



Suppose the coordinates of points P_1 , P_2 and P_3 are being measured. When a line perpendicular to the straight line between P_1 and P_2 is drawn from point P_3 , the range $(\overline{P_1 P})$ between P_1 and P can be measured.

The coordinates of P_1 , P_2 , P_3 and P are as follows:

$$\begin{cases} P_1 (x_1, y_1, z_1) \\ P_2 (x_2, y_2, z_2) \\ P_3 (x_3, y_3, z_3) \\ P(x, y, z) \end{cases}$$

According to the above figure, vectors a and b will be as follows:

$$\begin{cases} a = \overrightarrow{P_1 P_2} = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \\ b = \overrightarrow{P_1 P_3} = \begin{bmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \end{bmatrix} \end{cases}$$

Suppose the angle formed by straight lines P_1, P_2 and P_1, P_3 is θ . By orthogonalizing straight lines P_1, P_2 and P_1, P_3 , the equation to calculate the measurement of range $(\overline{P_1 P})$ between P_1 and P will be as follows:

$$\begin{aligned}\overline{P_1 P} &= |b| \cos(\theta) = \frac{|a||b| \cos(\theta)}{|a|} = \frac{a \cdot b}{|a|} \\ &= \frac{(x_2 - x_1)(x_3 - x_1) + (y_2 - y_1)(y_3 - y_1) + (z_2 - z_1)(z_3 - z_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}\end{aligned}$$