol	Estimation
Mirror reflectance	η_M	0.95
Focusing efficiency	η_f	0.85
Photon collection efficiency	η_Ω	0.045
Imaging lens transmittance	η_L	0.85
Detector quantum efficiency	η_D	0.15

Members

Sr

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Time Scale

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In+

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N. Nemitz

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K. Hayasaka

HHG

K. Wakui



Technical assistance

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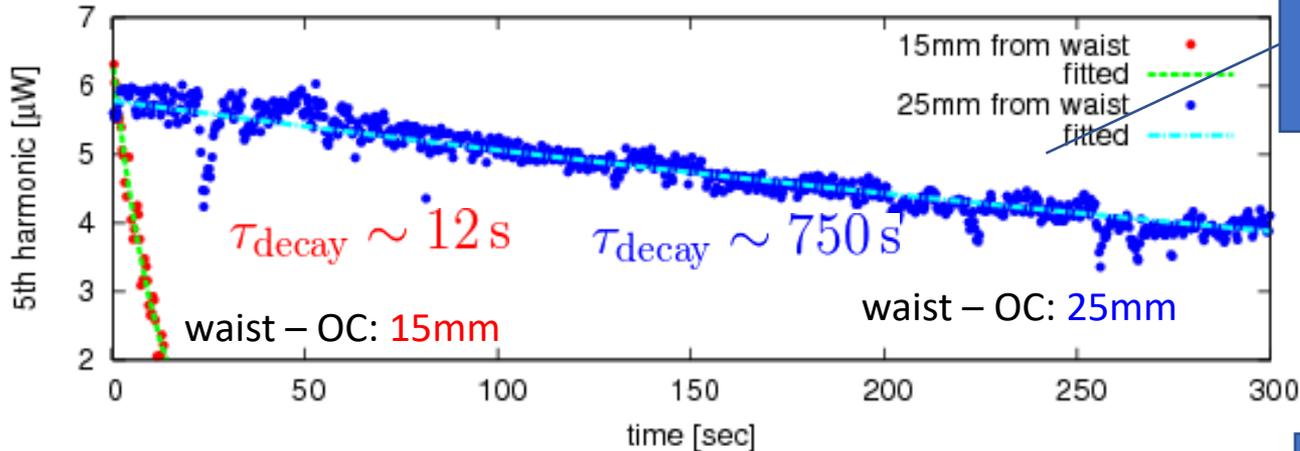
M. Mizuno

S. Ito

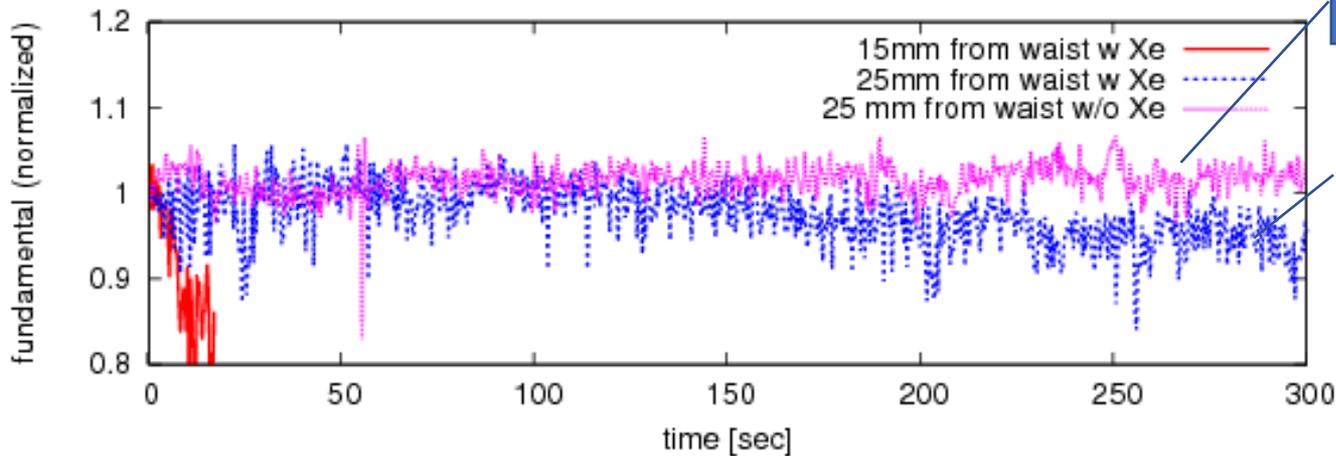
Special thanks to

1. **G. Petit** at BIPM for the fruitful discussions

Degradation of VUV power



- ✓ Larger beam spot mediated power degradation
- Tentative effect? (heat, VUV induced color center, etc.)



- ✓ No degradation w NIR only

- ✓ Loss increased w VUV (but recovered afterwards)

Wakui, Hayasaka & Ido, Appl. Phys. B 117, 957 (2014).

Requirement for HM and OFS operation rate

$$E[|\Delta\phi|] = (\varepsilon_p^2 + \varepsilon_F^2)^{1/2}$$

$$= \frac{T}{N} \left[\frac{(2N+1)(2N+3)}{N(N+1)(N+2)} \sigma_p^2 \right]$$

$$+ \underbrace{\frac{1}{\ln 2} \sigma_F^2}_{\text{Linear drift estimation error (from LSF)}} \underbrace{\left. \right]^{1/2}}_{\text{Stochastic phase excursion in Flicker noise (*)}}$$

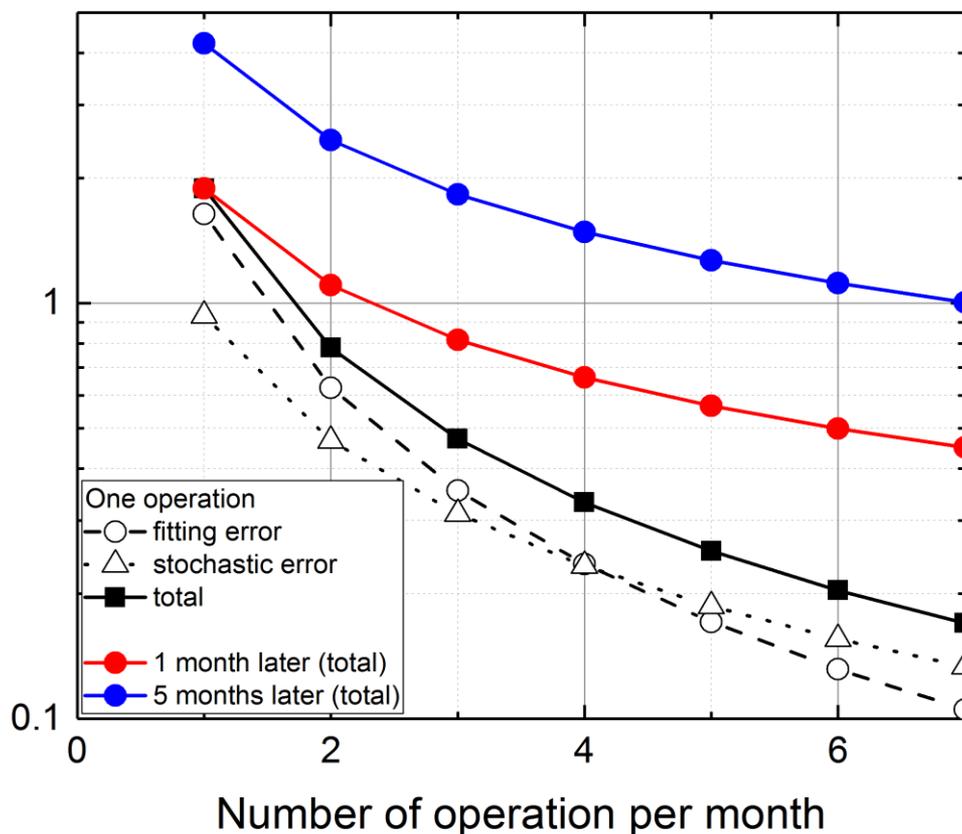
Stochastic phase excursion in Flicker noise (*)

$$\sigma_p = 4E - 16$$

(stability @ 10^4 s)

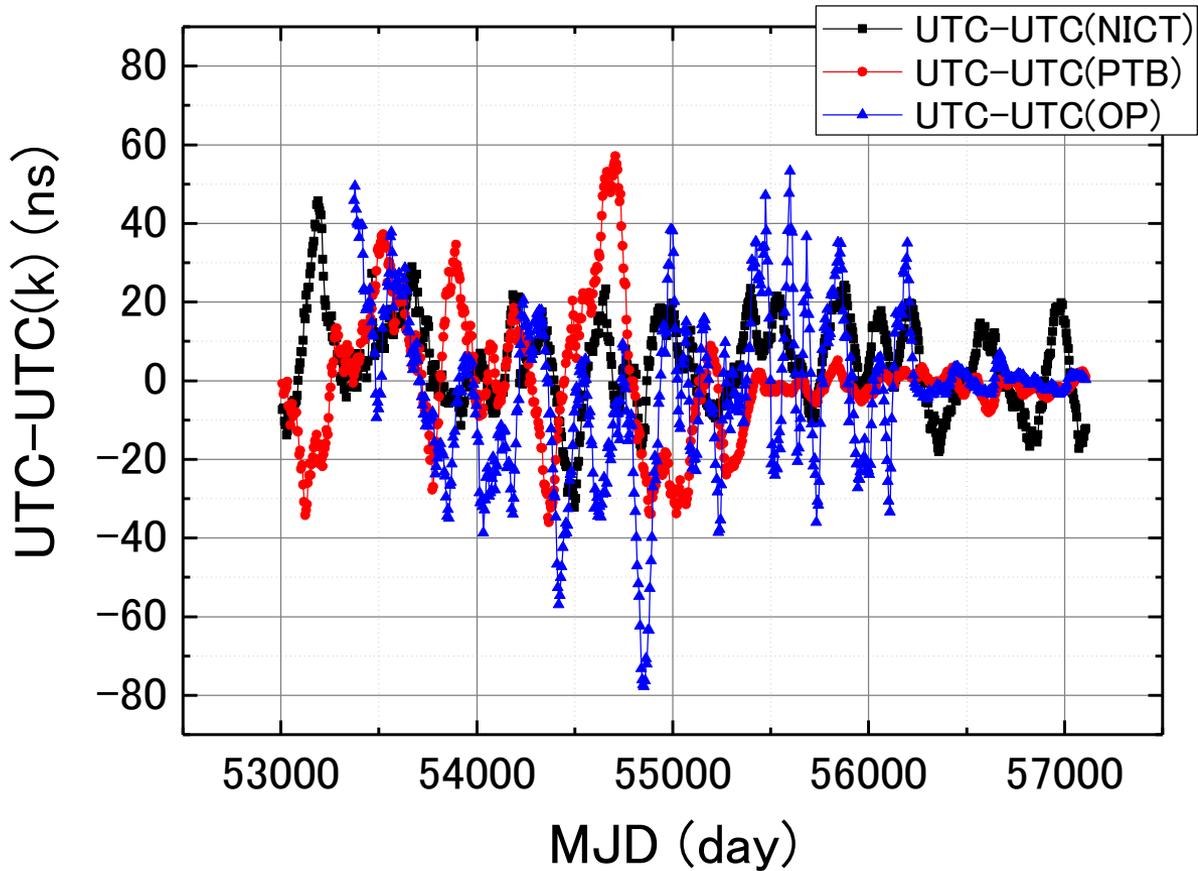
$$\sigma_F = 3E - 16$$

Flicker floor of Hadamard dev.



* D. Allan, IEEE Trans. Ultrasonic. Ferro. Freq. Cont., **34** 647 (1987)

Current status of UTC(k)



UTC(PTB):
Metrologia 49, 180 (2012).

UTC(OP):
Metrologia 53, S81 (2016).

Reliable fountains enable real-time steering of UTC(k).

Why not optical clocks?

Particularly for emerging countries, getting difficult to assign young talents to fountains.

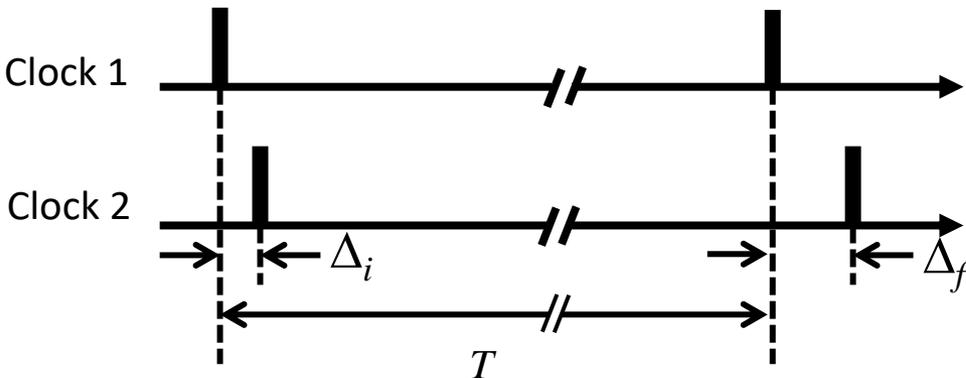
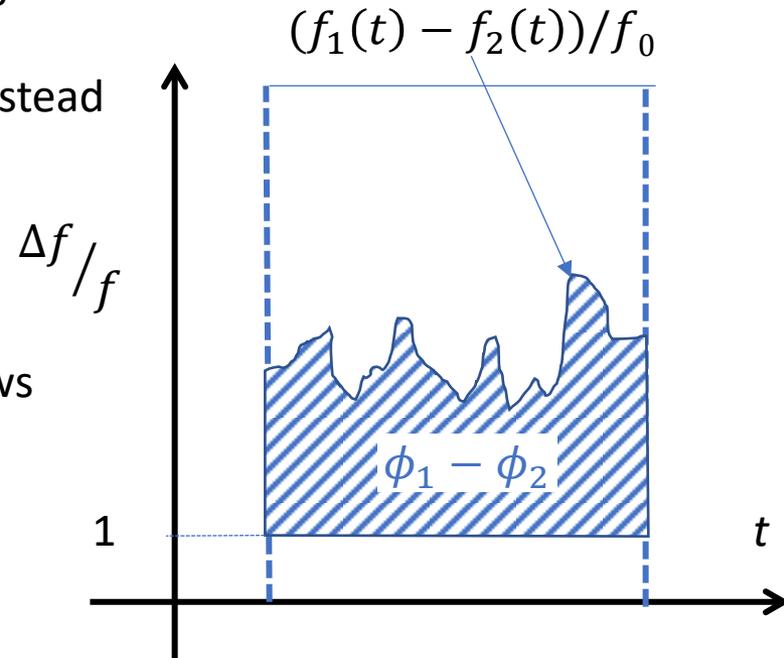
No absolute phase. Phase is always difference of two signals.

Why don't we think "fractional frequency difference" instead of absolute frequency when we compare two signals.

→

Time integration of fractional frequency difference shows relative phase in unit of second.

$$\phi_1 - \phi_2 = \Delta\phi = \int \frac{\Delta f}{f_0} dt = \int \frac{f_1 - f_2}{\bar{f}} dt$$



Time (phase) difference Δ_i and Δ_f at two instances which are separated for T

$$(f_2 - f_1)/f_0 = -\frac{\Delta_f - \Delta_i}{T}$$

Satellite link such as GPS provides time difference Δ not frequency difference

Circular T shows this $\Delta = \text{UTC} - \text{UTC}(k)$.

This formula lead fractional frequency difference between UTC & UTC(k)

Time = phase?

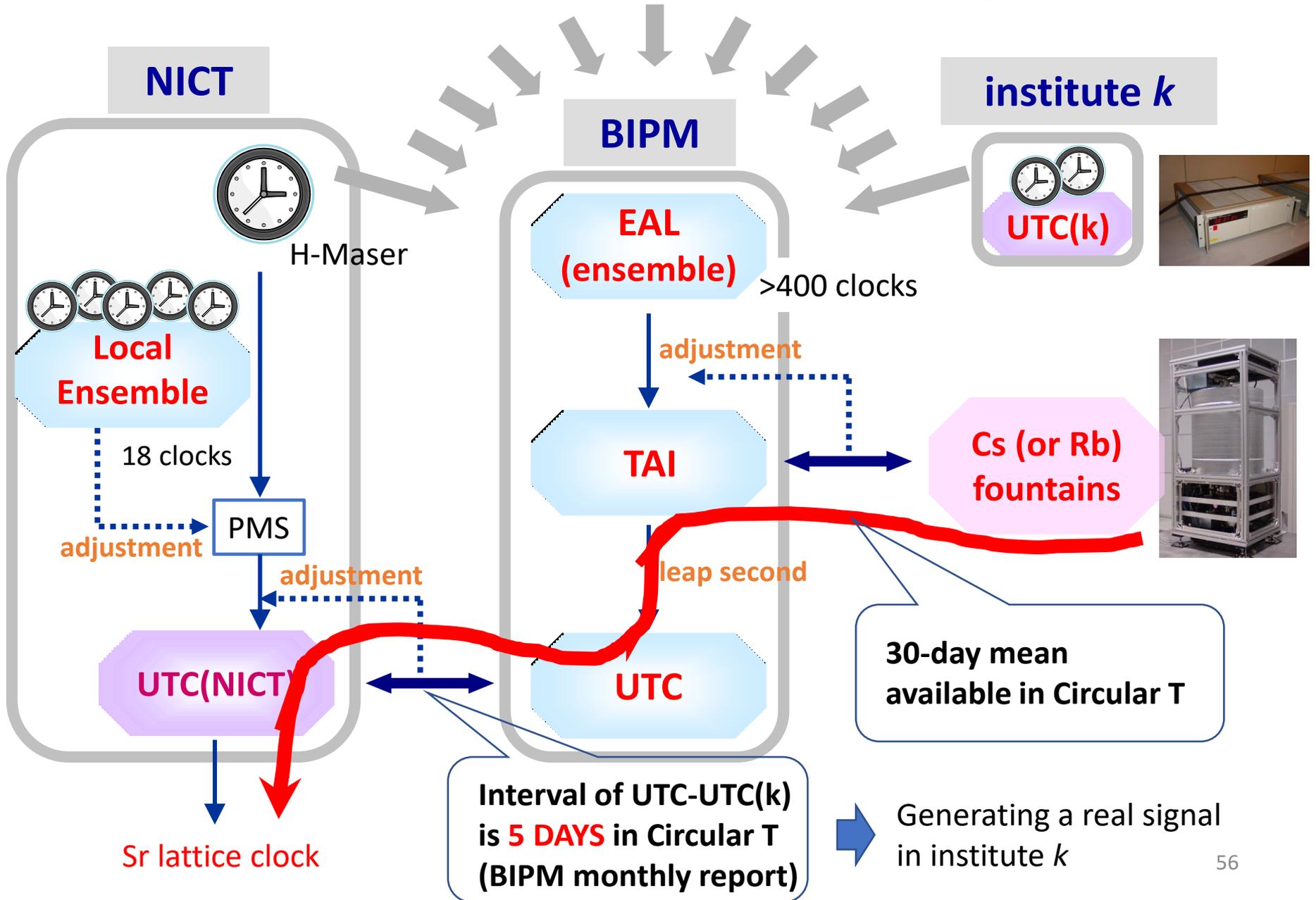
Atomic physicists, particularly in optical, always think frequency. When they draw graph, optical phase never show as vertical axis.

Why? Probably, optical phase is fragile. Easily jump and so on (coherence)

On the other hand, those in timescale like phase. Our phase always have the unit of “rad”. But their phase often has the unit of “s”. What’s happen?

Absolute frequency measurement using International Atomic Time

Absolute frequency measurement using TAI



TAI-based frequency measurement @ lab. "k"

Calibration of TAI
Reported from BIPM

$$\frac{\nu(Sr@k)}{\nu(SI)} = \frac{\nu(Sr@k)}{\nu(HM)} \frac{\nu(HM)}{\nu(UTC(k))} \frac{\nu(UTC(k))}{\nu(TAI)} \times \frac{\nu(TAI)}{\nu(SI)}$$

Suffers from link uncertainties
But not dependent on specific Cs fountain

Goal is to get this ratio

What lab k measures or calculate.

In future, this process will calibrate the TAI using a lattice clock

	$\frac{\nu(Sr@k)}{\nu(HM)}$	$\frac{\nu(HM)}{\nu(UTC(k))}$	$\frac{\nu(UTC(k))}{\nu(TAI)}$	$\frac{\nu(TAI)}{\nu(SI)}$
Link uncertainty	negligible	2e-16	9.8e-16@5day (satellite link)	~ 2-5e-16 (Cs, satellite)
Minimum Averaging Time	1 second	1 second	5 days	1 month



Suffers from dead time uncertainty

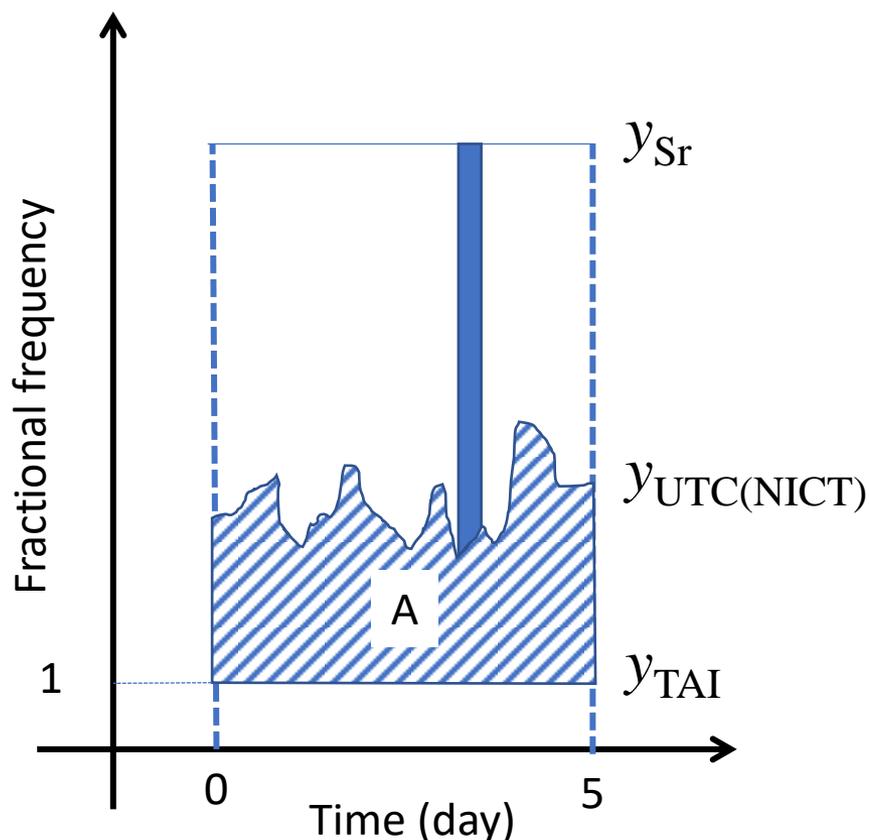
$$\overline{\nu(UTC(k))}_{10000s} \neq \overline{\nu(UTC(k))}_{5days}$$

$$\overline{\nu(TAI)}_{5days} \neq \overline{\nu(TAI)}_{1month}$$

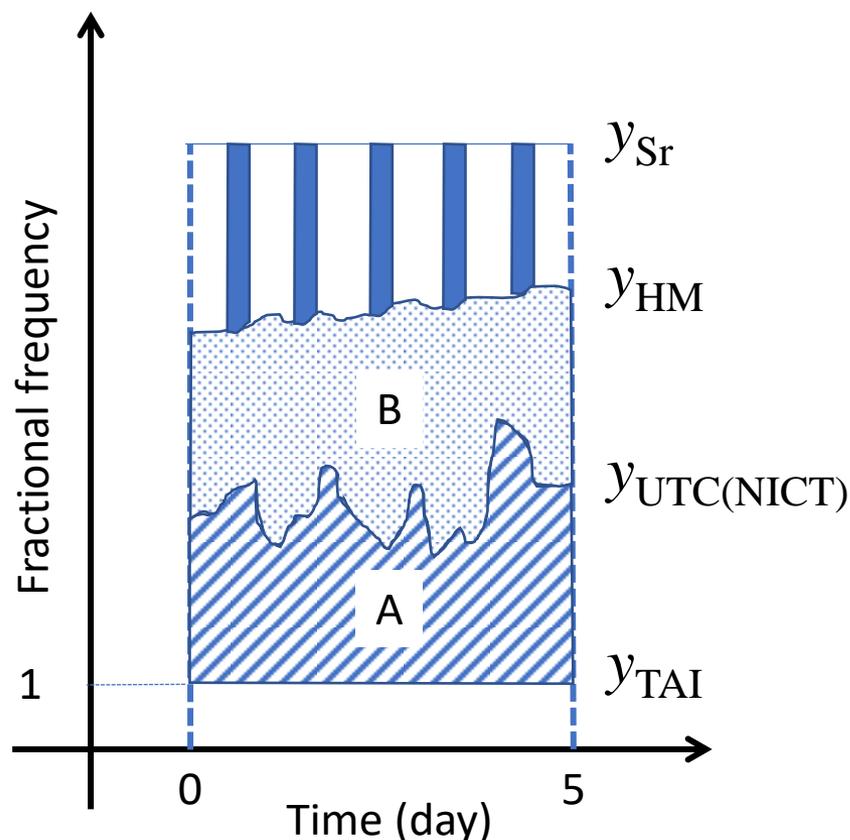
Link from Sr to TAI

$$A = \int_{Day0}^{Day5} (y_{UTC(k)} - 1) dt = [UTC(k) - UTC]_{Day5} - [UTC(k) - UTC]_{Day0}$$

Temporal average of frequency difference = Variation of the time difference

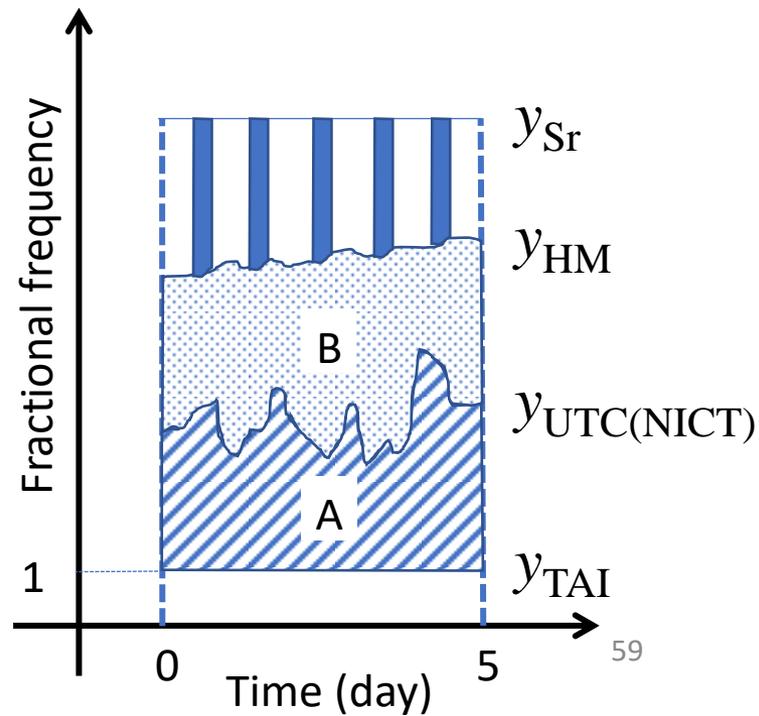
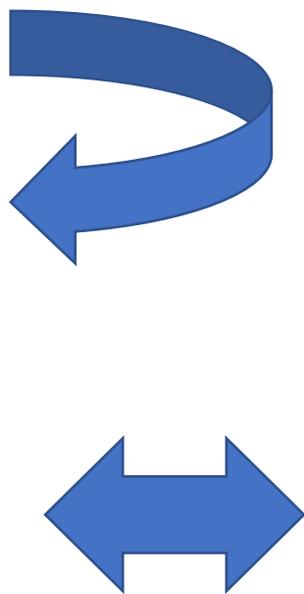
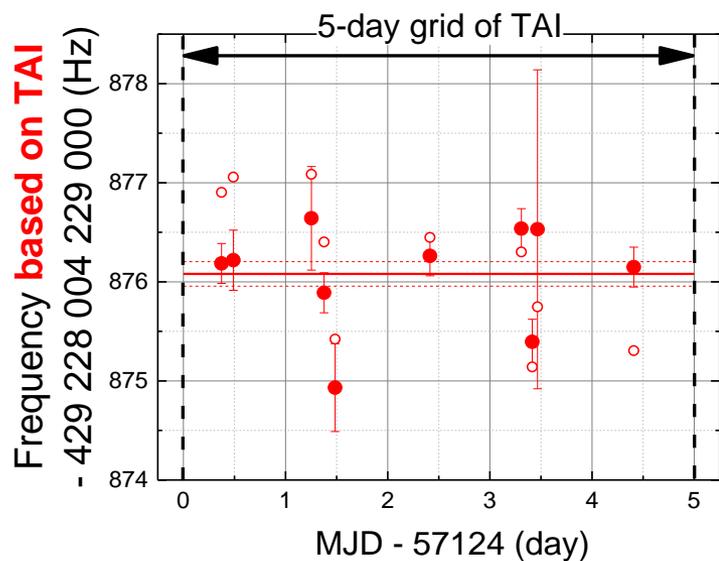
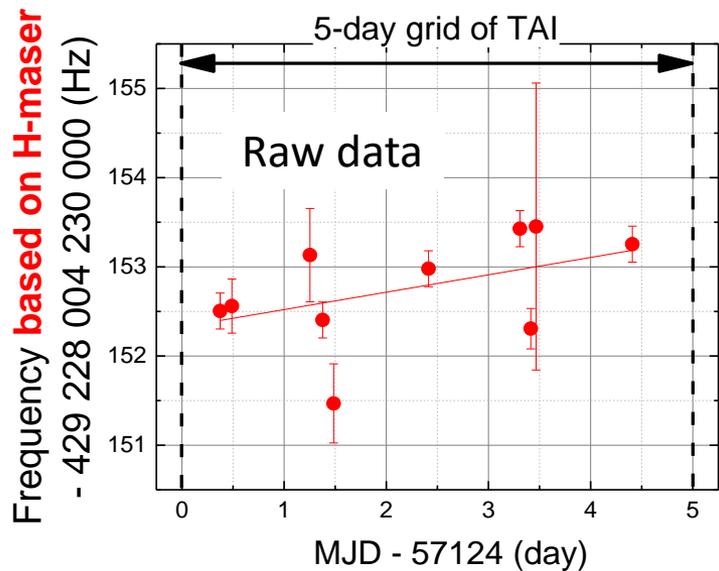


Previous measurement in 2012
Link uncertainty: $2.8e-15$



Measurement in 2015
Link uncertainty: $7.2e-16$

Measurement



Dead time uncertainty

Hadamard deviation (effective deviation of de-drift signal) of our HMs has flicker floor up to one month

Deviation of phase was investigated in 1980s.

(b)

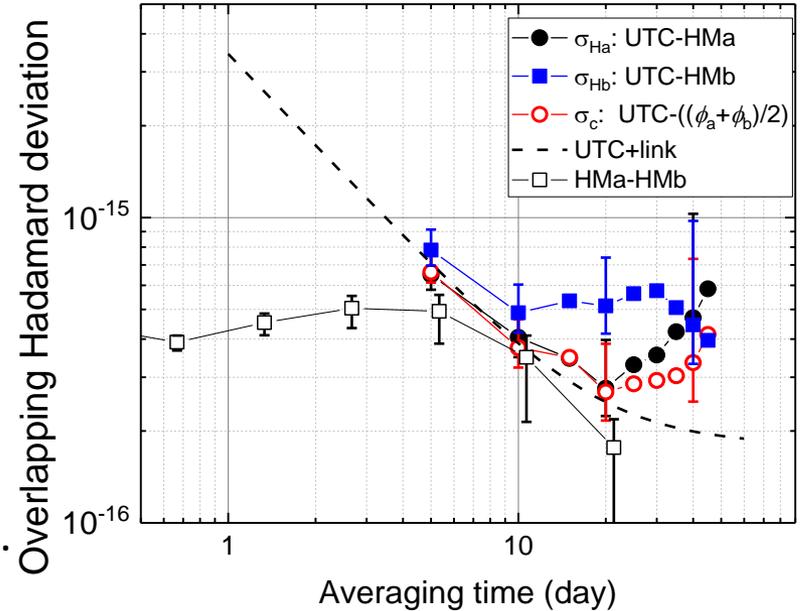


TABLE III

α	Typical Noise Types Name	Optimum Prediction $x(\tau_p)$ rms ^a	Time Error: Asymptotic Form
2	white-noise PM	$\tau_p \cdot \sigma_y(\tau_p) / \sqrt{3}$	constant
1	flicker-noise PM	$\sim \tau_p \cdot \sigma_y(\tau_p) \sqrt{\ln \tau_p / 2 \ln \tau_0}$	$\sqrt{\ln \tau_p}$
0	white-noise FM	$\tau_p \cdot \sigma_y(\tau_p)$	$\tau_p^{1/2}$
-1	flicker-noise FM	$\tau_p \cdot \sigma_y(\tau_p) / \sqrt{\ln 2}$	$\tau_p^{3/2}$
-2	random-walk FM	$\tau_p \cdot \sigma_y(\tau_p)$	$\tau_p^{3/2}$

^a τ_p is the prediction interval.

Ref. D. W. Allan, IEEE Trans. Ultrasonic, Ferro. Freq. Control UFFC-34, 647 (1987).

Phase noise during two operation separated τ : $\tau \sigma(\tau) / (\ln 2)^{1/2}$

Link from TAI to SI second

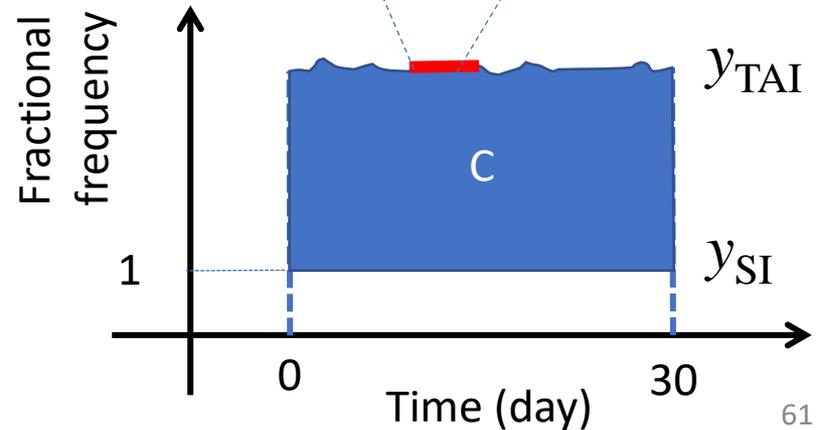
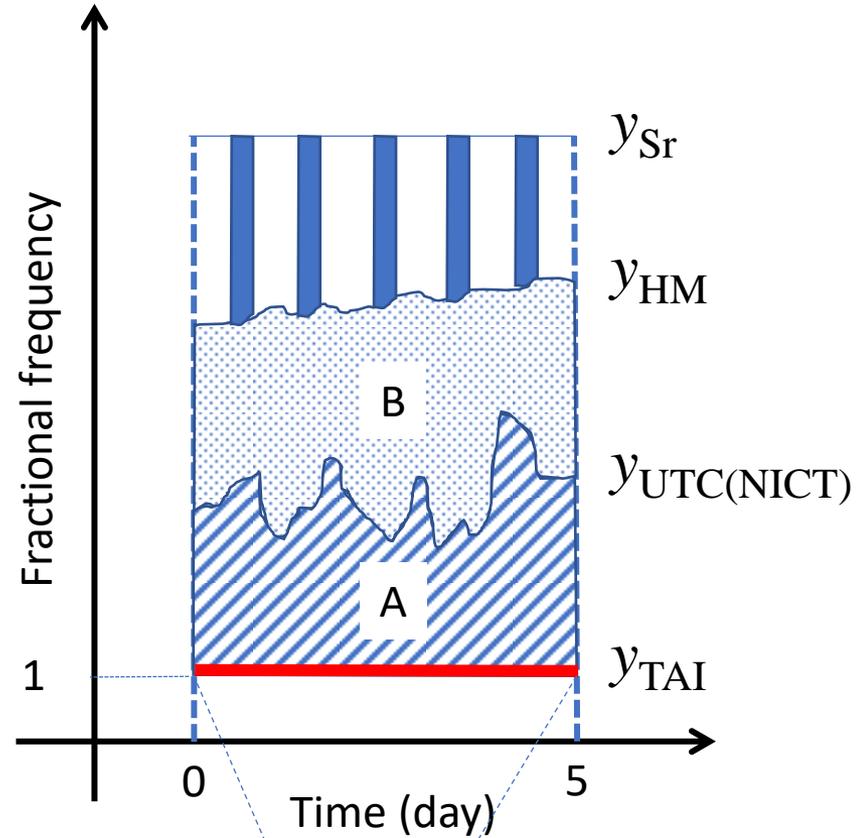
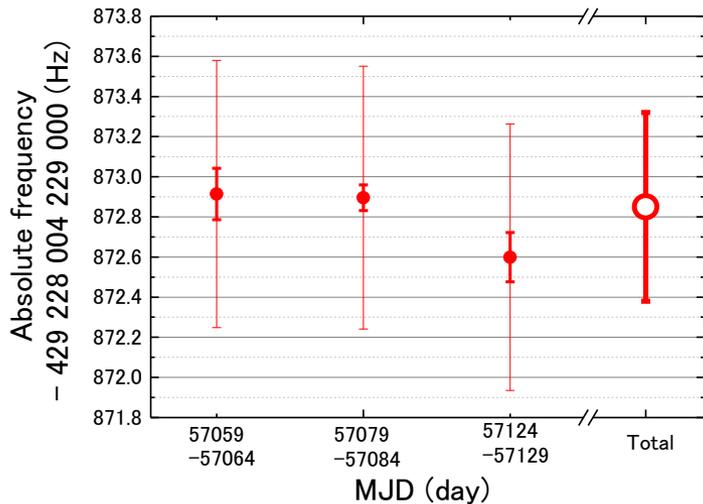
BIPM tells us the calibration of TAI
 in 1-month average basis.

The TAI in previous slide is 5-day average.

$$C = \int_{Day0}^{Day30} (y_{TAI(k)} - 1) dt$$

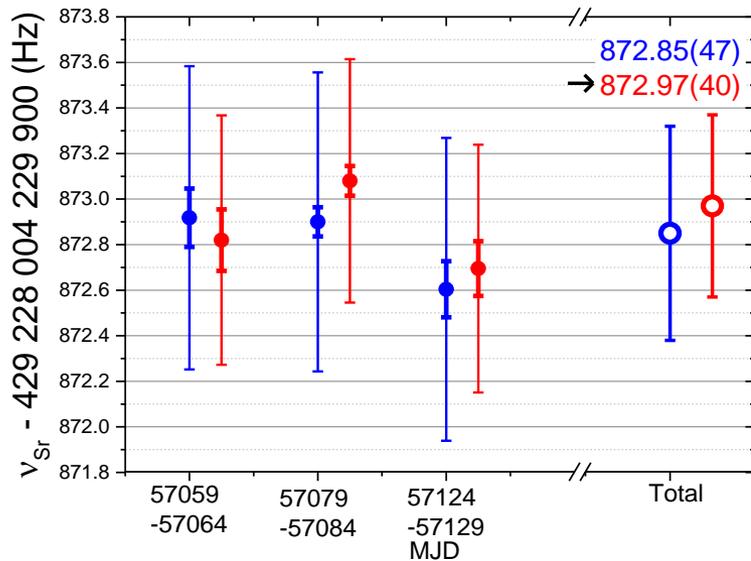
$$\overline{\nu(TAI)}_{5days} \neq \overline{\nu(TAI)}_{1month}$$

Uncertainty of this deviation
 $7.6e-16$



Evaluation using three 5-day campaigns

G. Petit at BIPM time department calculated the TAI calibration on three 5-day averages of our measurement campaigns



CIPM#(2017): ... 873.0

Appl. Phys. B **123**, 34 (2016).

Day of average	campaign #1	campaign #2	campaign #3
30 (Cir. T)	-4.4 (2.6)	-2.7(2.5)	-2.3 (2.6)
5 (by Petit)	-2.5 (8.1)	-6.8 (8.2)	-4.5 (7.4)

($\times 10^{-16}$)

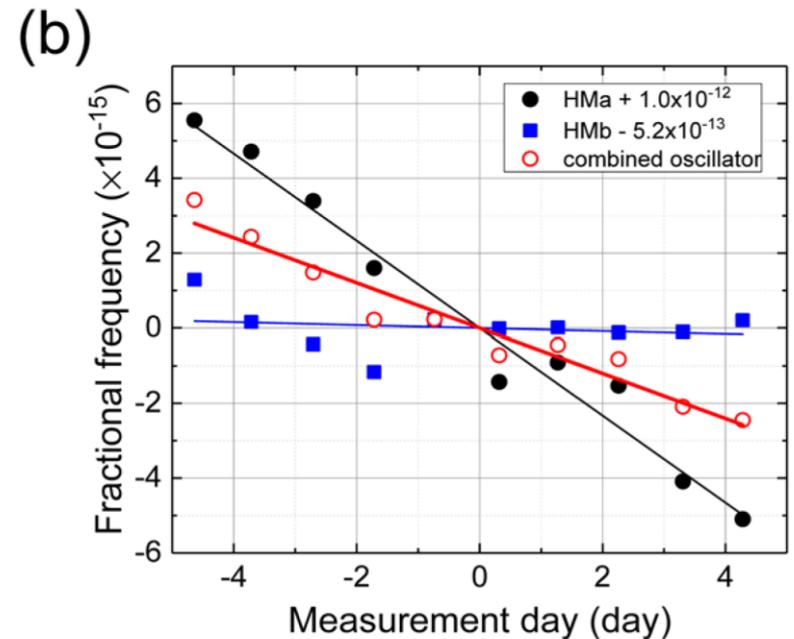
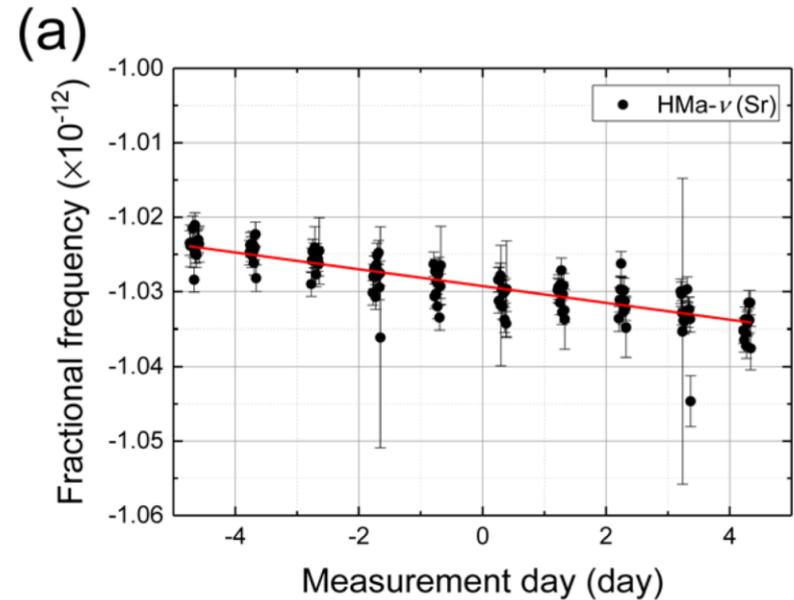
	30day	5day
Statistical	10	
Sr systematic	9	
Gravity	8	
Dead time (HM-UTC(k))	19	
Satellite link	69	
Dead time(TAI-SI second)	76	0
TAI-SI second	25	57
Total	109	93

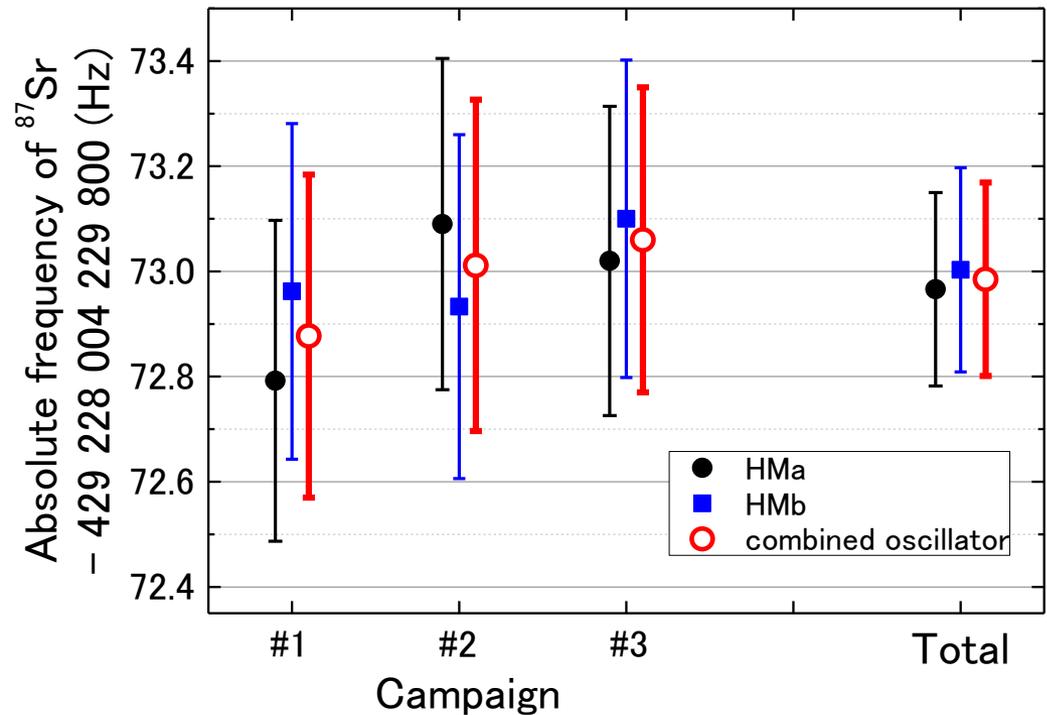
($\times 10^{-17}$)

Furthermore...
10 day campaign X 3
+ two HM ensemble
Benefit of 5-day \rightarrow 10-day

Fitting uncertainty \downarrow
UTC(NICT)-UTC error \downarrow

Ensemble of two HMs may mitigate
sporadic phase excursion of one HM





	Campaign # 2 (10^{-17})	Total (10^{-17})
Strontium		
statistical	2	1
systematic	6	6
Gravity	2	2
Local flywheel oscillator		
deterministic	18	10
stochastic (dead time)	10	6
Link		
UTC-UTC(NICT) link	49	28
UTC- SI second	(50)	
systematic uncertainty	15	14
rest of random part	48	26
Total	73	43

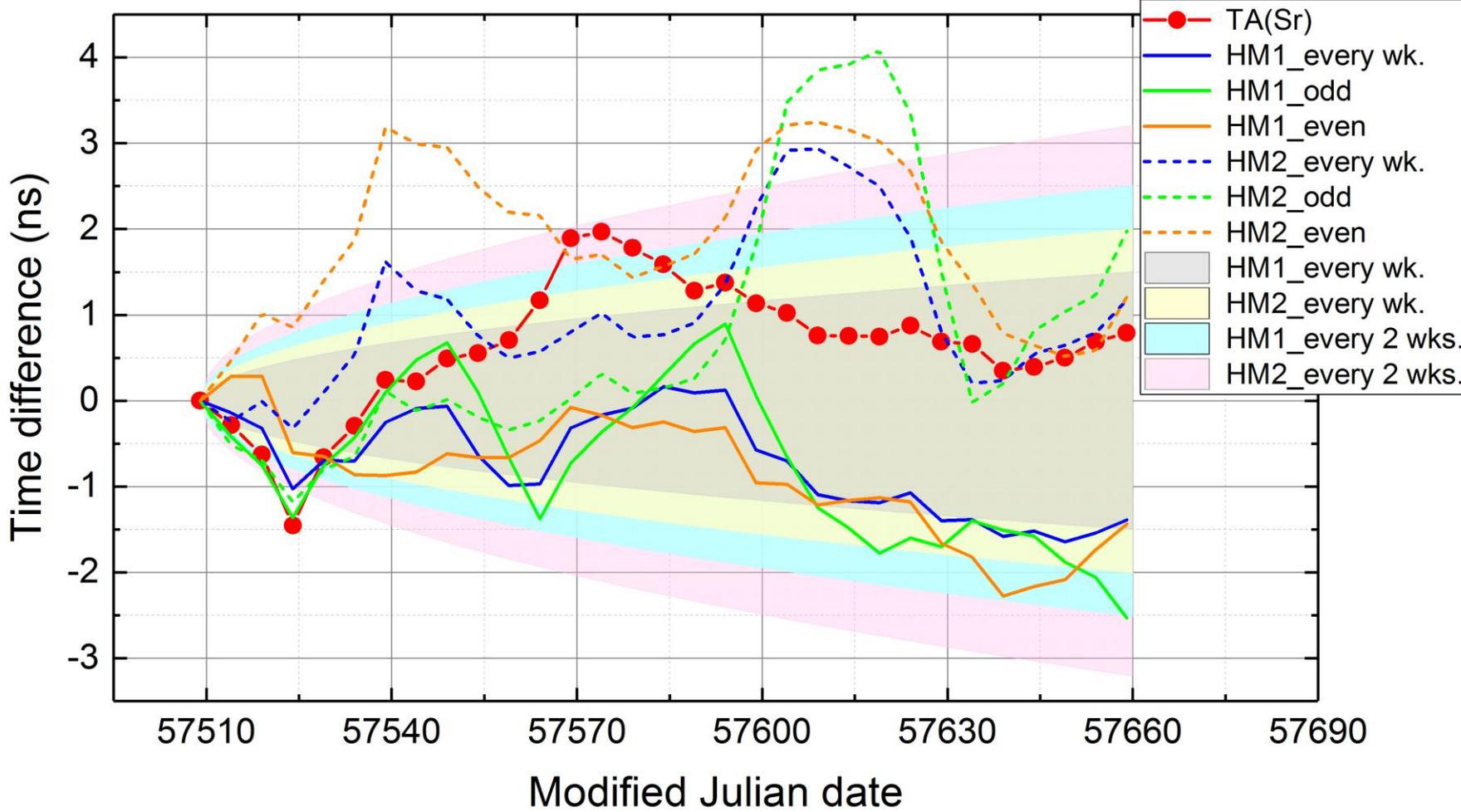
Due to the error in linear fitting

Due to the dead time error

These two uncertainty could be not independent.

< 5e-16

Simulation in worse HM & infrequent OFS operation



HM2 (dash) noisier than HM1 (solid) (HM2 Flicker floor: $5e-16$)

Blue: once per week, orange and green: once per two weeks (odd, even)

Secondary Representation of the Speed (2017)

Frequency / Hz	Fractional uncertainty	Transition	Status
6 834 682 610.904 312 6	6×10^{-16}	^{87}Rb	Revised 2017
310	7×10^{-16}	Ground state hfs	2015 value
429 228 004 229 873.0	4×10^{-16}	^{87}Sr neutral atom, $5s^2 \ ^1S_0$ - $5s5p \ ^3P_0$	Revised 2017
873.2	5×10^{-16}		2015 value
444 779 044 095 486.5	1.5×10^{-15}	$^{88}\text{Sr}^+$ ion, $5s \ ^2S_{1/2}$ - $4d \ ^2D_{5/2}$	Revised 2017
486.6	1.6×10^{-15}		2015 value
518 295 836 590 863.6	5×10^{-16}	^{171}Yb neutral atom, $6s^2 \ ^1S_0$ - $6s6p \ ^3P_0$	Revised 2017
864.0	2×10^{-15}		2015 value
642 121 496 772 645.0	6×10^{-16}	$^{171}\text{Yb}^+$ ion, $6s \ ^2S_{1/2}$ - $5d \ ^2F_{7/2}$	Not revised
688 358 979 309 308.3	6×10^{-16}	$^{171}\text{Yb}^+$ ion, $6s \ ^2S_{1/2}$ - $5d \ ^2D_{3/2}$	Not revised
1 064 721 609 899 145.3	1.9×10^{-15}	$^{199}\text{Hg}^+$ ion, $5d^{10}6s \ ^2S_{1/2}$ - $5d^96s^2 \ ^2D_{5/2}$	Not revised
1 121 015 393 207 857.3	1.9×10^{-15}	$^{27}\text{Al}^+$ ion, $3s^2 \ ^1S_0$ - $3s3p \ ^3P_0$	Not revised
1 128 575 290 808 154.4	5×10^{-16}	^{199}Hg neutral atom, $6s^2 \ ^1S_0$ - $6s6p \ ^3P_0$	New 2017