Lunar Time

Status

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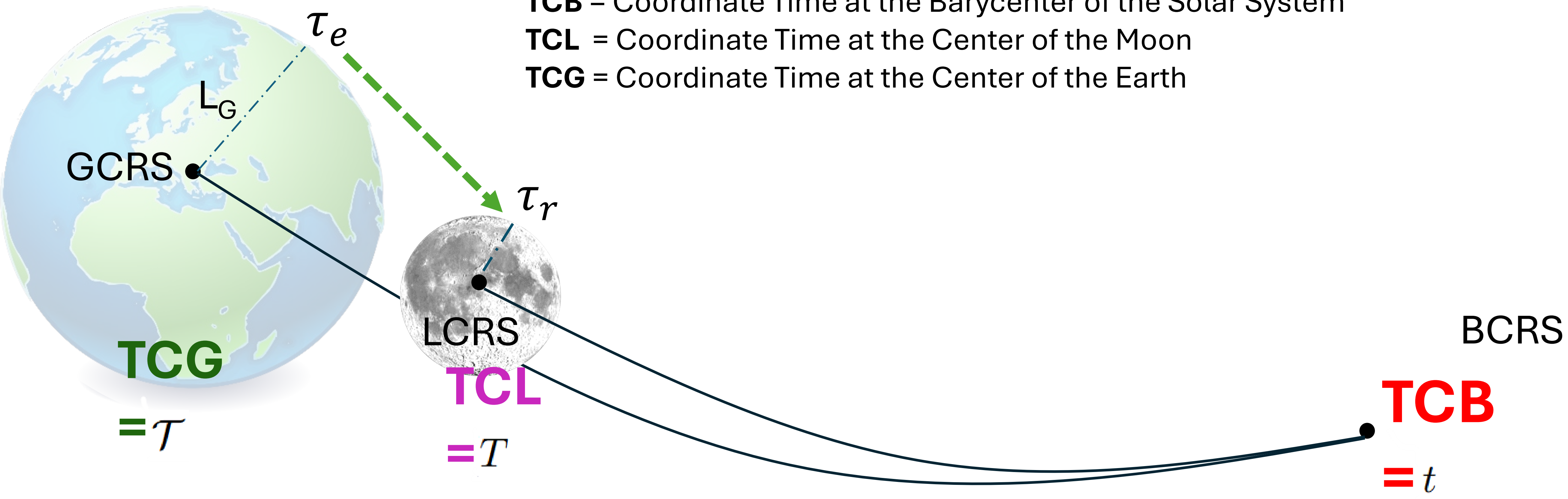
Time transfer between the Moon and Earth

$$\tau_\mathrm{r} - \tau_\mathrm{e} = [\tau - T]_{(T_\mathrm{r}, \boldsymbol{X}_\mathrm{r})} + [T - t]_{(t_\mathrm{r}, \boldsymbol{x}_\mathrm{r})} + [t_\mathrm{r} - t_\mathrm{e}]_{(t_\mathrm{e}, \boldsymbol{x}_\mathrm{e}, \boldsymbol{x}_\mathrm{r})} + [t - \mathcal{T}]_{(t_\mathrm{e}, \boldsymbol{x}_\mathrm{e})} + [\mathcal{T} - \tau]_{(\mathcal{T}_\mathrm{e}, \boldsymbol{\mathcal{X}}_\mathrm{e})}$$

TCB = Coordinate Time at the Barycenter of the Solar System
TCL = Coordinate Time at the Center of the Moon
TCG = Coordinate Time at the Center of the Earth



Reminder : Proper Time / Coordinate Time

PROPER TIME τ

The time as measured by a clock

A 

$$\tau_A - TCG = -\frac{1}{c^2}\int_{t_0}^{t}\left[\frac{{v_A}^2}{2} + U_E(\overrightarrow{x_A}) + W_{tidal}(\overrightarrow{x_A})\right]dt$$

2 events are synchronous in a space–time coordinate system if they have the same coordinate time.
Ex. 2 clocks on the geoid are synchronous in the GCRS
(!! if neglecting the tidal effects)

Synchronization is entirely dependent on the chosen space–time coordinate system

COORDINATE TIME t

One of the 4-D coordinates of an event in theory of relativity.

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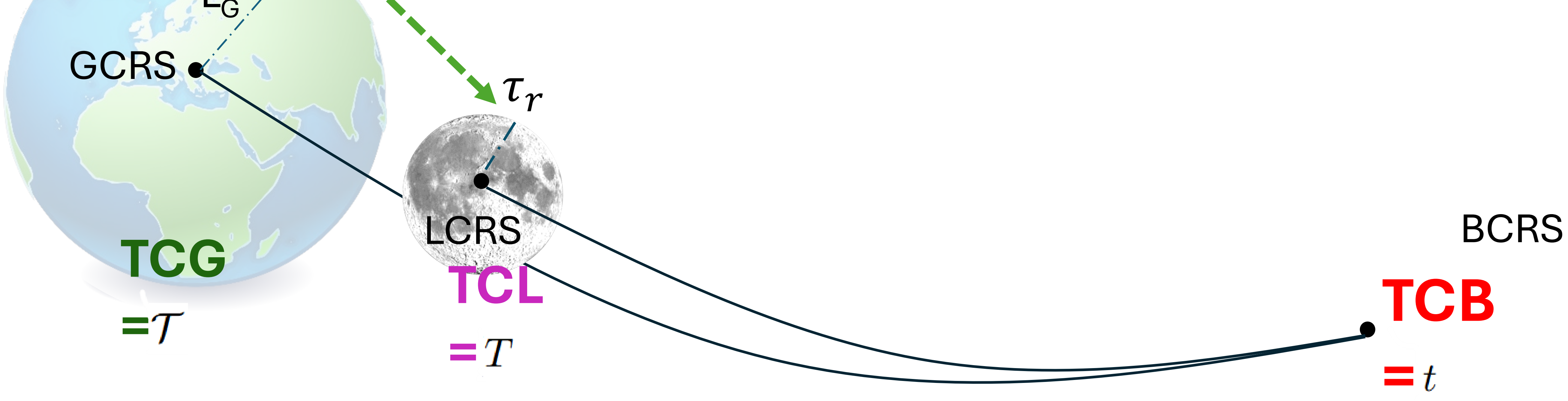
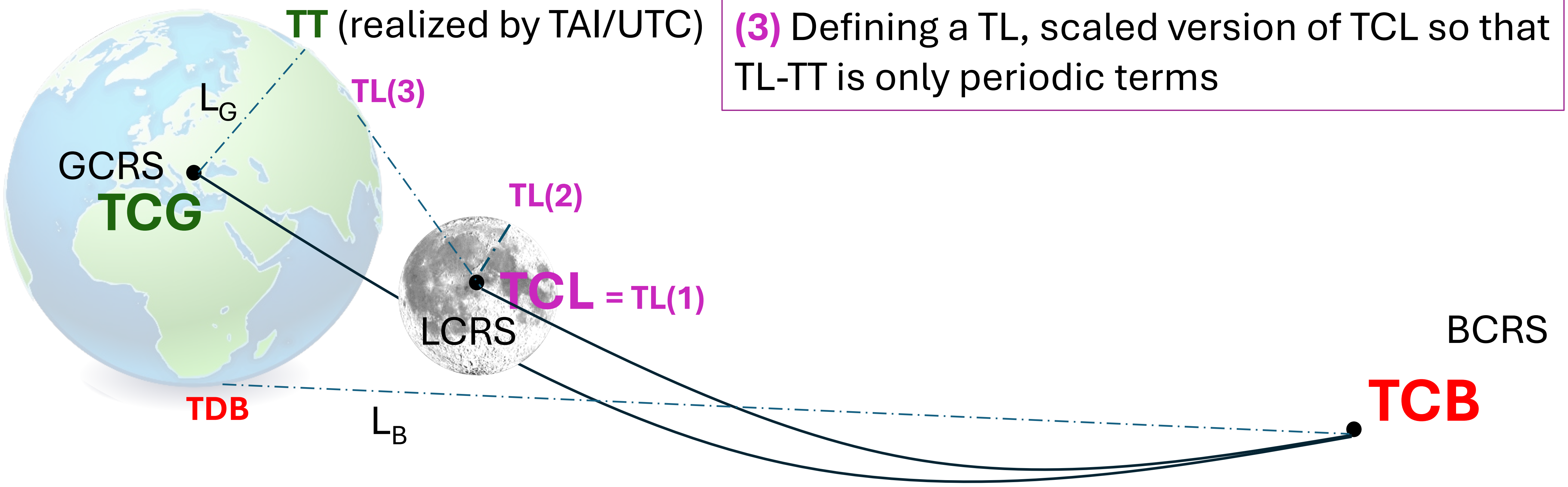
$$\tau_\mathrm{r} - \tau_\mathrm{e} = [\tau - T]_{(T_\mathrm{r}, \boldsymbol{X}_\mathrm{r})} + [T - t]_{(t_\mathrm{r}, \boldsymbol{x}_\mathrm{r})} + [t_\mathrm{r} - t_\mathrm{e}]_{(t_\mathrm{e}, \boldsymbol{x}_\mathrm{e}, \boldsymbol{x}_\mathrm{r})} + [t - \mathcal{T}]_{(t_\mathrm{e}, \boldsymbol{x}_\mathrm{e})} + [\mathcal{T} - \tau]_{(\mathcal{T}_\mathrm{e}, \boldsymbol{\mathcal{X}}_\mathrm{e})}$$

Term	freq. offset		remarks
$\tau - T = \tau_r - \mathrm{TCL}$	$\simeq 2.3 \times 10^{-11}$	$\simeq 2\,\mu\mathrm{s/d}$	on the surface
$T - t = \mathrm{TCL} - \mathrm{TCB}$	$\simeq 1.5 \times 10^{-8}$	$\simeq 1300\,\mu\mathrm{s/d}$	+ periodic terms
$t - \mathcal{T} = \mathrm{TCB} - \mathrm{TCG}$	$\simeq 1.5 \times 10^{-8}$	$\simeq -1300\,\mu\mathrm{s/d}$	+ periodic terms
$\mathcal{T} - \tau = \mathrm{TCG} - \tau_e$	$L_g = 6.969290134 \times 10^{-10}$	$\simeq -60\,\mu\mathrm{s/d}$	on the geoid

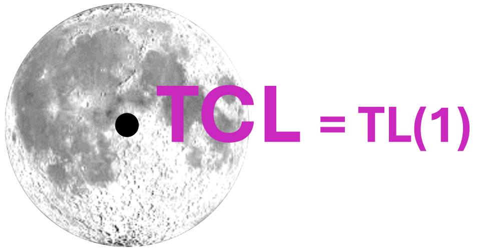


3 options for a reference time on the Moon

(1) Using TCL
(2) Defining a TL, scaled version of TCL, and defined on a given W_{L0}
(3) Defining a TL, scaled version of TCL so that TL-TT is only periodic terms



Option (1): TL as TCL, at lunar barycenter



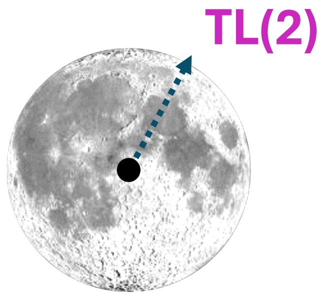
PROs

- No additional time scale defined, TCL (as time coordinate of the LCRS) is anyway needed for geodesy
- Also means that we do not introduce a new space-time reference system, which would imply another scaling of masses, distances (as is already the case with TCB, TDB, TT…)

CONs

- An ideal clock located on the Moon surface would have a frequency offset about 2μs/day (2.3×10^{-11}) with respect to the reference
- significant linear drift between the lunar reference and TT (or TDB): ~58 μs/day or 6.7×10^{-10}

Option (2): TL near the lunar surface



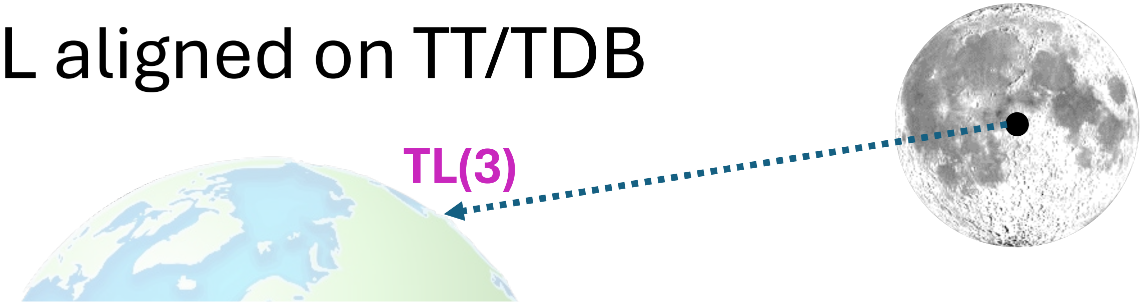
PROs

- An ideal clock located on the surface defined by a lunar W_{L0} would have no frequency offset with the reference time scale.
- Same logic as for Earth.

CONs

- The link between an Earth clock realizing TT and a Moon clock realizing TL would anyway depend on the clock location through TCL-TCG($\mathbf{x}$,t)
- For theoretical developments, this new scaling (TCL-TL) would induce a new scaling of masses, distances (E.g. the Earth Mass has already different values if using TCB, TDB, TT).
- There is still a significant linear drift (~ 56 μs/d or 6.5×10^{-10}) between the lunar reference and TT (or TDB)

Option (3): TL = scaled version of TCL aligned on TT/TDB



PROs

- Only periodic terms between TL and TT: no linear drift between the lunar reference and TT (or TDB)
- Direct compatibility between Lunar and Earth GNSS (same time reference, on average)

CONs

- An ideal clock located on the Moon would have a frequency offset about 56 µs/day (6.6×10^{-10}) with respect to the reference
- The link between an Earth clock realizing TT and a Moon clock realizing TL would anyway depend on the clock location through TCL-TCG(**x**,t)
- For theoretical developments, this new scaling (TCL-TL) would induce a new scaling of masses, distances (E.g. the Earth Mass has already different values if using TCB, TDB, TT).

Realization of the reference

In the first decade(s), a link with Earth will be unavoidable:

(τ_E -TCG) Earth ground clock to center of Earth

+ (TCG – TCL) (not unique ! Depends on the clock positions)

+ (TCL - τ_L)

… all depending on the exact position of the clocks (at the ns level for TCG –TCL)

If there is an autonomous network of clocks on the Moon:

Local synchronisation within this network “like UTC(k)s on Earth” is possible, but any time transfer with Earth will require the above calculations anyway.

This could then be a local realization of the Lunar reference time scale (TL(k) ?). LNSS satellites could broadcast their offset to it. The paper time scale of reference to compare all TL(k) could be called TL(Moon).

BIPM could publish a difference between TL(k) and UTC (with an ad-hoc time transfer convention) ?

In any case, UTC will remain the practical reference

- TL will be used for high-precision timing (e.g., synchronizing PNT constellations)
- A local (spatial + temporal) estimate of the UTC-TL difference should be calculated and broadcast by PNT systems (e.g. polynomials like the current UTC-GNSStime broadcast message)
- For most day-to-day operations it would be very tricky to use a different time scale on Earth and on the Moon, for no benefit if microseconds uncertainties are tolerated.
- As the other timescales, TL shall use Julian Days (of 86400 local seconds) as representation