PPS03-06

Development of Software for Precise LLR Data Analysis

and initial results of parameter estimation for lunar motion

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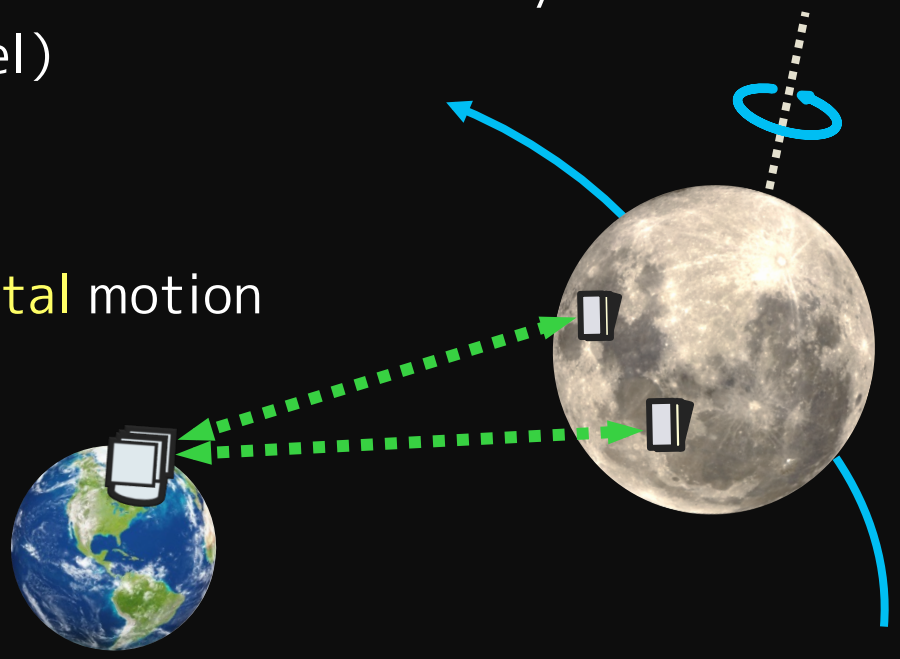
This study aims at...

Estimation of lunar interior structure, by analysis of lunar orbit, rotation & tidal deformation

Software development
- for precise determination of lunar motion, using LLR data
- comparable to JPL's software in terms of accuracy
  (range residuals ~ 5 cm level)

➢ Present status

simple estimation of lunar orbital motion



Lunar geophysical parameters estimated by LLR

Rotation

$I \, d\boldsymbol{\omega}/dt = \boldsymbol{N}$

- Dynamical oblateness β, γ
- Potential Love number k_2
- Gravity coefficients

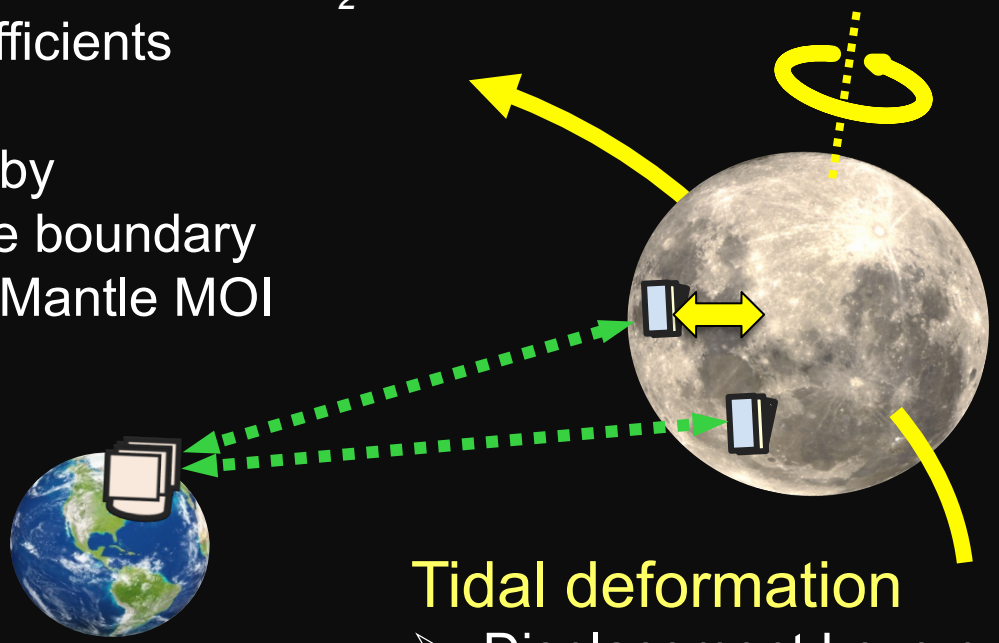
- Dissipation by core-mantle boundary
- Core MOI / Mantle MOI

etc.

Lunar orbit

$m \, d\boldsymbol{v}/dt = \boldsymbol{F}$

- GM of Earth + Moon
- Earth/Moon mass ratio



Tidal deformation

- Displacement Love number h_2 & l_2

Calculation Process

Set initial conditions

- Lunar position
- Lunar rotation angles
- const. parameters

Integrate dynamical equation
& variational equation

- Range residuals $r_i = \rho_i^{obs} - \rho_i^{calc}$
- Jacobian matrix $H_i = \partial(\rho_i^{calc}) / \partial(\text{init. conditions})$

Least-squares fitting of dynamics to observed value

- Update initial conditions $\Delta(\text{init. conditions}) = \sum_i^{} (H_i^T H_i)^{-1} H_i^T r_i$

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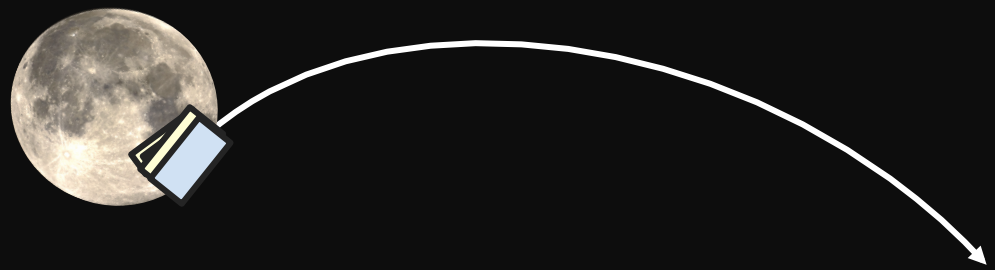

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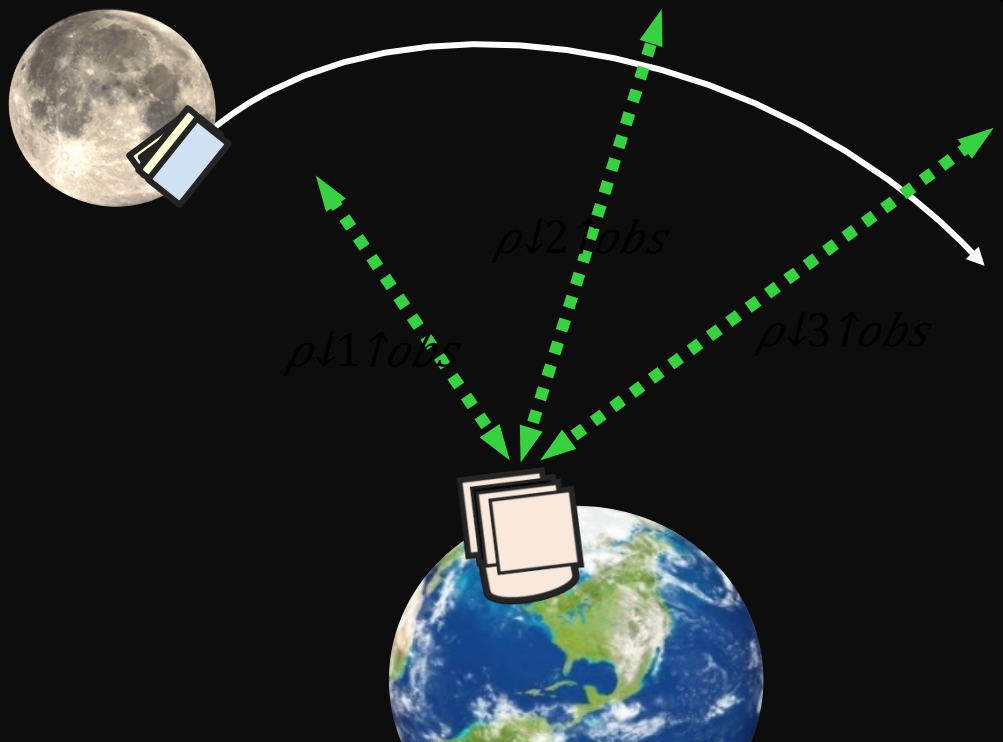

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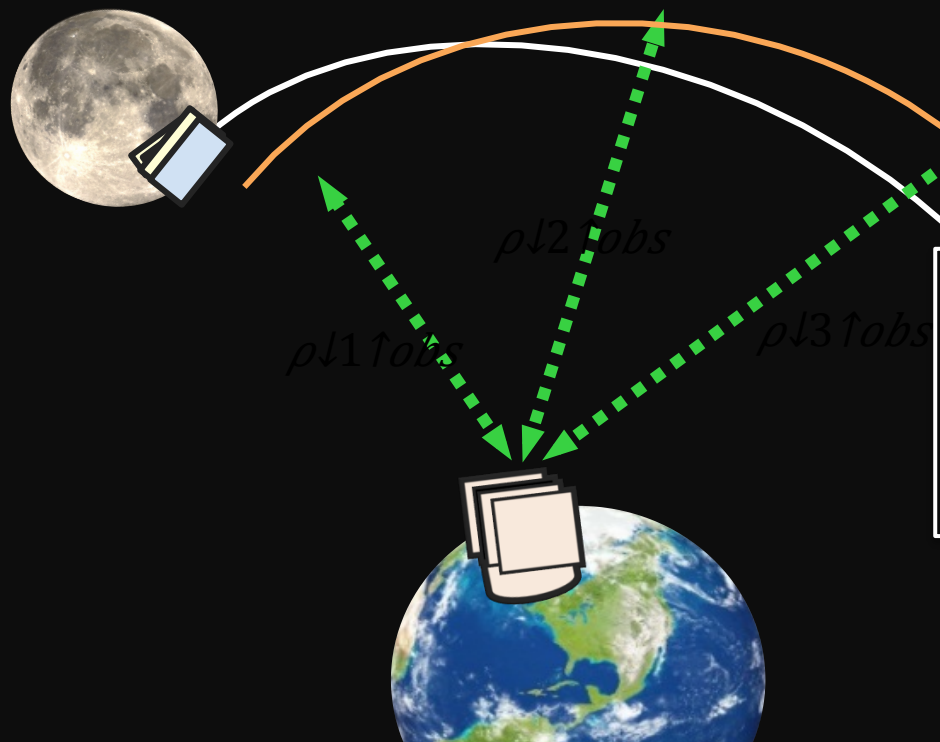

Least-squares fitting of dynamics to observed ranges

- Update initial conditions $\Delta(\text{init. conditions}) = \Sigma_i (H_i^T H_i)^{-1} H_i^T r_i$

Calculation Process

Set initial conditions

- Lunar position
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Integrate dynamical equation & variational equation

- Range residuals $r_i = \rho_i^{obs} - \rho_i^{calc}$
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Least-squares fitting of dynamics to observed ranges

- Update initial conditions $\Delta(\text{init. conditions}) = \sum_i (H_i^T H_i)^{-1} H_i^T r_i$

This process is repeated until the residuals converge.

LLR analysis softwares in the world

	JPL	IfE Hannover	IMCCE Paris	IAA RAS St. Petersburg	This study
Ephemeris	DE	not published	INPOP	EPM-ERA	
Residuals ("O-C" RMS)[1]	**4.2 cm**	**~5.0 cm** [pers. comm.]	**5.0 cm**	**5.8 cm**	**goal: 5.0 cm**
Partial derivatives	integration		numerical differences		integration
Integration method & order	Adams (Predictor + Corrector)				Runge-Kutta
	variable		12^{th} -order		8^{th}-order
Orbit determination	**Sun, Planets, Moon**				**Moon**
Coordinates	solar system barycentric				**geocentric**

Residuals are calculated by EPM-ERA software [Vasilyev & Yagudina, 2014]

3 Parts of Software Development

What we have considered

1. LLR observation model

Predicts the LLR observed values under the given ephemeris (here, DE430)

✔ Earth orientation, Signal propagation correction

2. Lunar dynamical motion models

Equation of lunar dynamical motion
Lunar ephemeris is created by integration

✔ Full relativistic point mass interaction, Figure Effect of Earth, Sun External torques

3. Least squares fitting

Fits integrated ephemeris to observed range Estimating initial parameters

Partial derivatives of “only” orbital motion

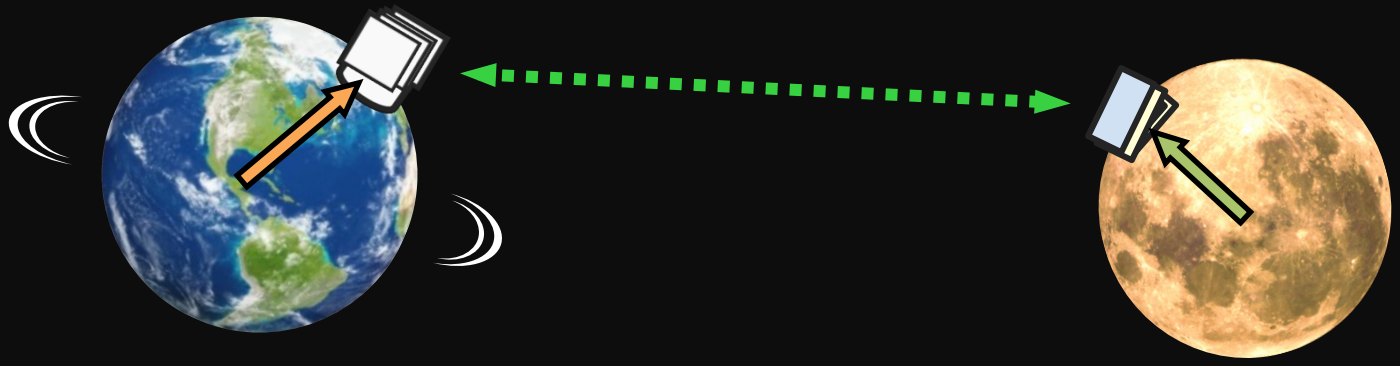
Components of observation model

Time variation of
Geocentric Station Position

Earth orientation	~ 100 m
Solid Earth tides	~ 0.1 m
Ocean Loading	~ 0.01 m

Time variation of
Selenocentric Reflector position

Solid Moon tides	~ 0.1 m



※ lunar orbit & rotation are treated as dynamical variables

Propagation delays by media

Relativistic effects	~ 10 m
Atomospheric delay	~ 1 m
Time scale transformation	~ 1 m

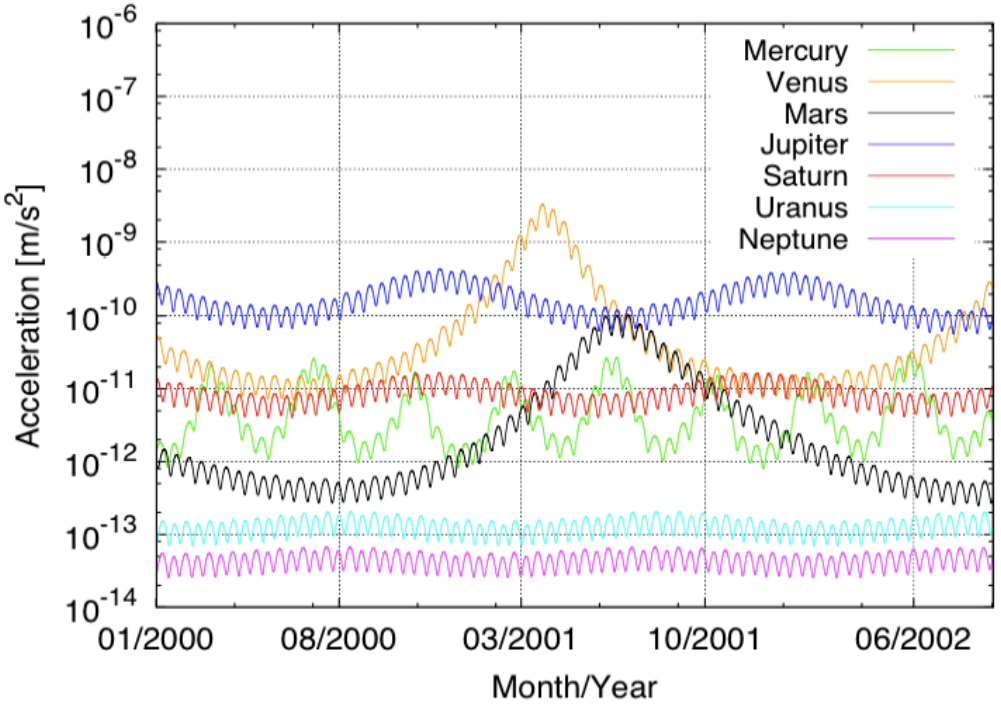
- TT (geocentric) to TDB (solar system barycentric)

Relativistic formalism of point mass interaction

$$\ddot{\boldsymbol{r}}_i = \sum_{j\neq i} \frac{\mu_j(\boldsymbol{r}_j - \boldsymbol{r}_i)}{r_{ij}^3} \left\{ 1 - \frac{2(\beta+\gamma)}{c^2} \sum_{k\neq i} \frac{\mu_k}{r_{ik}} - \frac{2\beta-1}{c^2} \sum_{k\neq j} \frac{\mu_k}{r_{jk}} + \gamma \left(\frac{v_i}{c}\right)^2 + (1+\gamma)\left(\frac{v_j}{c}\right)^2 - \frac{2(1+\gamma)}{c^2} \dot{\boldsymbol{r}}_i \cdot \dot{\boldsymbol{r}}_j - \frac{3}{2c^2}\left[\frac{(\boldsymbol{r}_i - \boldsymbol{r}_j)\cdot \dot{\boldsymbol{r}}_j}{r_{ij}}\right]^2 + \frac{1}{2c^2}(\boldsymbol{r}_i - \boldsymbol{r}_j)\cdot \ddot{\boldsymbol{r}}_j \right\} + \frac{1}{c^2} \sum_{j\neq i} \frac{\mu_j}{r_{ij}^3} \{[\boldsymbol{r}_i - \boldsymbol{r}_j]\cdot[(2+2\gamma)\dot{\boldsymbol{r}}_i - (1+2\gamma)\dot{\boldsymbol{r}}_j]\}(\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_j) + \frac{3+4\gamma}{2c^2} \sum_{j\neq i} \frac{\mu_j \ddot{\boldsymbol{r}}_j}{r_{ij}}$$

☝ Einstein-Infeld-Hoffmann formalism,

acceleration : SSB -> i-th body



Acceleration acting on the Moon as 3rd body perturbations of planets

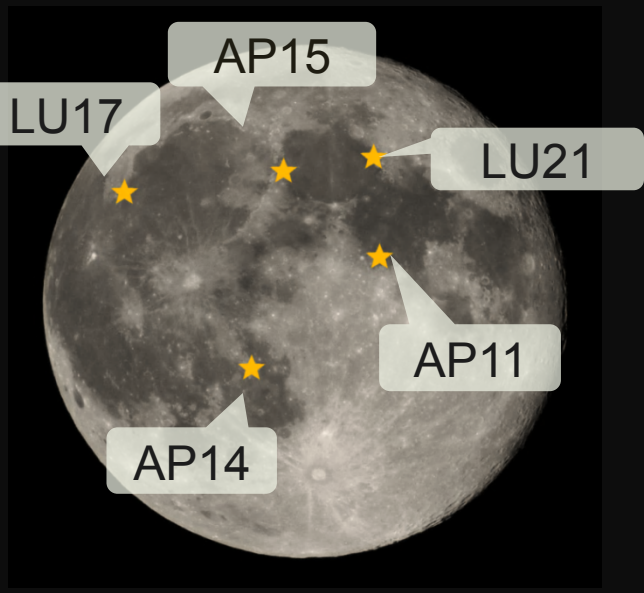
Observed data resources

LLR Normal Point

- CDDIS
  EUROLAS Data Center
  - McDonald (7080), Grasse (7845), Matera (7941)
  - Jun. 1996 – Dec. 2014
- APOLLO
  - Apache Point (7045)
  - Apr. 2006 – Aug. 2012

normal points we compared (3708 in total)

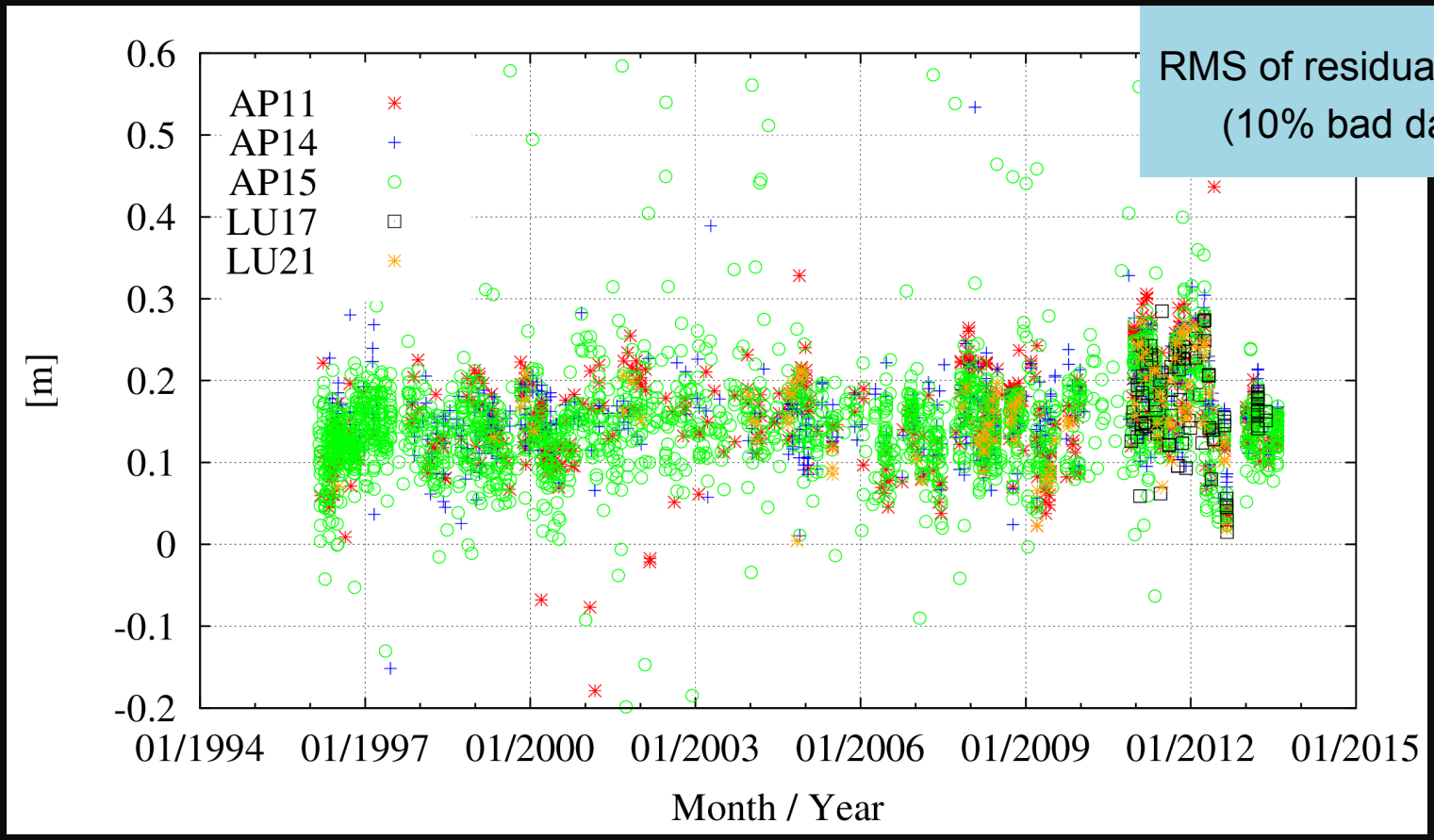
	AP11	AP14	AP15	LU17	LU21
Apache Point	255	264	668	75	81
Grasse	170	150	551	13	25
Matera	1	1	10	0	0
McDonald	181	174	1082	0	7



Result of part 1 : representation of observations

- lunar orbit & rotation: DE430 [Folkner+ 2014]

Residuals (obs. – calc.) in meters, colored by reflectors



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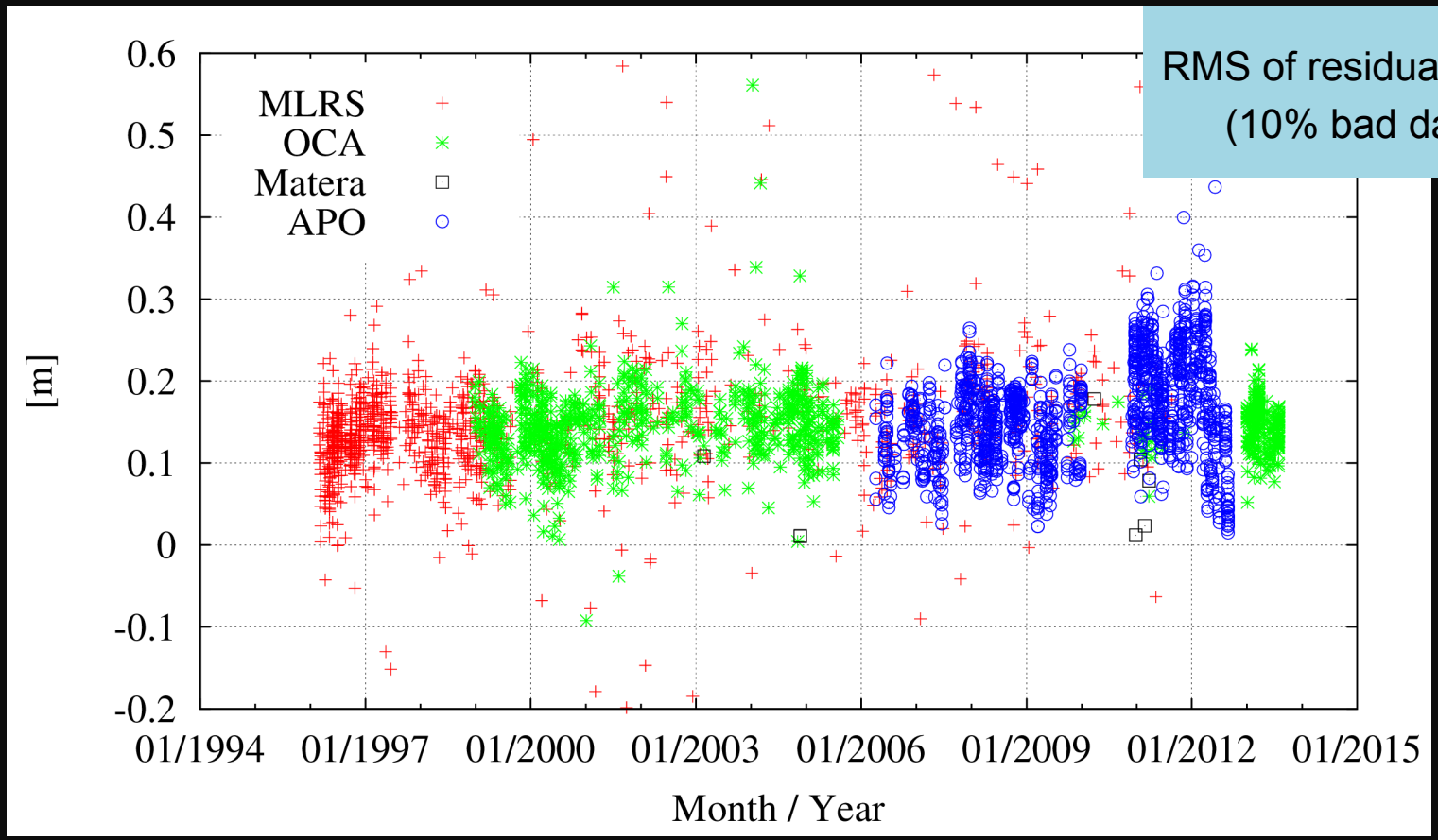

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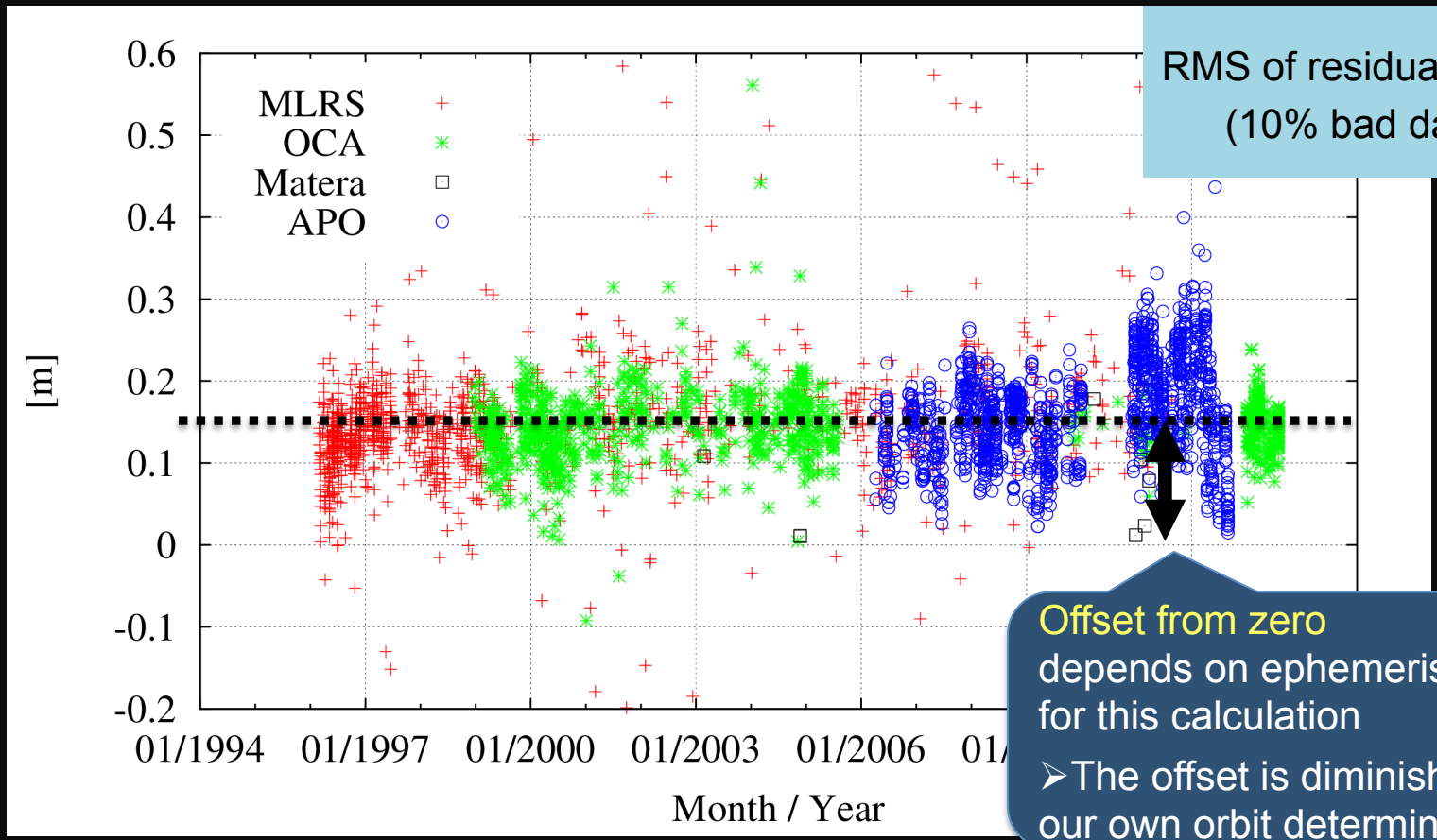


RMS of residuals : 5.1 cm
(10% bad data rejected)

Offset from zero
depends on ephemeris used for this calculation

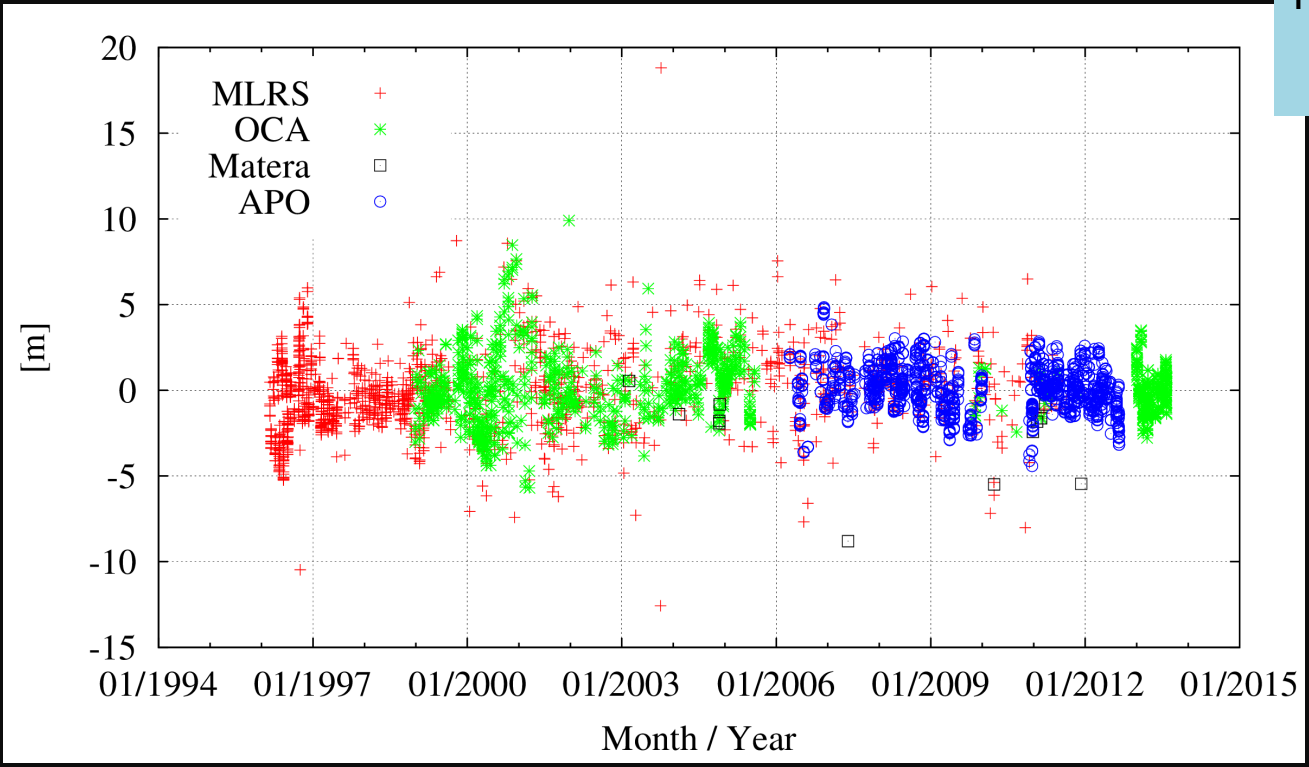
➢The offset is diminished by our own orbit determination

preliminary stage

Results of Least Squares Fitting

- Orbit : Free to be fit
  - Geopotential (up to 4th order), 3rd body perturbations, solar J_2, Earth Tides
- Rotation : fixed to DE430
  - At present, not applied to estimation

Post-fit residuals (obs. – calc.) in meters, colored by stations



RMS of residuals
Post-Fit = 1.77 m

preliminary stage

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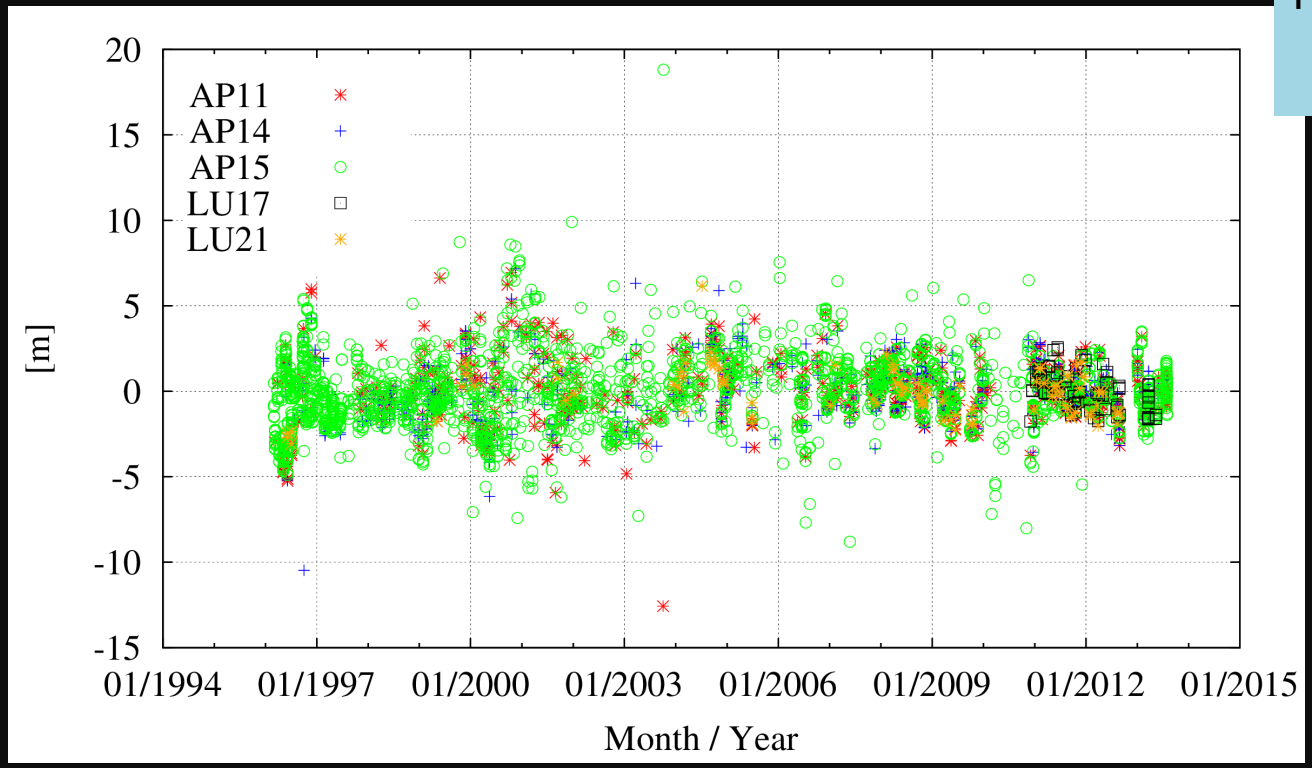
Post-fit residuals (obs. – calc.) in meters, colored by reflectors



RMS of residuals
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List of Estimated Parameters

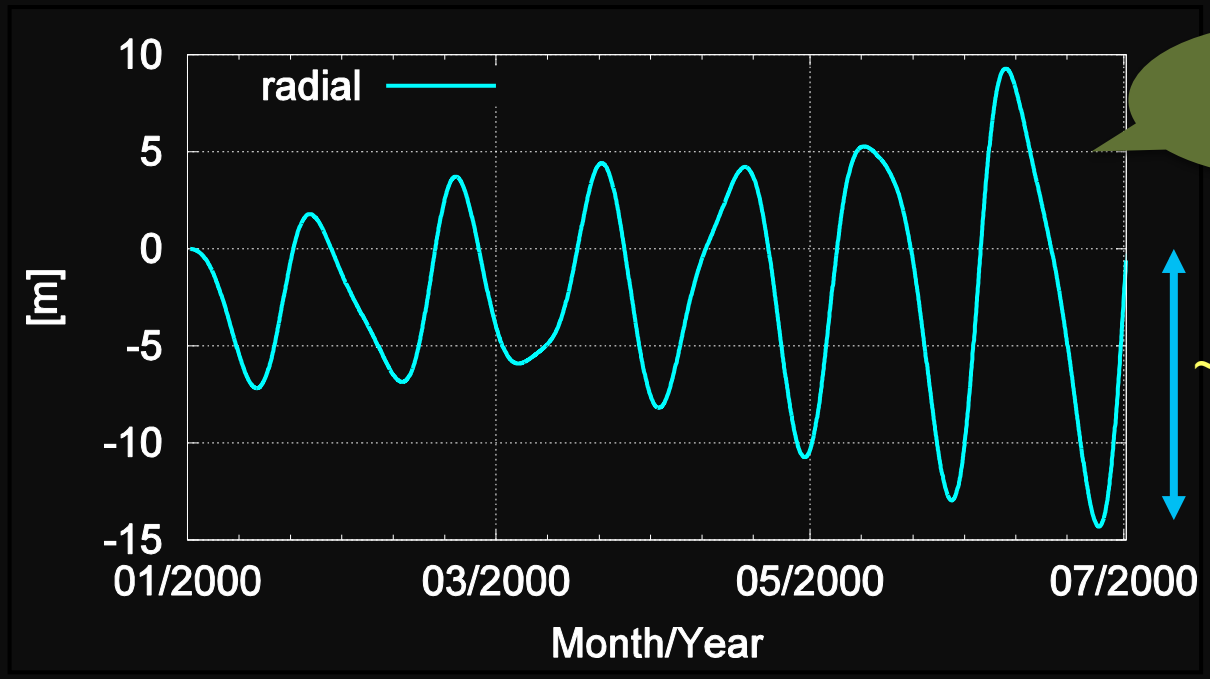
preliminary stage

Orbit : fit, Rotation : DE430

Parameter name	Unit	Estimated Value	Std. Dev. (estimated)	Std. Dev. (DE430)
GM of (Earth + Moon)	m^3/s^2	4.04×10^{14}	**3.57×10^{7}**	**5.00×10^{5}**
Earth/Moon mass ratio		81.18	**3.69×10^{-3}**	**2.40×10^{-6}**
h_2 of the Moon		0.372	**0.023**	**0.0064**
AP15 coordinates Lunar Principal Axes	m	X = 1.55×10^{6} Y = 9.71×10^{5} Z = 7.63×10^{5}	**X = 12.0** **Y = 3.30** **Z = 4.70**	
Lunar geocentric position at epoch	m	X = $- 3.18 \times 10^{8}$ Y = $- 2.37 \times 10^{8}$ Z = $- 6.27 \times 10^{7}$	**X = 9.21** **Y = 9.30** **Z = 18.0**	

- when h_2 and l_2 are both solved, h_2 = 2.26, l_2 = 1.75
- Love number l_2 may be strongly correlated with h_2 and reflector coordinates
  - ➢ JPL & IfE fix l_2 to a model value

Residuals: JPL DE430 minus model



- Integration method
  - 8th order Runge-Kutta [Prince & Dormand, 1981]
  - stepsize = 1 hour
- physical models ..basically the same as the JPL
  - Geopotential (up to 4th order), relativistic 3rd body perturbations, solar J_2, Earth Tides

Future Works
Requirements for further development

Necessity of creating planetary ephemeris ?

- Lack of another constraint of conserving mechanical energy drives SSB to wrong direction
- 200 m in radius (10 years in period)

[pers. comm. with Mai & Mueller, 2015]

Integration method : need to be fast, accurate

Dynamical system : numerically stable

- Remaining issues
  - Determination of the rotational motion
  - Lunar interior physics
    - e.g. core + mantle rotation modeling