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IV. EXPERIMENTAL RESULTS

IV. 8 APPLICATION OF VLBI DATA TO MEASUREMENTS OF IONOSPHERIC TOTAL ELECTRON CONTENT

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ABSTRACT

In very long baseline interferometry (VLBI) geodetic measurements, both X (8 GHz) and S (2 GHz) band radio signals from quasars are received in order to calibrate excess delays caused by charged particles existing along each ray path mainly due to the difference in the terrestrial ionospheric total electron content (TEC). A method of estimating TEC at each station from observed dual frequency band delays has been developed. In the estimation, daily TEC variation is modeled as the summation of Fourier components for a one day period up to 4th-order harmonics and consisting of linear terms with respect to time. Coefficients of each term are fitted by means of least squares estimation. TEC estimated from VLBI data was compared with that obtained by Faraday rotation measurements using a geostationary satellite beacon, and satisfactory agreement was found.

1. Introduction

In very long baseline interferometry (VLBI) geodetic measurements, it is important to compensate for delays caused by the propagation medium, i.e., atmospheric and ionospheric delays. The atmospheric delays are corrected by means of a posteriori parameter fitting of a model atmosphere(1). On the other hand, the ionospheric delays are directly corrected by receiving at two frequency bands, S-band (2 GHz) and X-band (8 GHz), and using the relationship that propagation delays in the ionosphere are proportional to the inverse square of the wave frequency. For more than a decade after development of geodetic VLBI systems, dual band delays have been used merely for the calibration of the ionospheric excess delays. However, information on ionospheric total electron content (TEC) is included in dual band delays, and an estimation method for obtaining daily TEC variation for each station in an experiment from the dual band delays has been recently developed(2). This paper describes a modified method for estimating TEC from VLBI data (modified to improve the residuals after the data fitting) and presents the results of a typical experiment using this method.

2. Ionospheric Excess Delays

The delay observed by VLBI, which is defined as the difference between the arrival time of a radio signal at one end of the baseline and its arrival at the other end, includes the difference in excess delays caused by the ionosphere. Since radio signals propagated through the magnetoionic media suffer excess delays depending on its frequency, the observed delay at a frequency f is expressed as

$$\tau_{obs}(f) = \tau_g + \tau_{ion1}(f) - \tau_{ion2}(f) + \Delta\tau, \qquad (1)$$

where τ_g is the geometrical delay, τ_{ion1} and τ_{ion2} are ionospheric excess delays at station 1 and 2, respectively (see Fig. 1), and $\Delta\tau$ includes other delays such as neutral atmospheric delays, instrumental delays, clock offset, etc. The difference in ionospheric delays, $\tau_{ion1}(f)-\tau_{ion2}(f)$, can be easily obtained by receiving at dual frequency bands.

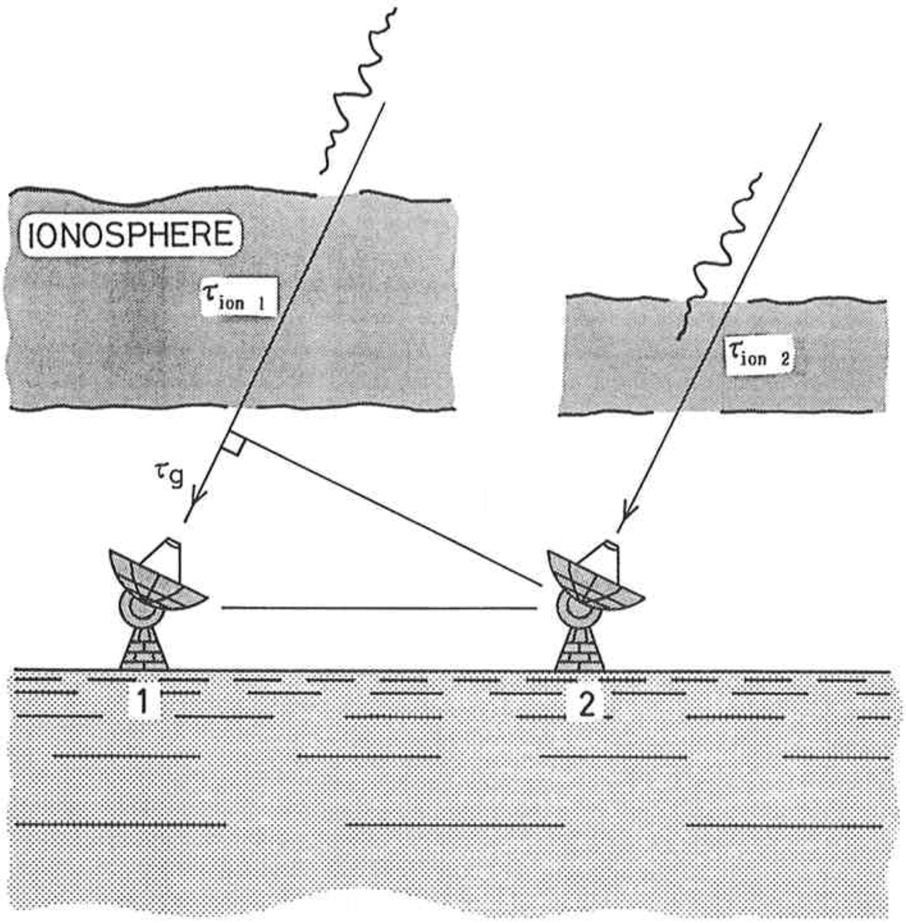

Fig. 1 Delay observed in a VLBI experiment. Radio waves arriving at stations 1 and 2 suffer ionospheric excess delays $\tau_{ion\ 1}$ and $\tau_{ion\ 2}$. τ_g is the geometrical delay.

In the geodetic VLBI measurements, both 8 GHz (X band) and 2 GHz (S band) radio signals are received. Because these frequencies are so high, we can apply a quasi-longitudinal approximation for a calculation of the refractive index[3] in the terrestrial ionosphere, where the plasma frequency and the cyclotron frequency are of the order of 10 MHz and 1 MHz, respectively. In this case, the excess group delay in the ionosphere can be expressed as

$$\tau_{\mathrm{ion}}(f) = 1.34 \times 10^{-7} N_{\mathrm{s}} f^{-2} (\mathrm{sec}), \qquad (2)$$

where N_s denotes TEC per unit area along the integrated line of sight (electron/m^2). We can, therefore, obtain the difference in ionospheric excess delay at X band, $\tau_{\mathrm{diff}}(f_{\mathrm{x}})$, by combining Eqs. (1) and (2) as follows,

$$\begin{aligned}\tau_{\mathrm{diff}}(f_{\mathrm{x}}) &= \tau_{\mathrm{ion1}}(f_{\mathrm{x}}) - \tau_{\mathrm{ion2}}(f_{\mathrm{x}}) \\ &= \left(\tau_{\mathrm{obs}}(f_{\mathrm{x}}) - \tau_{\mathrm{obs}}(f_{\mathrm{s}})\right) f_{\mathrm{s}}^2 / \left(f_{\mathrm{s}}^2 - f_{\mathrm{x}}^2\right)\end{aligned} \qquad (3)$$

where f_{x}, f_{s}, $\tau_{\mathrm{obs}}(f_{\mathrm{x}})$, and $\tau_{\mathrm{obs}}(f_{\mathrm{s}})$ are X band frequency, S band frequency, observed delay at X band, and that at S band, respectively. In the VLBI database, $\tau_{\mathrm{obs}}(f_{\mathrm{x}})$ and $\tau_{\mathrm{diff}}(f_{\mathrm{z}})$ are stored with other observables such as correlation amplitude, fringe phase, *etc.*, for further analysis. For geodetic analysis, a geometrical delay calculated by

$$\tau_{\mathrm{g}} = \tau_{\mathrm{obs}}(f_{\mathrm{x}}) - \tau_{\mathrm{diff}}(f_{\mathrm{x}}) \qquad (4)$$

is employed.

Here, $\tau_{\mathrm{diff}}(f_{\mathrm{x}})$ is employed for TEC estimation. From Eqs. (2) and (3), the difference between TEC at station 1 and station 2, ΔN_{t}, can be expressed as

$$\begin{aligned}\Delta N_{\mathrm{t}} &= N_{\mathrm{t1}} - N_{\mathrm{t2}} \\ &= 7.46 \times 10^6 f_{\mathrm{x}}^2 \tau_{\mathrm{diff}}(f_{\mathrm{x}}),\end{aligned} \qquad (5)$$

where N_{t1} and N_{t2} are TEC along the wave paths to the two stations. Thus, TEC directly obtained by VLBI observation is not that for each station but a difference in TEC between the two stations. However, if there is a sufficient number of τ_{diff} with respect to different source directions, we can obtain TEC for each station independently by means of least squares estimation. Our geodetic VLBI experiment sessions actually last for 24 hours with changing sources, and more than 150 observations in different directions were included in one session.

3. Least Squares Estimation of TEC

3.1 Linearized Least Squares Estimation

In least squares estimation N observable values are modeled as

$$Y = F(X) + e \qquad (6)$$

where Y denotes a measured observable vector (dimension N), X a parameter vector (dimension $M<N$), F a mathematical model for the effect of X on Y, and e an observation error vector (dimension N). $F(X)$ is linearized by expanding as

$$F(X) = F(X_0) + AX, \qquad (7)$$

where X_0 is a nominal parameter value vector (dimension M), which is assumed to be close to the true value vector, and A is a partial derivative (Jacobian) matrix (size $N \times M$), where an element of A is defined as

$$A_{nm} = \partial F_n(X) / \partial X_m (n = 1 \sim N, m = 1 \sim M). \qquad (8)$$

Thus, the observed values minus the calculated values, Y (dimension N), are expressed by N linearized equations as

$$\begin{aligned} \Delta Y &= Y - F(X_0) \\ &= AX + e. \end{aligned} \qquad (9)$$

The least squares solution $\hat{X}$ which minimizes the mean square observation error is computed by

$$\hat{X} = \left(\tilde{A}WA\right)^{-1} \tilde{A}W\Delta Y, \qquad (10)$$

where $\tilde{A}$ is the transposed matrix of A, W is an $N \times N$ weighting matrix (commonly diagonal) defined as

$$W_{ij} = \begin{cases} e_i^{-2} & (i = j) \\ 0 & (i \neq j) \end{cases} \qquad (i = 1 \sim N, j = 1 \sim N).$$

3.2 TEC Estimation

For the TEC estimation used here, an observable is $\tau_{diff}(f_x)$, and this differential ionospheric excess delay at X band is to be expressed by a mathematical function which includes TEC at each station. As for TEC variation with time, periodic variations are assumed. Figure 2 shows an example of the frequency spectrum of actual TEC variations observed at Kokubunji, Tokyo (35.7°N, 139.5°E), by Faraday rotation measurement. The figure clearly demonstrates that frequency with a period of one day and its harmonics up to the 4th order (i.e., 6-hour period) are dominant components of the spectrum. Hence, we use the following mathematical model to describe the variation of TEC at zenith at station i:

$$N_{ti}(t) = a_{i0} + \sum_{k=1}^{4} \left(a_{ik} \cos\left(\frac{kt\pi}{12}\right) + b_{ik} \sin\left(\frac{kt\pi}{12}\right) \right) + c_i t, \qquad (11)$$

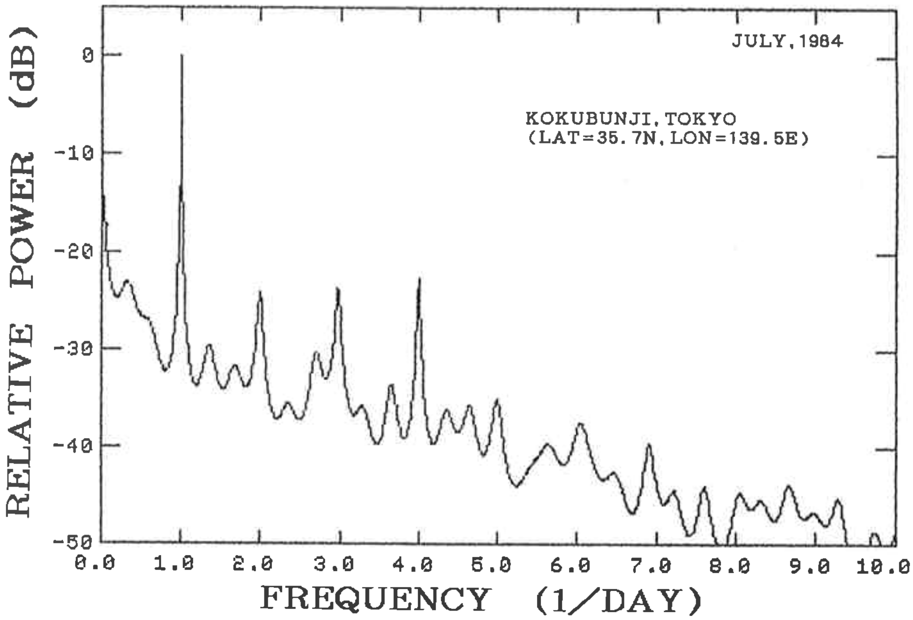

Fig. 2 Frequency spectrum of TEC variation observed in July 1984 at Kokubunji, Tokyo. Note that the one day component and its harmonics up to the 4th order are dominant.

where t is UT in hours and $c_i t$ is introduced to model long term (>1 day) variations. Therefore, TEC variation at a station is here modeled by ten parameters: a_{i0}, a_{i1}, ..., α_{i4}, b_{i1}, b_{i2}, ... b_{i4}, and c_i. A τ_{diff} observed by VLBI is the difference in delay along the line of sight for each station. Thus, by using a function $S(E_i)$, which describes the mapping function showing elevation-angle-dependence of the path length in the ionosphere at station i, the ionospheric excess delay at X band, $\tau_{ion\ i}$, can be modeled as

$$\tau_{ion\ i}(t) = 1.34 \times 10^{-7} f_x^{-2} N_{ti}(t) S(E_i). \quad (12)$$

The function $S(E_i)$ is given as

$$S(E_i) = 1/\cos\left\{\sin^{-1}\left[R\cos E_i / (R+h)\right]\right\}, \quad (13)$$

where E_i is the source elevation angle at station i, R is the earth's radius (6371.2 km), and h is the mean altitude of the ionosphere. In Eq. (13), a uniform thickness is assumed for the ionosphere over the station concerned, and h is taken to be 300 km. Consequently, the difference in X-band excess delays between station 1 and 2, $\tau_{diff}(f_x)$, can be modeled as

$$\tau_{model}(t) = \tau_{ion\ 1}(t) - \tau_{ion\ 2}(t) + \tau_{offset\ 1} - \tau_{offset\ 2}, \quad (14)$$

where $\tau_{\text{offset 1}}$ and $\tau_{\text{offset 2}}$ denote the instrumental delay offsets of station 1 and 2 and are introduced into the model to reflect actual observation conditions; they are also estimated from the data. In all, eleven parameters ($a_{i0}, a_{i1}, \ldots, b_{i4}, c_i$, and $\tau_{\text{offset i}}$) must be determined for one station by the estimation. For example, a total of 22 parameters should be adjusted in a single baseline experiment having two stations. However, it is impossible in such a situation to obtain instrumental delay offset independently at each station because of the interdependence between these instrumental delay offset parameters.

The partial derivatives of τ_{model} with respect to parameters, which are necessary for computing the least squares solution, are given by

$$
\begin{aligned}
&\frac{\partial\tau_{\text{model}}}{\partial\tau_{\text{offset}}} = \pm 1 \\
&\frac{\partial\tau_{\text{model}}}{\partial a_0} = \pm D \\
&\frac{\partial\tau_{\text{model}}}{\partial a_k} = \pm D\cos\left(\frac{kt\pi}{12}\right) \qquad (k = 1,2,3,4) \\
&\frac{\partial\tau_{\text{model}}}{\partial b_k} = \pm D\sin\left(\frac{kt\pi}{12}\right) \qquad (k = 1,2,3,4) \\
&\frac{\partial\tau_{\text{model}}}{\partial c} = \pm Dt
\end{aligned}
\qquad \cdots\cdots (15)
$$

with

$$D = 1.34\times10^{-7} f_x^{-2} S(E_i),$$

where positive and negative signs indicate remote and reference stations, respectively. We can then obtain the least squares solution of each parameter by computing Eq. (10). Substituting obtained parameters into Eq. (11), TEC at time t can be calculated.

The most simple estimation of TEC occurs for single baseline data. In this case, the size of the Jacobian matrix is $N\times 22$, where N is the number of observations. TECs for the two stations at either end of the baseline can be obtained simultaneously by means of the least squares estimation. As described previously, as long as single baseline data are used for the estimation, offset of at least one station must be fixed.

Multi-baseline data should be used for reliable estimation of TEC. In this case, the number of observable values becomes $N \times L$, where L is the number of baselines used, and the number of parameters becomes $11 \times I$, where I is the number of stations relevant to these baselines. Thus a partial derivative matrix consists of $N \times L \times 11 \times I$ elements. Each partial derivative relating to a baseline in which τ_{diff} is an observable is calculated by Eq. (15). Other partial derivatives not relevant to the baseline are set to zero.

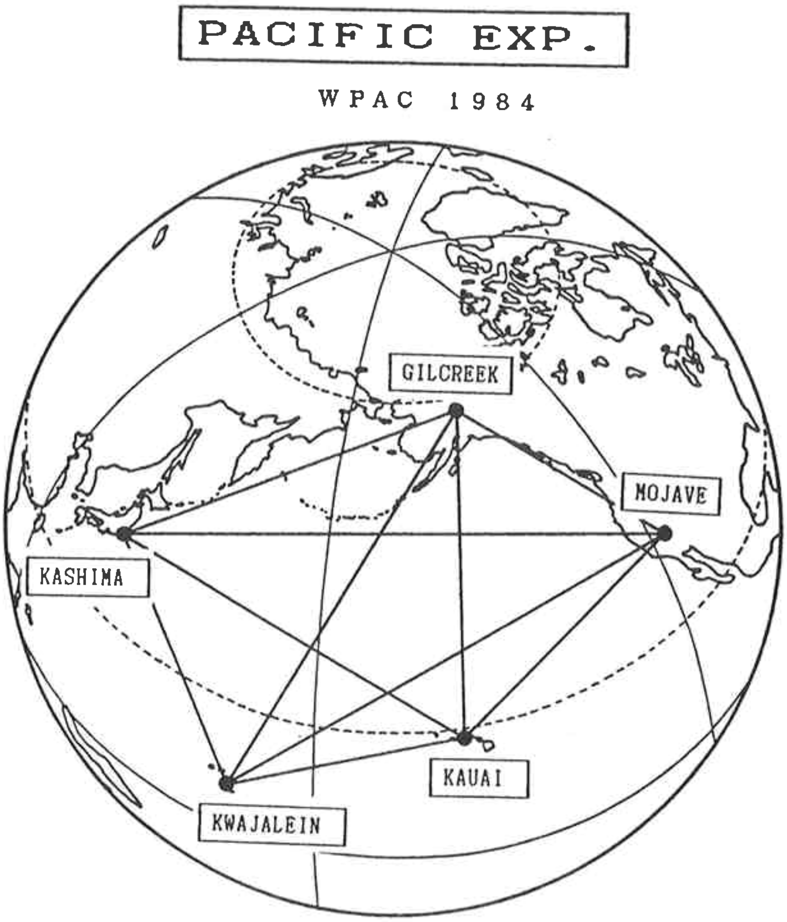

Fig. 3 Station configuration of VLBI experiment conducted on July 29, 1984.

4. Results

The data obtained by a VLBI experiment conducted on July 29, 1984 have been used to compare TEC estimated by the current method and that from another technique, such as Faraday rotation measurements of a geostationary satellite beacon. Five stations participated in the experiment forming ten baselines (Fig. 3). The TEC of each station was estimated using various combination of baselines. Figure 4 represents the TEC obtained for Kashima (35.9°N, 140.7°E) from four baselines of data (TEC_V). In the figure, TEC obtained from the Faraday rotation measurements (TEC_F) at Kokubunji (about 100 km west of Kashima) is shown by a dashed line for the sake of comparison. We can see good agreement between them: they take their minima around 18 UT and two maxima around 7 UT and 23

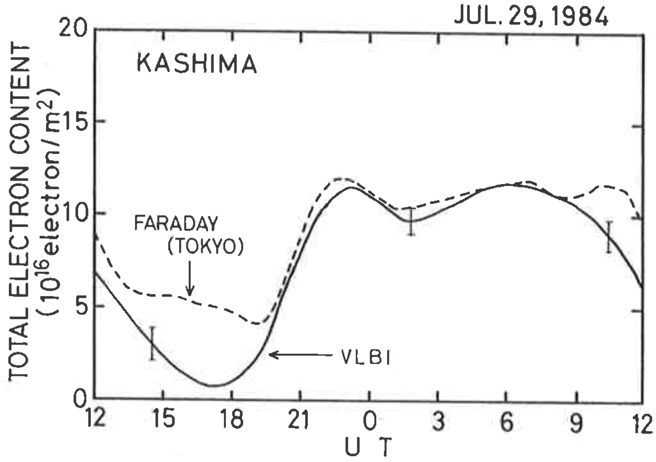

Fig. 4 TEC at Kashima estimated from VLBI data (solid line) and that observed at Kokubunji, Tokyo, by Faraday measurement of a geostationary satellite beacon (dashed line). Four baselines of data: Kashima (KAS)-Gilcreek, KAS-Kauai, KAS-Kwajalein, and KAS-Mojave baselines, were used for estimation. Error bars imposed on the solid line are one sigma error of estimation.

UT. They coincide within an error of less than 5×10^{16} electron/m^2. A systematic discrepancy larger than one sigma error ($> 1 \times 10^{16}$ electron/m^2) of estimation, however, remains between them; TEC_V is lower than TEC_F in the range of 11-20 UT.

5. Discussion

Based on the results described above, it can be concluded that TEC estimated from VLBI data is consistent with TEC obtained by conventional methods such as Faraday rotation measurement of a satellite beacon. There are, however, some discrepancies between them, as was shown in Fig. 4. Some possibilities have been considered to explain the discrepancies.

First, the spatial structure of the ionosphere must be taken into account. The Faraday measurements of the geostationary satellite beacon give us information about a fix point in the ionosphere. On the other hand, VLBI data give us TEC averaged out over a considerably extended region in both longitudinal and latitudinal directions in the ionosphere. Figure 5 shows the geographic locations of sub-ionospheric points (the point at which the path from the source to the observation station intersects an ionospheric altitude of 300 km) in the ray paths from the wave sources to Kashima during the VLBI experiment conducted on July 29, 1984. Each point denoted by "A", "B", ..., and "K" corresponds to a different observation in the VLBI experiment session. Their mean coordinates, indicated by "∗ ", are 36.6°N and 143.5°E. A sub-ionospheric point of the geostationary satellite ETS-II for the Faraday rotation measurement at Kokubinji, Tokyo, is also shown in the figure by "+", and its coordinates are 33.5°N and 138.4°E. As shown in Fig. 5 TEC_V is averaged over the region whose latitudinal and longitudinal extents are about 6° (700 km) and 12° (1100 km), respectively. Hence, ionospheric plasma distributions, in dimensions less than 700–1100 km are averaged out. On the other hand, TEC_F

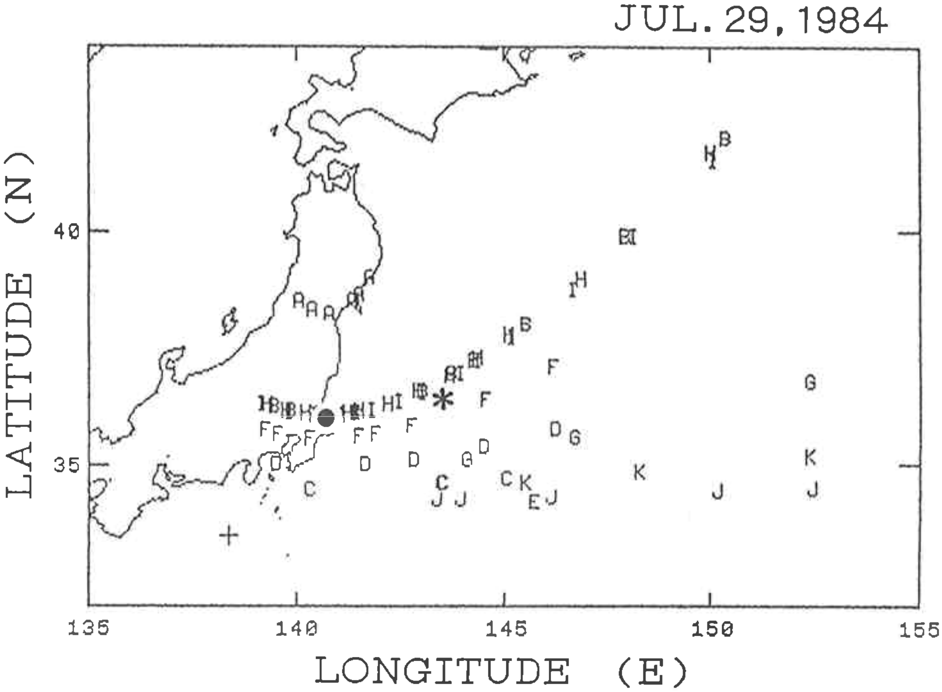

Fig. 5 Sub-ionospheric points ("A", "B", ...) of radio sources observed from Kashima for the VLBI experiment conducted on July 29, 1984. A mean ionospheric altitude of 300 km is assumed. Different symbols mean different radio sources. An averaged position over all observations is displayed by "*". The position of Kashima is shown by "•". The symbol "+" denotes the subionospheric point of the ray path from the geostationary satellite, ETS-II, to Kokubunji, Tokyo.

reflects the variation at a fixed point in the ionosphere. Therefore, it is thought that the discrepancies seen in Fig. 4 are due to the difference in ionospheric areas observed by the two techniques.

Next, the difference in time resolution between both techniques must be considered. In the current TEC model used for TEC_V estimation, fluctuations with periods shorter than 6 hours are omitted. On the other hand, the time resolution of TEC_F displayed in the figure is 15 minutes. This means that TEC_V is averaged not only over a spatial extent but also in a time domain. To express the rapid increase around 20 UT seen in TEC_F (see Fig. 4) by a Fourier series, higher order terms having a period shorter than 6 hours must be taken into the model. This could reduce the discrepancy seen at 12–20 UT. However, we must be careful when appending more parameters to the current TEC variation model as this may cause stronger coupling between parameters, making it more difficult to obtain a unique solution in the estimation.

As described above, TEC estimated from VLBI data is averaged over a spatial extent of several hundred kilometers, and fluctuations with a period shorter than 6 hours are omitted. These may explain the discrepancies between TEC_V and TEC_F.

6. Conclusion

We have described a method for estimating ionospheric total electron content from VLBI data, and have presented the results of an experiment. By comparing the TEC obtained from VLBI data with that derived by observing the Faraday rotation of a geostationary satellite beacon, it was confirmed that the TEC estimation method presented here gives reasonable results, particularly when multi-baseline data are used for the estimation. It is true, however, that there are some limitations due to the VLBI experiment itself and to the TEC variation model. In the model, TEC obtained from VLBI data is averaged over a spatial extent of several hundred kilometers and its time resolution is at most 6 hours. From these reasons, the model is inadequate for research that requires both high spatial resolution and high time resolution. However, this method allows the TEC of each station participating in the VLBI experiment to be obtained simultaneously with the same accuracy, which should be very beneficial in the investigation of the global ionosphere.

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